6. Factorization of Polynomials

Exercise 6.1

1. Question

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer:

(i) $3x^2-4x+15$

(ii) $y^2+2\sqrt{3}$

(iii) $3\sqrt{x}+\sqrt{2}\,x$

(iv) $x-\frac{4}{x}$

(v) $x^{12}+y^3+t^{50}$

Answer

(i) $3x^2-4x+15$ is a polynomial of one variable x.

(ii) $y^2+2\sqrt{3}$ is a polynomial of one variable y.

(iii) $3\sqrt{x}+\sqrt{2}\,x$ is not a polynomial as the exponent of $3\sqrt{x}$ is not a positive integer.

(iv) x $-\frac{4}{x}$ is not a polynomial as the exponent of $-\frac{4}{x}$ is not a positive integer.

(v) $x^{12}+y^3+t^{50}$ is a polynomial of three variables x, y, t.

2. Question

Write the coefficient of x^2 in each of the following:

(i) $17 - 2x + 7x^2$

(ii) $9-12x+x^3$

(iii) $\frac{\pi}{6}x^2 - 3x + 4$

(iv) $\sqrt{3}\,x-7$

Answer

Coefficient of x^2 in:

(i) $17 - 2x + 7x^2$ is 7

(ii) $9-12x+x^3$ is 0

(iii) $\frac{\pi}{6}x^2 - 3x + 4$ is $\frac{\pi}{6}$

(iv) $\sqrt{3}\,x-7$ is 0

3. Question

Write the degrees of each of the following polynomials:

(i) $7x^3+4x^2-3x+12$

(ii) $12 -x+2x^3$

(iii) $5y-\sqrt{2}$

(iv) 7

(v) 0

Answer

Degree of polynomial in:

(i) $7x^3+4x^2-3x+12$ is 3

(ii) $12-x+2x^3$ is 3

(iii) $5y-\sqrt{2}$ is 1

(iv) 7 is 0

(v) 0 is undefined

4. Question

Classify the following polynomials as linear, quadratic, cubic and biquadratic polynomials:

(i) $x+x^2+7y^2$ (ii) $3x-2$

(iii) $2x+x^2$

(iv) 3y (v) t^2+1

(vi) $7t^4+4t^3+3t-2$

Answer

Given polynomial,

(i) $x+x^2+7y^2$ is quadratic as degree of polynomial is 2.

(ii) $3x-2$ is linear as degree of polynomial is 1.

(iii) $2x+x^2$ is quadratic as degree of polynomial is 2.

(iv) 3y is linear as degree of polynomial is 1.

(v) t^2+1 is quadratic as degree of polynomial is 2.

(vi) $7t^4+4t^3+3t-2$ is bi-quadratic as degree of polynomial is 4.

5. Question

Classify the following polynomials as polynomials in one-variable, two variable etc:

(i) $x^2-xy+7y^2$ (ii) $x^2-2tx+7y^2-x+t$

(iii) t^3-3t^2+4t-5 (iv) $xy+yz+zx$

Answer

(i) $x^2-xy+7y^2$ is a polynomial in two variable x, y.

(ii) $x^2-2tx+7y^2-x+t$ is a polynomial in two variable x, t.

(iii) t^3-3t^2+4t-5 is a polynomial in one variable t.

(iv) $xy+yz+zx$ is a polynomial in three variable x, y, t.

6. Question

Identify polynomials in the following:

(i) $f(x)=4x^3-x^2-3x+7$

(ii) $g(x)=2x^3-3x^2+\sqrt{x}-1$

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7}{4}x + 9$

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$

(v) $h(x) = x^4 - x^{\frac{3}{2}} + x - 1$

(vi) $f(x) = 2 + \frac{3}{x} + 4x$

Answer

(i) $f(x) = 4x^3 - x^2 - 3x + 7$ is a polynomial.

(ii) $g(x) = 2x^3 - 3x^2 + \sqrt{x} - 1$ is not a polynomial as exponent of x in $\sqrt{x}$ is not a positive integer.

(iii) $p(x) = \frac{2}{3}x^2 - \frac{7}{4}x + 9$ is a polynomial as all the exponents are positive integer.

(iv) $q(x) = 2x^2 - 3x + \frac{4}{x} + 2$ is not a polynomial as the exponent of x in $\frac{4}{x}$ is not a positive integer.

(v) $h(x) = x^4 - x^{\frac{3}{2}} + x - 1$ is not a polynomial as the exponent of x in $-x^{3/2}$ is not a positive integer.

(vi) $f(x) = 2 + \frac{3}{x} + 4x$ is not a polynomial as the exponent of x in $\frac{3}{x}$ is not a positive integer.

7. Question

Identify constant, linear, quadratic and cubic polynomials from the following polynomials:

(i) $f(x) = 0$ (ii) $g(x) = 2x^3 - 7x + 4$

(iii) $h(x) = -3x + \frac{1}{2}$

(iv) $p(x) = 2x^2 - x + 4$

(v) $q(x) = 4x + 3$ (vi) $r(x) = 3x^3 + 4x^2 + 5x - 7$

Answer

Given polynomial,

(i) $f(x) = 0$ is a constant polynomial as 0 is constant.

(ii) $g(x) = 2x^3 - 7x + 4$ is a cubic polynomial as degree of the polynomial is 3.

(iii) $h(x) = -3x + \frac{1}{2}$ is a linear polynomial as the degree of polynomial is 1.

(iv) $p(x) = 2x^2 - x + 4$ is a quadratic polynomial as the degree of polynomial is 2.

(v) $q(x) = 4x + 3$ is a linear polynomial as the degree of polynomial is 1.

(vi) $r(x) = 3x^3 + 4x^2 + 5x - 7$ is a cubic polynomial as the degree of polynomial is 3.

8. Question

Give one example each of a binomial of degree 35, and of a monomial of degree 100

Answer

Example of a binomial with degree 35 is $7x^{35} – 5$.

Example of a monomial with degree 100 is $2t^{100}$.

Exercise 6.2

1. Question

If f(x) = $2x^3-13x^2+17x+12$, find

(i) $f(2)$ (ii) $f(-3)$ (iii) $f(0)$

Answer

We have,

$f(x) = 2x^3-13x^2+17x+12$

(i) f(2) = 2 $(2)^3$ – 13 $(2)^2$ + 17 (2) + 12

= (2 * 8) – (13 * 4) + (17 * 2) + 12

= 16 – 52 + 34 + 12

= 10

(ii) f (-3) = 2 $(-3)^3$ – 13 $(-3)^2$ + 17 (-3) + 12

= (2 * -27) – (13 * 9) + (17 * -3) + 12

= -54 – 117 – 51 + 12

= - 210

(iii) f (0) = 2 $(0)^3$ – 13 $(0)^2$ + 17 (0) + 12

= 0 – 0 + 0 + 12

= 12

2. Question

Verify whether the indicated numbers are zeros of the polynomials corresponding to them in the following cases:

(i) $f(x) = 3x+1$; $x = -\frac{1}{3}$

(ii) $f(x) = x^2-1$; $x = 1, -1$

(iii) $g(x) = 3x^2-2$; $x = \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$

(iv) $p(x) = x^3-6x^2+11x-6$, $x = 1,2,3$

(v) $f(x) = 5x-\pi$, $x = \frac{4}{5}$

(vi) $f(x) = x^2$, $x=0$

(vii) $f(x) = 1x+m$, $x = -\frac{m}{1}$

(viii) $f(x) = 2x+1$, $x = \frac{1}{2}$

Answer

(i) f(x) = 3x + 1

Put x = -1/3

f (-1/3) = 3 * (-1/3) + 1

= -1 + 1

= 0

Therefore, x = -1/3 is a root of f (x) = 3x + 1

(ii) We have,

$f(x) = x^2 - 1$

Put x = 1 and x = -1

$f(1) = (1)^2 - 1$ and $f(-1) = (-1)^2 - 1$

= 1 – 1 = 1- 1

= 0 = 0

Therefore, x = -1 and x = 1 are the roots of $f(x) = x^2 - 1$

(iii) $g(x) = 3x^2 - 2$

Put $x = \frac{2}{\sqrt{3}}$ and $x = \frac{-2}{\sqrt{3}}$

$g(\frac{2}{\sqrt{3}}) = 3(\frac{2}{\sqrt{3}})^2 - 2$ and $g(\frac{-2}{\sqrt{3}}) = 3(\frac{-2}{\sqrt{3}})^2 - 2$

$= 3 * \frac{4}{3} - 2 = 3 * \frac{4}{3} - 2$

$= 2 \neq 0 = 2 \neq 0$

Therefore, $x = \frac{2}{\sqrt{3}}$ and $x = \frac{-2}{\sqrt{3}}$ are not the roots of $g(x) = 3x^2 - 2$

(iv) $p(x) = x^3 - 6x^2 + 11x - 6$

Put x = 1

$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

= 1 – 6 + 11 – 6

= 0

Put x = 2

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6$

= 8 – 24 + 22 – 6

= 0

Put x = 3

$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6$

= 27 – 54 + 33 – 6

= 0

Therefore, x = 1, 2, 3 are roots of $p(x) = x^3 - 6x^2 + 11x - 6$

(v) $f(x) = 5x - \pi$

Put $x = \frac{4}{5}$

$f(\frac{4}{5}) = 5 * \frac{4}{5} - \pi$

$= 4 - \pi \neq 0$

Therefore, $x = \frac{4}{5}$ is not a root of $f(x) = 5x - \pi$

(vi) $f(x) = x^2$

Put x = 0

f (0) = $(0)^2$

= 0

Therefore, x = 0 is not a root of f (x) = x^2

(vii) f (x) = lx + m

Put x = $\frac{-m}{l}$

f ($\frac{-m}{l}$) = l * ($\frac{-m}{l}$) + m

= -m + m

= 0

Therefore, x = $\frac{-m}{l}$ is a root of f (x) = lx + m

(viii) f (x) = 2x + 1

Put x = $\frac{1}{2}$

f ($\frac{1}{2}$) = 2 * $\frac{1}{2}$ + 1

= 1 + 1

= 2 $\neq$ 0

Therefore, x = $\frac{1}{2}$ is not a root of f (x) = 2x + 1

3. Question

If x = 2 is a root of the polynomial $f(x)$= $2x^2$-3x+7a, find the value of a.

Answer

We have,

f (x) = $2x^2$ – 3x + 7a

Put x = 2

f (2) = 2 $(2)^2$ – 3 (2) + 7a

= 2 * 4 – 6 + 7a

= 8 – 6 + 7a

= 2 + 7a

Given, x = 2 is a root of f (x) = $2x^2$ – 3x + 7a

f (2) = 0

Therefore, 2 + 7a = 0

7a = -2

a = $\frac{-2}{7}$

4. Question

If x = -1/2 is a zero of the polynomial $p(x)$ = $8x^3$ - ax^2 - x + 2, find the value of a.

Answer

We have,

$p\,(x) = 8x^3 - ax^2 - x + 2$

Put x $= -\frac{1}{2}$

p $(-\frac{1}{2}) = 8\,(-\frac{1}{2})^3 - a\,(-\frac{1}{2})^2 - (-\frac{1}{2}) + 2$

$= 8 \times \frac{-1}{8} - a \times \frac{1}{4} + \frac{1}{2} + 2$

$= -1 - \frac{a}{4} + \frac{1}{2} + 2$

$= \frac{3}{2} - \frac{a}{4}$

Given that,

x $= -\frac{1}{2}$ is a root of p (x)

p $(-\frac{1}{2}) = 0$

Therefore,

$\frac{3}{2} - \frac{a}{4} = 0$

$\frac{3}{2} = \frac{a}{4}$

2a = 12

a = 6

5. Question

If $x = 0$ and $x = -1$ are the roots of the polynomial $f\,(x) = 2x^3-3x^2+ax+b$, find the value of a and b.

Answer

we have,

f (x) $= 2x^3-3x^2+ax+b$

Put,

x = 0

f (0) $= 2\,(0)^3 - 3\,(0)^2 + a\,(0) + b$

= 0 – 0 + 0 + b

= b

x = -1

f (-1) $= 2\,(-1)^3 - 3\,(-1)^2 + a\,(-1) + b$

= -2 – 3 – a + b

= -5 – a + b

Since, x = 0 and x = -1 are roots of f (x)

f (0) = 0 and f (-1) = 0

b = 0 and -5 – a + b = 0

= a – b = -5

$= a – 0 = -5$

$= a = -5$

Therefore, $a = -5$ and $b = 0$

6. Question

Find the integral roots of the polynomial $f(x) = x^3+6x^2+11x+6$.

Answer

We have,

$f(x) = x^3+6x^2+11x+6$

Clearly, f (x) is a polynomial with integer coefficient and the coefficient of the highest degree term i.e., the leading coefficient is 1.

Therefore, integer root of f (x) are limited to the integer factors of 6, which are:

$\pm1, \pm2, \pm3, \pm6$

We observe that

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 -11 + 6$

$= 0$

$f(-2) = (-2)^3 + 6(-2)^2 + 11(-2) + 6$

$= -8 + 24 – 22 + 6$

$= 0$

$f(-3) = (-3)^3 + 6(-3)^2 + 11(-3) + 6$

$= -27 + 54 – 33 + 6$

$= 0$

Therefore, integral roots of f (x) are -1, -2, -3.

7. Question

Find rational roots of the polynomial $f(x) = 2x^3+x^2-7x-6$.

Answer

We have,

$f(x) = 2x^3+x^2-7x-6$

Clearly, f (x) is a cubic polynomial with integer coefficients. If $\frac{b}{c}$ is a rational root in lowest term, then the value of b are limited to the factors of 6 which are $\pm1, \pm2, \pm3, \pm6$ and values of c are limited to the factors of 2 which are $\pm1, \pm2$.

Hence, the possible rational roots of f(x) are:

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}$

We observe that,

$f(-1) = 2(-1)^3 + (-1)^2 – 7(-1) – 6$

$= -2 + 1 + 7 – 6$

$= 0$

f (2) = 2 $(2)^3$ + $(2)^2$ – 7 (2) – 6

= 16 + 4 – 14 – 6

= 0

f ($\frac{-3}{2}$) = 2 $(\frac{-3}{2})^3$ + $(\frac{-3}{2})^2$ – 7 ($\frac{-3}{2}$) – 6

= $\frac{-27}{4} + \frac{9}{4} + \frac{21}{2}$ – 6

= 0

Hence, -1, 2, $\frac{-3}{2}$ are the rational roots of f (x).

Exercise 6.3

1. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = x^3+4x^2-3x+10$, $g(x) = x+4$

Answer

We have,

$f(x) = x^3+4x^2-3x+10$ and g (x) = x + 4

Therefore, by remainder theorem when f (x) is divided by g (x) = x – (-4), the remainder is equal to f (-4)

Now, $f(x) = x^3+4x^2-3x+10$

f (-4) = $(-4)^3$ + 4 $(-4)^2$ – 3 (-4) + 10

= -64 + 4 * 16 + 12 + 10

= 22

Hence, required remainder is 22.

2. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = 4x^4-3x^3-2x^2+x-7$, $g(x) = x-1$

Answer

We have,

$f(x) = 4x^4-3x^3-2x^2+x-7$ and $g(x) = x-1$

Therefore, by remainder theorem when f (x) is divided by g (x) = x – 1, the remainder is equal to f (+1)

Now, $f(x) = 4x^4-3x^3-2x^2+x-7$

f (1) = 4 $(1)^4$ – 3 $(1)^3$ – 2 $(1)^2$ + 1 – 7

= 4 – 3 – 2 + 1 – 7

= -7

Hence, required remainder is -7.

3. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = 2x^4-6x^3+2x^2-x+2$, $g(x) = x+2$

Answer

We have,

$f(x) = 2x^4\text{-}6x^3\text{+}2x^2\text{-}x\text{+}2$ and $g(x) = x\text{+}2$

Therefore, by remainder theorem when f (x) is divided by g (x) = x – (-2), the remainder is equal to f (-2)

Now, $f(x) = 2x^4\text{-}6x^3\text{+}2x^2\text{-}x\text{+}2$

f (-2) = 2 $(-2)^4$ – 6 $(-2)^3$ + 2 $(-2)^2$ – (-2) + 2

= 2 * 16 + 48 + 8 + 2 + 2

= 32 + 48 + 12

= 92

Hence, required remainder is 92.

4. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = 4x^3\text{-}12x^2\text{+}14x\text{-}3$, $g(x) = 2x\text{-}1$

Answer

We have,

$f(x) = 4x^3\text{-}12x^2\text{+}14x\text{-}3$ and $g(x) = 2x\text{-}1$

Therefore, by remainder theorem when f (x) is divided by g (x) = 2 (x - $\frac{1}{2}$), the remainder is equal to f ($\frac{1}{2}$)

Now, $f(x) = 4x^3\text{-}12x^2\text{+}14x\text{-}3$

f ($\frac{1}{2}$) = 4 $(\frac{1}{2})^3$ – 12 $(\frac{1}{2})^2$ + 14 ($\frac{1}{2}$) – 3

= (4 * $\frac{1}{8}$) – (12 * $\frac{1}{4}$) + 7 – 3

= $\frac{1}{2}$ – 3 + 7 – 3

= $\frac{3}{2}$

Hence, required remainder is $\frac{3}{2}$

5. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = x^3\text{-}6x^2\text{+}2x\text{-}4$, $g(x) = 1\text{-}2x$

Answer

We have,

$f(x) = x^3\text{-}6x^2\text{+}2x\text{-}4$ and $g(x) = 1\text{-}2x$

Therefore, by remainder theorem when f (x) is divided by g (x) = -2 (x - $\frac{1}{2}$), the remainder is equal to f ($\frac{1}{2}$)

Now, $f(x) = x^3\text{-}6x^2\text{+}2x\text{-}4$

f ($\frac{1}{2}$) = $(\frac{1}{2})^3$ – 6 $(\frac{1}{2})^2$ + 2 ($\frac{1}{2}$) – 4

= $\frac{1}{8}$ - $\frac{3}{2}$ + 1 – 4

$= \frac{-35}{8}$

Hence, required remainder is $\frac{-35}{8}$

6. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = x^4 - 3x^2 + 4$, $g(x) = x - 2$

Answer

We have,

$f(x) = x^4 - 3x^2 + 4$ and $g(x) = x - 2$

Therefore, by remainder theorem when f (x) is divided by g (x) = x – 2, the remainder is equal to f (2)

Now, $f(x) = x^4 - 3x^2 + 4$

$f(2) = (2)^4 - 3(2)^2 + 4$

$= 16 - 12 + 4$

$= 8$

Hence, required remainder is 8.

7. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = 9x^3 - 3x^2 + x - 5$, $g(x) = x = -\frac{2}{3}$

Answer

We have,

$f(x) = 9x^3 - 3x^2 + x - 5$ and $g(x) = x = -\frac{2}{3}$

Therefore, by remainder theorem when f (x) is divided by g (x) = x - $\frac{2}{3}$, the remainder is equal to f ($\frac{2}{3}$)

Now, $f(x) = 9x^3 - 3x^2 + x - 5$

$f(\frac{2}{3}) = 9(\frac{2}{3})^3 - 3(\frac{2}{3})^2 + \frac{2}{3} - 5$

$= (9 * \frac{8}{27}) - (3 * \frac{4}{9}) + \frac{2}{3} - 5$

$= \frac{8}{3} - \frac{4}{3} + \frac{2}{3} - 5$

$= 2 - 5 = -3$

Hence, the required remainder is -3.

8. Question

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$:

$f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$, $g(x) = x + \frac{2}{3}$

Answer

We have,

$f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ and $g(x) = x + \frac{2}{3}$

Therefore, by remainder theorem when f (x) is divided by g (x) = x – (- $\frac{2}{3}$), the remainder is equal to f ($\frac{-2}{3}$)

Now, $f(x) = 3x^4+2x^3-\frac{x^2}{3}-\frac{x}{9}+\frac{2}{27}$

f ($\frac{-2}{3}$) = 3 $(\frac{-2}{3})^4$ + 2 $(\frac{-2}{3})^3$ – $\left(\frac{\frac{-2}{3}*\frac{-2}{3}}{3}\right)$ - $\frac{-\frac{2}{3}}{9}$ + $\frac{2}{27}$

= 3 * $\frac{16}{81}$ + 2 * $\frac{-8}{27}$ - $\frac{4}{9*3}$ - $\frac{-2}{3*9}$ + $\frac{2}{27}$

= $\frac{16}{27}$ - $\frac{16}{27}$ - $\frac{4}{27}$ + $\frac{2}{27}$ + $\frac{2}{27}$

= $\frac{16-16-4+2+2}{27}$ = $\frac{0}{27}$

= 0

Hence, required remainder is 0.

9. Question

If the polynomials $2x^3+ax^2+3x-5$ and x^3+x^2-4x+a leave the same remainder when divided by $x-2$, find the value of a.

Answer

Let, p (x) = $2x^3+ax^2+3x-5$ and q (x) = x^3+x^2-4x+a be the given polynomials.

The remainders when p (x) and q (x) are divided by (x – 2) and p (2) and q (2) respectively.

By the given condition, we have:

p (2) = q (2)

2 $(2)^3$ + a $(2)^2$ + 3 (2) – 5 = $(2)^3$ + $(2)^2$ – 4 (2) + a

16 + 4a + 6 – 5 = 8 + 4 – 8 + a

3a + 13 = 0

3a = - 13

a = $\frac{-13}{3}$

10. Question

If the polynomials ax^3+3x^2-3x and $2x^3-5x+a$ when divided by (x-4) leave the remainder R_1 and R_2 respectively. Find the value of a in each of the following cases, if

(i) $R_1 = R_2$ (ii) $R_1 + R_2=0$

(iii) $2R_1-R_2 = 0$.

Answer

Let, p (x) = ax^3+3x^2-3 and q (x) = $2x^3-5x+a$ be the given polynomials.

Now,

R_1 = Remainder when p (x) is divided by (x – 4)

= p (4)

= a $(4)^3$ + 3 $(4)^2$ – 3 [Therefore, p (x) = ax^3+3x^2-3]

= 64a + 48 – 3

R_1 = 64a + 45

And,

R_2 = Remainder when q (x) is divided by (x – 4)

= q (4)

= 2 $(4)^3$ – 5 (4) + a [Therefore, q (x) = $2x^3-5x+a$]

= 128 – 20 + a

R_2 = 108 + a

(i) Given condition is,

$R_1 = R_2$

64a + 45 = 108 + a

63a – 63 = 0

63a = 63

a = 1

(ii) Given condition is $R_1 + R_2 = 0$

64a + 45 + 108 + a = 0

65a + 153 = 0

65a = -153

a = $\frac{-153}{65}$

(iii) Given condition is $2R_1 - R_2 = 0$

2 (64a + 45) – (108 + a) = 0

128a + 90 – 108 – a

127a – 18 = 0

127a = 18

a = $\frac{18}{127}$

11. Question

If the polynomials ax^3+3x^2-13 and $2x^3-5x+a$ when divided by (x-2) leave the same remainder, find the value of a.

Answer

Let p (x) = ax^3+3x^2-13 and q (x) = $2x^3-5x+a$ be the given polynomials.

The remainders when p (x) and q (x) are divided by (x – 2) and p (2) and q (2) respectively.

By the given condition, we have:

p (2) = q (2)

a $(2)^3$ + 3 $(2)^2$ – 13 = 2 $(2)^3$ – 5 (2) + a

8a + 12 – 13 = 16 – 10 + a

7a – 7 = 0

7a = 7

a = $\frac{7}{7}$

= 1

12. Question

Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$ (ii) $x-\frac{1}{2}$

(iii) x (iv) $x+\pi$

(v) $5+2x$

Answer

Let, f(x) = x^3+3x^2+3x+1

(i) x + 1

Apply remainder theorem

⇒ x + 1 =0

⇒ x = - 1

Replace x by – 1 we get

⇒ $x^3+3x^2 + 3x + 1$

⇒ $(-1)^3 + 3(-1)^2 + 3(-1) + 1$

⇒ -1 + 3 - 3 + 1

⇒ 0

Hence, the required remainder is 0.

(ii) $x-\frac{1}{2}$

Apply remainder theorem

⇒ x – 1/2 =0

⇒ x = 1/2

Replace x by 1/2 we get

⇒ $x^3+3x^2 + 3x + 1$

⇒ $(1/2)^3 + 3(1/2)^2 + 3(1/2) + 1$

⇒ 1/8 + 3/4 + 3/2 + 1

Add the fraction taking LCM of denominator we get

⇒ (1 + 6 + 12 + 8)/8

⇒ 27/8

Hence, the required remainder is 27/8

(iii) x = x – 0

By remainder theorem required remainder is equal to f (0)

Now, f (x) = x^3+3x^2+3x+1

f (0) = $(0)^3 + 3\ (0)^2 + 3\ (0) + 1$

= 0 + 0 + 0 + 1

= 1

Hence, the required remainder is 1.

(iv) $x+\pi = x - (-\pi)$

By remainder theorem required remainder is equal to f (-π)

Now, f (x) = $x^3 + 3x^2 + 3x + 1$

$f(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

Hence, required remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1$.

(v) $5 + 2x = 2\,[x - (\frac{-5}{2})]$

By remainder theorem required remainder is equal to f $(\frac{-5}{2})$

Now, f (x) = $x^3 + 3x^2 + 3x + 1$

$f(\frac{-5}{2}) = (\frac{-5}{2})^3 + 3(\frac{-5}{2})^2 + 3(\frac{-5}{2}) + 1$

$= \frac{-125}{8} + 3 * \frac{25}{4} + 3 * \frac{-5}{2} + 1$

$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

$= \frac{-27}{8}$

Hence, the required remainder is $\frac{-27}{8}$.

Exercise 6.4

1. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = x^3-6x^2+11x-6$, $g(x) = x-3$

Answer

We have,

$f(x) = x^3-6x^2+11x-6$ and $g(x) = x-3$

In order to find whether polynomials g (x) = x – 3 is a factor of f (x), it is sufficient to show that f (3) = 0

Now,

$f(x) = x^3-6x^2+11x-6$

$f(3) = 3^3 - 6(3)^2 + 11(3) - 6$

$= 27 - 54 + 33 - 6$

$= 60 - 60$

$= 0$

Hence, g (x) is a factor of f (x).

2. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = 3x^4+17x^3+9x^2-7x-10$, $g(x) = x+5$

Answer

We have,

$f(x) = 3x^4+17x^3+9x^2-7x-10$ and $g(x) = x+5$

In order to find whether the polynomials g (x) = x – (-5) is a factor of f (x) or not, it is sufficient to show that f (-5) = 0

Now,

$f(x) = 3x^4+17x^3+9x^2-7x-10$

$f(-5) = 3(-5)^4 + 17(-5)^3 + 9(-5)^2 - 7(-5) - 10$

$= 3 * 625 + 17 * (-125) + 9 * 25 + 35 - 10$

$= 1875 - 2125 + 225 + 35 - 10$

$= 0$

Hence, g (x) is a factor of f (x).

3. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = x^5+3x^4-x^3-3x^2+5x+15$, $g(x) = x+3$

Answer

We have,

$f(x) = x^5+3x^4-x^3-3x^2+5x+15$ and $g(x) = x+3$

In order to find whether g (x) = x – (-3) is a factor of f (x) or not, it is sufficient to prove that f (-3) = 0

Now,

$f(x) = x^5+3x^4-x^3-3x^2+5x+15$

$f(-3) = (-3)^5 + 3(-3)^4 - (-3)^3 - 3(-3)^2 + 5(-3) + 15$

$= -243 + 243 - (-27) - 3(9) + 5(-3) + 15$

$= -243 + 243 + 27 - 27 - 15 + 15$

$= 0$

Hence, g (x) is a factor of f (x).

4. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = x^3-6x^2-19x+84$, $g(x) = x-7$

Answer

We have,

$f(x) = x^3-6x^2-19x+84$ and $g(x) = x-7$

In order to find whether g (x) = x – 7 is a factor of f (x) or not, it is sufficient to show that f (7) = 0

Now,

$f(x) = x^3-6x^2-19x+84$

$f(7) = (7)^3 - 6(7)^2 - 19(7) + 84$

$= 343 - 294 - 133 + 84$

= 0

Hence, g (x) is a factor of f (x).

5. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = 3x^3+x^2-20x+12$, $g(x) = 3x-2$

Answer

We have,

$f(x) = 3x^3+x^2-20x+12$ and $g(x) = 3x-2$

In order to find whether g (x) is = 3x – 2 is a factor of f (x) or not, it is sufficient to show that f $(\frac{2}{3})$ = 0

Now,

$f(x) = 3x^3+x^2-20x+12$

f $(\frac{2}{3})$ = 3 $(\frac{2}{3})^3$ + $(\frac{2}{3})^2$ – 20 $(\frac{2}{3})$ + 12

= $\frac{12}{9}$ - $\frac{40}{3}$ + 12

= $\frac{120-120}{9}$

= 0

Hence, g (x) is a factor of f (x).

6. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = 2x^3-9x^2+x+12$, $g(x) = 3-2x$

Answer

We have,

$f(x) = 2x^3-9x^2+x+12$ and $g(x) = 3-2x$

In order to find g (x) = 3 – 2x = 2 (x - $\frac{3}{2}$) is a factor of f (x) or not, it is sufficient to prove that f $(\frac{3}{2})$ = 0

Now,

$f(x) = 2x^3-9x^2+x+12$

f $(\frac{3}{2})$ = 2 $(\frac{3}{2})^3$ – 9 $(\frac{3}{2})^2$ + $\frac{3}{2}$ + 12

= $\frac{27}{4}$ - $\frac{81}{4}$ + $\frac{3}{2}$ + 12

= $\frac{81-81}{4}$

= 0

Hence, g (x) is a factor of f (x).

7. Question

In each of the following, use factor theorem to find whether polynomial $g(x)$ is a factor of polynomial $f(x)$ or, not:

$f(x) = x^3-6x^2+11x-6$, $g(x) = x^2-3x+2$

Answer

We have,

$f(x) = x^3-6x^2+11x-6$ and $g(x) = x^2-3x+2$

In order to find g (x) = x^2-3x+2 = (x – 1) (x – 2) is a factor of f (x) or not, it is sufficient to prove that (x – 1) and (x – 2) are factors of f (x)

i.e. We have to prove that f (1) = 0 and f (2) = 0

f (1) = $(1)^3$ – 6 $(1)^2$ + 11 (1) – 6

= 1 – 6 + 11 – 6

= 12 – 12

= 0

f (2) = $(2)^3$ – 6 $(2)^2$ + 11 (2) – 6

= 8 – 24 + 22 – 6

= 30 – 30

= 0

Since, (x – 1) and (x – 2) are factors of f (x).

Therefore, g (x) = (x – 1) (x – 2) are the factors of f (x).

8. Question

Show that (x-2), (x+3) and (x-4) are factors of $x^3-3x^2-10x+24$.

Answer

Let, f (x) = $x^3-3x^2-10x+24$ be the given polynomial.

In order to prove that (x – 2) (x + 3) (x – 4) are the factors of f (x), it is sufficient to show that f (2) = 0, f (-3) = 0 and f (4) = 0 respectively.

Now,

f (x) = $x^3-3x^2-10x+24$

f (2) = $(2)^3$ – 3 $(2)^2$ – 10 (2) + 24

= 8 – 12 – 20 + 24

= 0

f (-3) = $(-3)^3$ – 3 $(-3)^2$ – 10 (-3) + 24

= -27 – 27 + 30 + 24

= 0

f (4) = $(4)^3$ – 3 $(4)^2$ – 10 (4) + 24

= 64 – 48 – 40 + 24

= 0

Hence, (x – 2), (x + 3) and (x – 4) are the factors of the given polynomial.

9. Question

Show that (x+4),(x-3) and (x-7) are factors of $x^3-6x^2-19x+84$.

Answer

Let f (x) = x^3-6x^2-19x+84 be the given polynomial.

In order to prove that (x + 4), (x – 3) and (x – 7) are factors of f (x), it is sufficient to prove that f (-4) = 0, f (3) = 0 and f (7) = 0 respectively.

Now,

f (x) = x^3-6x^2-19x+84

f (-4) = $(-4)^3$ – 6 $(-4)^2$ – 19 (-4) + 84

= -64 – 96 + 76 + 84

= 0

f (3) = $(3)^3$ – 6 $(3)^2$ – 19 (3) + 84

= 27 – 54 – 57 + 84

= 0

f (7) = $(7)^3$ – 6 $(7)^2$ – 19 (7) + 84

= 343 – 294 – 133 + 84

= 0

Hence, (x – 4), (x – 3) and (x -7) are the factors of the given polynomial x^3-6x^2-19x+84.

10. Question

For what value of *a* is (x-5) a factor of x^3-3x^2+ax-10.

Answer

Let, f (x) = x^3-3x^2+ax-10 be the given polynomial.

By factor theorem,

If (x – 5) is a factor of f (x) then f (5) = 0

Now,

f (x) = x^3-3x^2+ax-10

f (5) = $(5)^3$ – 3 $(5)^2$ + a (5) – 10

0 = 125 – 75 + 5a – 10

0 = 5a + 40

a = -8

Hence, (x – 5) is a factor of f (x), if a = - 8.

11. Question

Find the value of a such that (x-4) is a factor of 5x^3-7x^2-ax-28.

Answer

Let f(x) = 5x^3-7x^2-ax-28 be the given polynomial.

From factor theorem,

If (x -4) is a factor of f (x) then f (4) = 0

f (4) = 0

0 = 5 $(4)^3$ – 7 $(4)^2$ – a (4) – 28

0 = 320 – 112 – 4a – 28

$0 = 180 – 4a$

$4a = 180$

$a = 45$

Hence, $(x – 4)$ is a factor of $f(x)$ when $a = 45$.

12. Question

Find the value of a, if $x+2$ is a factor of $4x^4+2x^3-3x^2+8x+5a$.

Answer

Let, $f(x) = 4x^4+2x^3-3x^2+8x+5a$

$f(-2) = 0$

$4(-2)^4 + 2(-2)^3 – 3(-2)^2 + 8(-2) + 5a = 0$

$64 – 16 – 12 – 16 + 5a = 0$

$5a = -20$

$a = -4$

Hence, $(x + 2)$ is a factor $f(x)$ when $a = -4$.

13. Question

Find the value of k if $x-3$ is a factor of $k^2x^3 - kx^2 + 3kx - k$.

Answer

Let, $f(x) = k^2x^3 - kx^2 + 3kx - k$

By factor theorem,

If $(x – 3)$ is a factor of $f(x)$ then $f(3) = 0$

$k^2(3)^3 – k(3)^2 + 3k(3) – k = 0$

$27k^2 – 9k + 9k – k = 0$

$k(27k – 1) = 0$

$k = 0$ or $(27k – 1) = 0$

$k = 0$ or $k = \frac{1}{27}$

Hence, $(x – 3)$ is a factor of $f(x)$ when $k = 0$ or $k = \frac{1}{27}$.

14. Question

Find the value is of a and b, if x^2-4 is a factor of $ax^4+2x^3-3x^2+bx-4$.

Answer

Let, $f(x) = ax^4+2x^3-3x^2+bx-4$ and $g(x) = x^2 – 4$

We have,

$g(x) = x^2 – 4$

$= (x – 2)(x + 2)$

Given,

$g(x)$ is a factor of $f(x)$

$(x – 2)$ and $(x + 2)$ are factors of $f(x)$.

From factor theorem if (x – 2) and (x + 2) are factors of f (x) then f (2) = 0 and f (-2) = 0 respectively.

f (2) = 0

a * $(-2)^4 + 2\ (2)^3 - 3\ (2)^2$ + b (2) – 4 = 0

16a – 16 – 12 + 2b – 4 = 0

16a + 2b = 0

2 (8a + b) = 0

8a + b = 0 (i)

Similarly,

f (-2) = 0

a * $(-2)^4 + 2\ (-2)^3 - 3\ (-2)^2$ + b (-2) – 4 = 0

16a – 16 – 12 - 2b – 4 = 0

16a - 2b – 32 = 0

16a – 2b – 32 = 0

2 (8a - b) = 32

8a – b = 16 (ii)

Adding (i) and (ii), we get

8a + b + 8a – b = 16

16a = 16

a = 1

Put a = 1 in (i), we get

8 * 1 + b = 0

b = -8

Hence, a = 1 and b = -8.

15. Question

Find α and β if x+1 and x+2 are factors of $x^3+3x^2-2\alpha x+\beta$.

Answer

Let, f (x) = $x^3+3x^2-2\alpha x+\beta$ be the given polynomial,

From factor theorem,

If (x + 1) and (x + 2) are factors of f (x) then f (-1) = 0 and f (-2) = 0

f (-1) = 0

$(-1)^3 + 3\ (-1)^2 - 2\ \alpha\ (-1) + \beta = 0$

-1 + 3 + 2 α + β = 0

2 α + β + 2 = 0 (i)

Similarly,

f (-2) = 0

$(-2)^3 + 3\ (-2)^2 - 2\ \alpha\ (-2) + \beta = 0$

-8 + 12 + 4 α + β = 0

$4\alpha + \beta + 4 = 0$ (ii)

Subtract (i) from (ii), we get

$4\alpha + \beta + 4 - (2\alpha + \beta + 2) = 0 - 0$

$4\alpha + \beta + 4 - 2\alpha - \beta - 2 = 0$

$2\alpha + 2 = 0$

$\alpha = -1$

Put $\alpha = -1$ in (i), we get

$2(-1) + \beta + 2 = 0$

$\beta = 0$

Hence, $\alpha = -1$ and $\beta = 0$.

16. Question

Find the value of p and q so that $x^4+px^3+2x^2-3x+q$ is divisible by (x^2-1).

Answer

Let, f (x) = $x^4+px^3+2x^2-3x+q$ be the given polynomial.

And, let g (x) = $(x^2 - 1) = (x - 1)(x + 1)$

Clearly,

(x – 1) and (x + 1) are factors of g (x)

Given, g (x) is a factor of f (x)

(x – 1) and (x + 1) are factors of f (x)

From factor theorem

If (x – 1) and (x + 1) are factors of f (x) then f (1) = 0 and f (-1) = 0 respectively.

f (1) = 0

$(1)^4 + p(1)^3 + 2(1)^2 - 3(1) + q = 0$

$1 + p + 2 - 3 + q = 0$

$p + q = 0$ (i)

Similarly,

f (-1) = 0

$(-1)^4 + p(-1)^3 + 2(-1)^2 - 3(-1) + q = 0$

$1 - p + 2 + 3 + q = 0$

$q - p + 6 = 0$ (ii)

Adding (i) and (ii), we get

$p + q + q - p + 6 = 0$

$2q + 6 = 0$

$2q = -6$

$q = -3$

Putting value of q in (i), we get

$p - 3 = 0$

p = 3

Hence, x^2 – 1 is divisible by f (x) when p = 3 and q = - 3.

17. Question

Find the value is of a and b, so that $(x+1)$ and $(x-1)$ are factors of $x^4+ax^3-3x^2+2x+b$.

Answer

Let, f (x) = $x^4+ax^3-3x^2+2x+b$ be the given polynomial

From factor theorem

If (x + 1) and (x – 1) are factors of f (x) then f (-1) = 0 and f (1) = 0 respectively.

f (-1) = 0

$(-1)^4 + a(-1)^3 - 3(-1)^2 + 2(-1) + b = 0$

1 – a – 3 – 2 + b = 0

b – a – 4 = 0 (i)

Similarly, f (1) = 0

$(1)^4 + a(1)^3 - 3(1)^2 + 2(1) + b = 0$

1 + a – 3 + 2 + b = 0

a + b = 0 (ii)

Adding (i) and (ii), we get

2b – 4 = 0

2b = 4

b = 2

Putting the value of b in (i), we get

2 – a – 4 = 0

a = -2

Hence, a = -2 and b = 2.

18. Question

If $x^3+ax^2-bx+10$ is divisible by x^2-3x+2, find the values of a and b.

Answer

Let f (x) = $x^3+ax^2-bx+10$ and g (x) = x^2-3x+2 be the given polynomials.

We have g (x) = x^2-3x+2 = (x – 2) (x – 1)

Clearly, (x -1) and (x – 2) are factors of g (x)

Given that f (x) is divisible by g (x)

g (x) is a factor of f (x)

(x – 2) and (x – 1) are factors of f (x)

From factor theorem,

If (x – 1) and (x – 2) are factors of f (x) then f (1) = 0 and f (2) = 0 respectively.

f (1) = 0

$(1)^3 + a(1)^2 - b(1) + 10 = 0$

$1 + a – b + 10 = 0$

$a – b + 11 = 0$ (i)

$f (2) = 0$

$(2)^3 + a (2)^2 - b (2) + 10 = 0$

$8 + 4a – 2b + 10 = 0$

$4a – 2b + 18 = 0$

$2 (2a – b + 9) = 0$

$2a – b + 9 = 0$ (ii)

Subtract (i) from (ii), we get

$2a – b + 9 – (a – b + 11) = 0$

$2a – b + 9 – a + b – 11 = 0$

$a – 2 = 0$

$a = 2$

Putting value of a in (i), we get

$2 – b + 11 = 0$

$b = 13$

Hence, $a = 2$ and $b = 13$

19. Question

If both x+1 and x-1 are factors of ax^3+x^2-2x+b, find the value of a and b.

Answer

Let, f (X) = ax^3+x^2-2x+b be the given polynomial.

Given (x + 1) and (x – 1) are factors of f (x).

From factor theorem,

If (x + 1) and (x – 1) are factors of f (x) then f (-1) = 0 and f (1) = 0 respectively.

$f (-1) = 0$

$a (-1)^3 + (-1)^2 – 2 (-1) + b = 0$

$-a + 1 + 2 + b = 0$

$-a + 3 + b = 0$

$b – a + 3 = 0$ (i)

$f (1) = 0$

$a (1)^3 + (1)^2 – 2 (1) + b = 0$

$a + 1 – 2 + b = 0$

$a + b – 1 = 0$

$b + a – 1 = 0$ (ii)

Adding (i) and (ii), we get

$b – a + 3 + b + a – 1 = 0$

$2b + 2 = 0$

$2b = -2$

$b = -1$

Putting value of b in (i), we get

$-1 - a + 3 = 0$

$-a + 2 = 0$

$a = 2$

Hence, the value of a = 2 and b = -1.

20. Question

What must be added to $x^3 - 3x^2 - 12x + 19$ so that the result is exactly divisibly by $x^2 + x - 6$?

Answer

Let p (x) = $x^3-3x^2-12x+19$ and q (x) = x^2+x-6

By division algorithm, when p (x) is divided by q (x), the remainder is a linear expression in x.

So, let r (x) = ax + b is added to p (x) so that p (x) + r (x) is divisible by q (x).

Let,

f (x) = p (x) + r (x)

$= x^3 - 3x^2 - 12x + 19 + ax + b$

$= x^3 – 3x^2 + x (a – 12) + b + 19$

We have,

q (x) = x^2+x-6

$= (x + 3) (x – 2)$

Clearly, q (x) is divisible by (x – 2) and (x + 3) i.e. (x – 2) and (x + 3) are factors of q (x)

We have,

f (x) is divisible by q (x)

(x – 2) and (x + 3) are factors of f (x)

From factor theorem,

If (x – 2) and (x + 3) are factors of f (x) then f (2) = 0 and f (-3) = 0 respectively.

f (2) = 0

$(2)^3 – 3 (2)^2 + 2 (a – 12) + b + 19 = 0$

$\Rightarrow 8 – 12 + 2a – 24 + b + 19 = 0$

$\Rightarrow 2a + b – 9 = 0$ (i)

Similarly,

f (-3) = 0

$(-3)^3 – 3 (-3)^2 + (-3) (a – 12) + b + 19 = 0$

$\Rightarrow -27 – 27 – 3a + 36 + b + 19 = 0$

$\Rightarrow b – 3a + 1 = 0$ (ii)

Subtract (i) from (ii), we get

$b – 3a + 1 – (2a + b – 9) = 0 – 0$

$\Rightarrow$ b – 3a + 1 – 2a – b + 9 = 0

$\Rightarrow$ - 5a + 10 = 0

$\Rightarrow$ 5a = 10

$\Rightarrow$ a = 2

Put a = 2 in (ii), we get

b – 3 × 2 + 1 = 0

$\Rightarrow$ b – 6 + 1 = 0

$\Rightarrow$ b – 5 = 0

$\Rightarrow$ b = 5

Therefore, r (x) = ax + b

= 2x + 5

Hence, $x^3 – 3x – 12x + 19$ is divisible by $x^2 + x – 6$ when 2x + 5 is added to it.

21. Question

What must be subtracted from $x^3 - 6x^2 - 15x + 80$, so that the result is exactly divisible by $x^2 + x - 12$?

Answer

Let p (x) = $x^3 - 6x^2 - 15x + 80$ and q (x) = $x^2 + x - 12$

By division algorithm, when p (x) is divided by q (x), the remainder is a linear expression in x.

So, let r (x) = ax + b is subtracted to p (x) so that p (x) + r (x) is divisible by q (x).

Let, f (x) = p (x) – r (x)

$\Rightarrow$ f(x) = $x^3 - 6x^2 - 15x + 80 – (ax + b)$

$\Rightarrow$ f(x) = $x^3 - 6x^2 – (a + 15)x + (80 – b)$

We have,

q(x) = $x^2 + x – 12$

$\Rightarrow$ q(x) = (x + 4) (x - 3)

Clearly, q (x) is divisible by (x + 4) and (x - 3) i.e. (x + 4) and (x - 3) are factors of q (x)

Therefore, f (x) will be divisible by q (x), if (x + 4) and (x - 3) are factors of f (x).

i.e. f(-4) = 0 and f(3) = 0

f (3) = 0

$\Rightarrow (3)^3 – 6(3)^2 – 3 (a + 15) + 80 – b = 0$

$\Rightarrow$ 27 – 54 – 3a – 45 + 80 – b = 0

$\Rightarrow$ 8 – 3a – b = 0 (i)

f (-4) = 0

$\Rightarrow (-4)^3 – 6 (-4)^2 – (-4) (a + 15) + 80 – b = 0$

$\Rightarrow$ -64 – 96 + 4a + 60 + 80 – b = 0

$\Rightarrow$ 4a – b – 20 = 0 (ii)

Subtract (i) from (ii), we get

$\Rightarrow$ 4a – b – 20 – (8 – 3a – b) = 0

⇒ 4a – b – 20 – 8 + 3a + b = 0

⇒ 7a = 28

⇒ a = 4

Put value of a in (ii), we get

⇒ b = -4

Putting the value of a and b in r (x) = ax + b, we get

r (x) = 4x – 4

Hence, p (x) is divisible by q (x), if r (x) = 4x – 4 is subtracted from it.

22. Question

What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$?

Answer

Let p (x) = $3x^3 + x^2 - 22x + 9$ and q (x) = $3x^2 + 7x - 6$.

By division algorithm,

When p (x) is divided by q (x), the remainder is a linear expression in x.

So, let r (x) = ax + b is added to p (x) so that p (x) + r (x) is divisible by q (x).

Let, f (x) = p (x) + r (x)

= $3x^3 + x^2 - 22x + 9 + (ax + b)$

= $3x^3 + x^2 + x(a - 22) + b + 9$

We have,

q (x) = $3x^2 + 7x - 6$

q (x) = 3x (x + 3) – 2 (x + 3)

q (x) = (3x – 2) (x + 3)

Clearly, q (x) is divisible by (3x – 2) and (x + 3). i.e. (3x – 2) and (x + 3) are factors of q(x),

Therefore, f(x) will be divisible by q(x), if (3x – 2) and (x + 3) are factors of f(x).

i.e. f (2/3) = 0 and f (-3) = 0 [∵ 3x – 2 = 0, x = 2/3 and x + 3 = 0, x = -3]

f (2/3) = 0

$\Rightarrow 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 + \frac{2}{3}(a - 2x) + b + 9 = 0$

$\Rightarrow \frac{12}{9} + \frac{2}{3a} - \frac{44}{3} + b + 9 = 0$

$\Rightarrow \frac{12+6a-132+9b+81}{9} = 0$

⇒ 6a + 9b – 39 = 0

⇒ 3 (2a + 3b – 13) = 0

⇒ 2a + 3b – 13 = 0 (i)

Similarly,

f (-3) = 0

⇒ $3(-3)^3 + (-3)^2 + (-3)(a - 2x) + b + 9 = 0$

⇒ -81 + 9 – 3a + 66 + b + 9 = 0

⇒ b – 3a + 3 = 0

⇒ 3 (b – 3a + 3) = 0

⇒ 3b – 9a + 9 = 0   (ii)

Subtract (i) from (ii), we get

3b – 9a + 9 – (2a + 3b – 13) = 0

3b – 9a + 9 – 2a – 3b + 13 = 0

⇒ -11a + 22 = 0

⇒ a = 2

Putting value of a in (i), we get

⇒ b = 3

Putting the values of a and b in r (x) = ax + b, we get

r (x) = 2x + 3

Hence, p (x) is divisible by q (x) if r (x) = 2x + 3 is divisible by it.

23. Question

If x-2 is a factor of each of the following two polynomials, find the values of a in each case.

(i) x^3-2ax^2+ax-1

(ii) $x^5-3x^4-ax^3+3ax^2+2ax+4$

Answer

(i) Let, f (x) = x^3-2ax^2+ax-1 be the given polynomial

From factor theorem,

If (x – 2) is a factor of f (x) then f (2) = 0 [Therefore, x – 2 = 0, x = 2]

f (2) = 0

$(2)^3 – 2\,a\,(2)^2 + a\,(2) – 1 = 0$

8 – 8a + 2a – 1 = 0

7 – 6a = 0

6a = 7

a = $\frac{7}{6}$

Hence, (x – 2) is a factor of f (x) when a = $\frac{7}{6}$.

(ii) Let f (x) = $x^5-3x^4-ax^3+3ax^2+2ax+4$ be the given polynomial

From factor theorem,

If (x – 2) is a factor of f (x) then f (2) = 0 [Therefore, x – 2= 0, x = 2]

f (2) = 0

$(2)^5 – 3\,(2)^4 – a\,(2)^3 + 3\,a\,(2)^2 + 2\,a\,(2) + 4 = 0$

32 – 48 – 8a + 12a + 4a + 4 = 0

-12 + 8a = 0

8a = 12

$a = \frac{3}{2}$

Hence, (x – 2) is a factor of f (x) when $a = \frac{3}{2}$.

24. Question

In each of the following two polynomials, find the value of a, if x-a is a factor:

(i) $x^6-ax^5+x^4-ax^3+3x-a+2$.

(ii) $x^5-a^2x^3+2x+a+1$.

Answer

(i) Let f (x) = $x^6-ax^5+x^4-ax^3+3x-a+2$ be the given polynomial

From factor theorem,

If (x – a) is a factor of f (x) then f (a) = 0 [Therefore, x – a = 0, x = a]

f (a) = 0

$(a)^6 – a (a)^5 + (a)^4 – a (a)^3 + 3 (a) – a + 2 = 0$

$a^6 – a^6 + a^4 – a^4 + 3a – a + 2 = 0$

2a + 2 = 0

a = -1

Hence, (x – a) is a factor f (x) when a = -1.

(ii) Let, f (x) = $x^5-a^2x^3+2x+a+1$ be the given polynomial

From factor theorem,

If (x – a) is a factor of f (x) then f (a) = 0 [Therefore, x – a = 0, x = a]

f (a) = 0

$(a)^5 – a^2 (a)^3 + 2 (a) + a + 1 = 0$

$a^5 – a^5 + 2a + a + 1 = 0$

3a + 1 = 0

3a = -1

$a = \frac{-1}{3}$

Hence, (x – a) is a factor f (x) when $a = \frac{-1}{3}$.

25. Question

In each of the following two polynomials, find the value of a, if x+a is a factor:

(i) $x^3+ax^2-2x+a+4$

(ii) $x^4-a^2x^2+3x-a$

Answer

(i) Let, f (x) = $x^3+ax^2-2x+a+4$ be the given polynomial

From factor theorem,

If (x + a) is a factor of f (x) then f (-a) = 0 [Therefore, x + a = 0, x = -a]

f (-a) = 0

$(-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$

$- a^3 + a^3 + 2a + a + 4 = 0$

$3a + 4 = 0$

$3a = -4$

$a = \frac{-4}{3}$

Hence, (x + a) is a factor f (x) when $a = \frac{-4}{3}$.

(ii) Let, f (x) = $x^4 - a^2x^2 + 3x - a$ be the given polynomial

From factor theorem,

If (x + a) is a factor of f (x) then f (-a) = 0 [Therefore, x + a = 0, x = -a]

f (-a) = 0

$(-a)^4 – a^2 (-a)^2 + 3(-a) - a = 0$

$a^4 – a^4 - 3a - a = 0$

$-4a = 0$

$a = 0$

Hence, (x + a) is a factor f (x) when a = 0.

Exercise 6.5

1. Question

Using factor theorem, factorize each of the following polynomial:

$x^3 + 6x^2 + 11x + 6$

Answer

Let f (x) = $x^3 + 6x^2 + 11x + 6$ be the given polynomial.

The constant term in f (x) is 6 and factors of 6 are $\pm 1, \pm 2, \pm 3 \ and \ \pm 6$

Putting x = - 1 in f (x) we have,

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 – 11 + 6$

$= 0$

Therefore, (x + 1) is a factor of f (x)

Similarly, (x + 2) and (x + 3) are factors of f (x).

Since, f (x) is a polynomial of degree 3. So, it cannot have more than three linear factors.

Therefore, f (x) = k (x + 1) (x + 2) (x + 3)

$x^3 + 6x^2 + 11x + 6 = k(x + 1)(x + 2)(x + 3)$

Putting x = 0, on both sides we get,

$0 + 0 + 0 + 6 = k(0 + 1)(0 + 2)(0 + 3)$

$6 = 6k$

$k = 1$

Putting k = 1 in f (x) = k (x + 1) (x + 2) (x + 3), we get f (x) = (x + 1) (x + 2) (x + 3)

Hence,

$x^3+6x^2+11x+6$ = (x + 1) (x + 2) (x + 3)

2. Question

Using factor theorem, factorize each of the following polynomial:

x^3+2x^2-x-2

Answer

Let, f (x) = x^3+2x^2-x-2

The constant term in f (x) is equal to -2 and factors of -2 are $\pm 1, \pm 2$.

Putting x = 1 in f (x), we have

f (1) = $(1)^3$ + 2 $(1)^2$ – 1 – 2

= 1 + 2 – 1 – 2

= 0

Therefore, (x – 1) is a factor of f (x).

Similarly, (x + 1) and (x + 2) are the factors of f (x).

Since, f (x) is a polynomial of degree 3. So, it cannot have more than three linear factors.

Therefore, f (x) = k (x – 1) (x + 1) (x + 2)

x^3+2x^2-x-2 = k (x – 1) (x + 1) (x + 2)

Putting x = 0 on both sides, we get

0 + 0 – 0 – 2 = k (0 – 1) (0 + 1) (0 + 2)

-2 = -2k

k = 1

Putting k = 1 in f (x) = k (x – 1) (x + 1) (x + 2), we get

f (x) = (x – 1) (x + 1) (x + 2)

Hence,

x^3+2x^2-x-2 = (x – 1) (x + 1) (x + 2)

3. Question

Using factor theorem, factorize each of the following polynomial:

$x^3-6x^2+3x+10$

Answer

Let, f (x) = $x^3-6x^2+3x+10$

The constant term in f (x) is equal to 10 and factors of 10 are $\pm 1, \pm 2, \pm 5 \ and \ \pm 10$

Putting x = - 1 in f (x), we have

f (-1) = $(-1)^3$ – 6 $(-1)^2$ + 3 (-1) + 10

= -1 – 6 – 3 + 10

= 0

Therefore, (x + 1) is a factor of f (x).

Similarly, (x - 2) and (x - 5) are the factors of f (x).

Since, f (x) is a polynomial of degree 3. So, it cannot have more than three linear factors.

Therefore, f (x) = k (x + 1) (x - 2) (x - 5)

$x^3-6x^2+3x+10$ = k (x + 1) (x - 2) (x - 5)

Putting x = 0 on both sides, we get

0 + 0 – 0 + 10 = k (0 + 1) (0 - 2) (0 - 5)

10 = 10k

k = 1

Putting k = 1 in f (x) = k (x + 1) (x - 2) (x - 5), we get

f (x) = (x + 1) (x - 2) (x - 5)

Hence,

$x^3-6x^2+3x+10$ = (x + 1) (x - 2) (x - 5)

4. Question

Using factor theorem, factorize each of the following polynomial:

$x^4-7x^3+9x^2+7x-10$

Answer

Let, f (x) = $x^4-7x^3+9x^2+7x-10$

The constant term in f (x) is equal to -10 and factors of -10 are $\pm1, \pm2, \pm5 \ and \ \pm 10$

Putting x = 1 in f (x), we have

f (1) = $(1)^4$ – 7 $(1)^3$ + 9 $(1)^2$ + 7 (1) - 10

= 1 – 7 + 9 + 7 - 10

= 0

Therefore, (x - 1) is a factor of f (x).

Similarly, (x + 1), (x - 2) and (x - 5) are the factors of f (x).

Since, f (x) is a polynomial of degree 4. So, it cannot have more than four linear factors.

Therefore, f (x) = k (x – 1) (x + 1) (x - 2) (x - 5)

$x^4-7x^3+9x^2+7x-10$ = k (x – 1) (x + 1) (x - 2) (x - 5)

Putting x = 0 on both sides, we get

0 + 0 – 0 - 10 = k (0 – 1) (0 + 1) (0 - 2) (0 - 5)

-10 = -10k

k = 1

Putting k = 1 in f (x) = k (x – 1) (x + 1) (x - 2) (x - 5), we get

f (x) = (x – 1) (x + 1) (x - 2) (x - 5)

Hence,

$x^4-7x^3+9x^2+7x-10$ = (x – 1) (x + 1) (x - 2) (x - 5)

5. Question

Using factor theorem, factorize each of the following polynomial:

x^4-2x^3-7x^2+8x+12

Answer

Let, f (x) = x^4-2x^3-7x^2+8x+12

The constant term in f (x) is equal to +12 and factors of +12 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6\ and\ \pm 12$

Putting x = - 1 in f (x), we have

f (-1) = $(-1)^4$ – 2 $(-1)^3$ – 7 $(-1)^2$ + 8 (-1) + 12

= 1 + 2 – 7 – 8 + 12

= 0

Therefore, (x + 1) is a factor of f (x).

Similarly, (x + 2), (x – 2) and (x - 3) are the factors of f (x).

Since, f (x) is a polynomial of degree 4. So, it cannot have more than four linear factors.

Therefore, f (x) = k (x + 1) (x + 2) (x - 2) (x - 3)

x^4-2x^3-7x^2+8x+12 = k (x + 1) (x + 2) (x - 2) (x - 3)

Putting x = 0 on both sides, we get

0 - 0 – 0 + 0 + 12 = k (0 + 1) (0 + 2) (0 - 2) (0 - 3)

12 = 12k

k = 1

Putting k = 1 in f (x) = k (x + 1) (x + 2) (x - 2) (x - 3), we get

f (x) = (x + 1) (x + 2) (x - 2) (x - 3)

Hence,

x^4-2x^3-7x^2+8x+12 = (x + 1) (x + 2) (x - 2) (x - 3)

6. Question

Using factor theorem, factorize each of the following polynomial:

x^4+10x^3+35x^2+50x +24

Answer

Let, f (x) = x^4+10x^3+35x^2+50x +24

The constant term in f (x) is equal to +24 and factors of +24 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12\ and\ \pm 18$

Putting x = - 1 in f (x), we have

f (-1) = $(-1)^4$ + 10 $(-1)^3$ + 35 $(-1)^2$ + 50 (-1) + 24

= 1 – 10 + 35 – 50 + 24

= 0

Therefore, (x + 1) is a factor of f (x).

Similarly, (x + 2), (x + 3) and (x + 4) are the factors of f (x).

Since, f (x) is a polynomial of degree 4. So, it cannot have more than four linear factors.

Therefore, f (x) = k (x + 1) (x + 2) (x + 3) (x + 4)

$x^4+10x^3+35x^2+50x+24$ = k (x + 1) (x + 2) (x + 3) (x + 4)

Putting x = 0 on both sides, we get

0 + 0 + 0 + 0 + 24 = k (0 + 1) (0 + 2) (0 + 3) (0 + 4)

24 = 24k

k = 1

Putting k = 1 in f (x) = k (x + 1) (x + 2) (x + 3) (x + 4), we get

f (x) = (x + 1) (x + 2) (x + 3) (x + 4)

Hence,

$x^4+10x^3+35x^2+50x+24$ = (x + 1) (x + 2) (x + 3) (x + 4)

7. Question

Using factor theorem, factorize each of the following polynomial:

$2x^4-7x^3-13x^2+63x-45$

Answer

Let, f (x) = $2x^4-7x^3-13x^2+63x-45$

The factors of the constant term – 45 are $\pm1, \pm3, \pm5, \pm9, \pm15 \; and \pm 45$

The factor of the coefficient of x^4 is 2. Hence, possible rational roots of f (x) are:

$\pm1, \pm3, \pm5, \pm9, \pm15, \pm45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

We have,

f (1) = 2 $(1)^4$ – 7 $(1)^3$ – 13 $(1)^2$ + 63 (1) – 45

= 2 – 7 – 13 + 63 – 45

= 0

And,

f (3) = 2 $(3)^4$ – 7 $(3)^3$ – 13 $(3)^2$ + 63 (3) – 45

= 162 – 189 – 117 + 189 – 45

= 0

So, (x – 1) and (x + 3) are the factors of f (x)

(x – 1) (x + 3) is also a factor of f (x)

Let us now divide

f (x) = $2x^4-7x^3-13x^2+63x-45$ by (x^2 – 4x + 3) to get the other factors of f (x)

Using long division method, we get

$2x^4-7x^3-13x^2+63x-45$ = (x^2 – 4x + 3) ($2x^2$ + x – 15)

$2x^4-7x^3-13x^2+63x-45$ = (x – 1) (x – 3) ($2x^2$ + x – 15)

Now,

$2x^2$ + x – 15 = $2x^2$ + 6x – 5x – 15

= 2x (x + 3) – 5 (x + 3)

= (2x – 5) (x + 3)

Hence, $2x^4-7x^3-13x^2+63x-45$ = (x – 1) (x – 3) (x + 3) (2x – 5)

8. Question

Using factor theorem, factorize each of the following polynomial:

$3x^3-x^2-3x+1$

Answer

Let, f (x) = $3x^3-x^2-3x+1$

The factors of the constant term $\pm 1\ is \pm 1$.

The factor of the coefficient of x^3 is 3. Hence, possible rational roots of f (x) are:

$\pm 1, \pm \frac{1}{3}$

We have,

f (1) = 3 $(1)^3$ – $(1)^2$ – 3 (1) + 1

= 3 – 1 – 3 + 1

= 0

So, (x – 1) is a factor of f (x)

Let us now divide

f (x) = $3x^3-x^2-3x+1$ by (x - 1) to get the other factors of f (x)

Using long division method, we get

$3x^3-x^2-3x+1$ = (x – 1) ($3x^2$ + 2x – 1)

Now,

$3x^2$ + 2x - 1 = $3x^2$ + 3x – x – 1

= 3x (x + 1) – 1 (x + 1)

= (3x – 1) (x + 1)

Hence, $3x^3-x^2-3x+1$ = (x – 1) (x + 1) (3x – 1)

9. Question

Using factor theorem, factorize each of the following polynomial:

$x^3-23x^2+142x-120$

Answer

Let, f (x) = $x^3-23x^2+142x-120$

The factors of the constant term – 120 are
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 40, \pm 60\ and \pm 120$

Putting x = 1, we have

f (1) = $(1)^3$ – 23 $(1)^2$ + 142 (1) – 120

= 1 – 23 + 142 – 120

= 0

So, (x – 1) is a factor of f (x)

Let us now divide

f (x) = x^3-23x^2+142x-120 by (x - 1) to get the other factors of f (x)

Using long division method, we get

x^3-23x^2+142x-120 = (x – 1) (x^2 – 22x + 120)

x^2 – 22x + 120 = x^2 – 10x – 12x + 120

= x (x – 10) – 12 (x – 10)

Hence, x^3-23x^2+142x-120 = (x – 1) (x - 10) (x - 12)

10. Question

Using factor theorem, factorize each of the following polynomial:

y^3-7y+ 6

Answer

Let, f (y) = y^3-7y+ 6

The constant term in f (y) is equal to + 6 and factors of + 6 are $\pm 1, \pm 2, \pm 3\ and\ \pm 6$

Putting y = 1 in f (y), we have

f (1) = $(1)^3$ – 7 (1) + 6

= 1 – 7 + 6

= 0

Therefore, (y - 1) is a factor of f (y).

Similarly, (y - 2) and (y + 3) are the factors of f (y).

Since, f (y) is a polynomial of degree 3. So, it cannot have more than three linear factors.

Therefore, f (y) = k (y – 1) (y - 2) (y + 3)

y^3-7y+ 6 = k (y – 1) (y - 2) (y + 3)

Putting x = 0 on both sides, we get

0 – 0 + 6 = k (0 – 1) (0 - 2) (0 + 3)

6 = 6k

k = 1

Putting k = 1 in f (y) = k (y – 1) (y - 2) (y + 3), we get

f (y) = (y – 1) (y - 2) (y + 3)

Hence,

y^3-7y+ 6 = (y – 1) (y - 2) (y + 3)

11. Question

Using factor theorem, factorize each of the following polynomial:

x^3-10x^2-53x-42

Answer

Let, f (x) = x^3-10x^2-53x-42

The factors of the constant term – 42 are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21\ and \pm 42$

Putting x = - 1, we have

f (-1) = $(-1)^3$ – 10 $(-1)^2$ – 53 (-1) - 42

= -1 – 10 + 53 - 42

= 0

So, (x + 1) is a factor of f (x)

Let us now divide

f (x) = $x^3-10x^2-53x-42$ by (x + 1) to get the other factors of f (x)

Using long division method, we get

$x^3-10x^2-53x-42$ = (x + 1) (x^2 – 11x – 42)

x^2 – 11x - 42 = x^2 – 14x + 3x - 42

= x (x – 14) + 3 (x – 14)

= (x – 14) (x + 3)

Hence, $x^3-10x^2-53x-42$ = (x + 1) (x - 14) (x + 3)

12. Question

Using factor theorem, factorize each of the following polynomial:

$y^3-2y^2-29y-42$

Answer

Let, f (y) = $y^3-2y^2-29y-42$

The factors of the constant term – 42 are $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21\ and \pm 42$

Putting y = - 2, we have

f (-2) = $(-2)^3$ – 2 $(-2)^2$ – 29 (-2) - 42

= - 8 – 8 + 58 - 42

= 0

So, (y + 2) is a factor of f (y)

Let us now divide

f (y) = $y^3-2y^2-29y-42$ by (y + 2) to get the other factors of f (x)

Using long division method, we get

$y^3 - 2y^2 - 29y - 42$ = (y + 2) (y^2 – 4y – 21)

y^2 – 4y - 21 = y^2 – 7y + 3y - 21

= y (y – 7) + 3 (y – 7)

= (y – 7) (y + 3)

Hence, $y^3-2y^2-29y-42$ = (y + 2) (y - 7) (y + 3)

13. Question

Using factor theorem, factorize each of the following polynomial:

$2y^3-5y^2-19y+42$

Answer

Let, f (y) = $2y^3-5y^2-19y+42$

The factors of the constant term + 42 are $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21 \text{ and } \pm 42$

Putting y = 2, we have

f (2) = 2 $(2)^3$ – 5 $(2)^2$ – 19 (2) + 42

= 16 – 20 - 38 + 42

= 0

So, (y - 2) is a factor of f (y)

Let us now divide

f (y) = $2y^3-5y^2-19y+42$ by (y - 2) to get the other factors of f (x)

Using long division method, we get

$2y^3-5y^2-19y+42$ = (y - 2) ($2y^2$ – y – 21)

$2y^2$ – y - 21 = (y + 3) (2y – 7)

Hence, $2y^3-5y^2-19y+42$ = (y - 2) (2y - 7) (y + 3)

14. Question

Using factor theorem, factorize each of the following polynomial:

$x^3+13x^2+32x+20$

Answer

Let, f (x) = $x^3+13x^2+32x+20$

The factors of the constant term + 20 are $\pm1, \pm2, \pm4, \pm5, \pm10 \text{ and } \pm 20$

Putting x = -1, we have

f (-1) = $(-1)^3$ + 13 $(-1)^2$ + 32 (-1) + 20

= -1 + 13 – 32 + 20

= 0

So, (x + 1) is a factor of f (x)

Let us now divide

f (x) = $x^3+13x^2+32x+20$ by (x + 1) to get the other factors of f (x)

Using long division method, we get

$x^3+13x^2+32x+20$ = (x + 1) (x^2 + 12x + 20)

x^2 + 2x + 20 = x^2 + 10x + 2x + 20

= x (x + 10) + 2 (x + 10)

= (x + 10) (x + 2)

Hence, $x^3+13x^2+32x+20$ = (x + 1) (x + 10) (x + 2)

15. Question

Using factor theorem, factorize each of the following polynomial:

x^3-3x^2-9x-5

Answer

Let, f (x) = x^3-3x^2-9x-5

The factors of the constant term - 5 are $\pm 1, \pm 5$

Putting x = -1, we have

f (-1) = $(-1)^3$ – 3 $(-1)^2$ – 9 (-1) - 5

= -1 – 3 + 9 - 5

= 0

So, (x + 1) is a factor of f (x)

Let us now divide

f (x) = x^3-3x^2-9x-5 by (x + 1) to get the other factors of f (x)

Using long division method, we get

x^3-3x^2-9x-5 = (x + 1) (x^2 - 4x 5)

x^2 - 4x - 5 = x^2 - 5x + x - 5

= x (x - 5) + 1 (x - 5)

= (x + 1) (x - 5)

Hence, x^3+13x^2+32x+20 = (x + 1) (x + 1) (x - 5)

= $(x + 1)^2$ (x – 5)

16. Question

Using factor theorem, factorize each of the following polynomial:

2y^3+y^2-2y-1

Answer

Let, f (y) = 2y^3+y^2-2y-1

The factors of the constant term - 1 are ± 1

The factor of the coefficient of y^3 is 2. Hence, possible rational roots are $\pm 1, \pm \frac{1}{2}$

We have

f (1) = 2 $(1)^3$ + $(1)^2$ – 2 (1) - 1

= 2 + 1 – 2 - 1

= 0

So, (y - 1) is a factor of f (y)

Let us now divide

f (y) = 2y^3+y^2-2y-1 by (y - 1) to get the other factors of f (x)

Using long division method, we get

2y^3+y^2-2y-1 = (y - 1) (2y^2 + 3y + 1)

2y^2 + 3y + 1 = 2y^2 + 2y + y + 1

= 2y (y + 1) + 1 (y + 1)

= (2y + 1) (y + 1)

Hence, 2y^3+y^2-2y-1 = (y - 1) (2y + 1) (y + 1)

17. Question

Using factor theorem, factorize each of the following polynomial:

x^3-2x^2-x+2

Answer

Let, f (x) = x^3-2x^2-x+2

The factors of the constant term +2 are $\pm 1, \pm 2$

Putting x = 1, we have

f (1) = $(1)^3$ – 2 $(1)^2$ – (1) + 2

= 1 – 2 – 1 + 2

= 0

So, (x - 1) is a factor of f (x)

Let us now divide

f (x) = x^3-2x^2-x+2 by (x - 1) to get the other factors of f (x)

Using long division method, we get

x^3-2x^2-x+2 = (x - 1) (x^2 - x - 2)

x^2 - x - 2 = x^2 - 2x + x - 2

= x (x - 2) + 1 (x - 2)

= (x + 1) (x - 2)

Hence, x^3-2x^2-x+2 = (x - 1) (x + 1) (x - 2)

= (x - 1) (x + 1) (x – 2)

18. Question

Factorize each of the following polynomials:

(i) x^3+13x^2+31x-45 given that x+9 is a factor

(ii) 4x^3+20x^2+33x+18 given that 2x+3 is a factor.

Answer

(i) Let, f (x) = x^3+13x^2+31x-45

Given that (x + 9) is a factor of f (x)

Let us divide f (x) by (x + 9) to get the other factors

By using long division method, we have

f (x) = x^3+13x^2+31x-45

= (x + 9) (x^2 + 4x – 5)

Now,

x^2 + 4x – 5 = x^2 + 5x – x – 5

= x (x + 5) – 1 (x + 5)

= (x – 1) (x + 5)

f (x) = (x + 9) (x + 5) (x – 1)

Therefore, $x^3+13x^2+31x-45 = (x + 9) (x + 5) (x - 1)$

(ii) Let, f (x) = $4x^3+20x^2+33x+18$

Given that (2x + 3) is a factor of f (x)

Let us divide f (x) by (2x + 3) to get the other factors

By long division method, we have

$4x^3+20x^2+33x+18 = (2x + 3) (2x^2 + 7x + 6)$

$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$

= 2x (x + 2) + 3 (x + 2)

= (2x + 3) (x + 2)

$4x^3+20x^2+33x+18 = (2x + 3) (2x + 3) (x + 2)$

$= (2x + 3)^2 (x + 2)$

Hence,

$4x^3+20x^2+33x+18 = (2x + 3)^2 (x + 2)$

CCE - Formative Assessment

1. Question

Define zero or root of a polynomial.

Answer

The zeros are the roots, or where the polynomial crosses the axis. A polynomial will have 2 roots that mean it has 2 zeros. To find the roots you can graph and look where it crosses the axis, or you can use the quadratic equation. This is also known as the solution.

2. Question

If $x = \frac{1}{2}$ is a zero of the polynomial $f(x) = 8x^3+ax^2-4x+2$, find the value of a.

Answer

If $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 2$

$0 = 1 + \frac{a}{4} - 2 + 2$

a = - 4

3. Question

Write the remainder when the polynomial $f(x) = x^3+x^2-3x+2$ is divided by $x+1$.

Answer

$f(x) = x^3 + x^2 - 3x + 2$

Given,

f (x) divided by (x+1), so reminder is equal to f (-1)

$f(-1) = (-1)^3 + (-1)^2 - 3(-1) + 2$

= -1 +1 +3 +2

=5

Thus, remainder is 5.

4. Question

Find the remainder when x^3+4x^2+4x-3 is divided by x.

Answer

Let, f (x) = $x^3 + 4x^2 + 4x - 3$

Given f (x) is divided by x so remainder is equal to f (0)

$f(0) = 0^3 + 4(0)^2 + 4(0) - 3$

= 0 - 3

= - 3

Thus, remainder is - 3

5. Question

If $x+1$ is a factor of x^3+a, then write the value of a.

Answer

Let, f (x) = $x^3 + a$

(x +1) is a factor of f(x), so f (-1) = 0

f (-1) =0

$(-1)^3 + a = 0$

-1 + a = 0

a = 1

6. Question

If $f(x) = x^4-2x^3+3x^2-ax-b$ when divided by $x-1$, the remainder is 6, then find the value of $a + b$

Answer

$f(x) = x^4 - 2x^2 + 3x^2 - ax - b$

Given f(x) is divided by (x-1), then remainder is 6

f (1) = 6

$1^4 - 2(1)^3 - 3(1)^2 - a(1) - b = 6$

1 -2 +3 -a -b = 6

-a -b = 4

a + b = - 4

1. Question

If $x-2$ is factor of $x^2-3ax-2a$, then a =

A. 2

B. -2

C. 1

D. -1

Answer

Let f(x) = x^2 -3ax -2a

Since, x-2 is a factor of f(x) so,

f (2) = 0

2^2+ 3 a (2) - 2a =0

4 + 6a -2a = 0

a = -1

2. Question

If x^3+6x^2+4x+k is exactly divisible by $x+2$, then k =

A. -6

B. -7

C. -8

D. -10

Answer

Since, x+2 is exactly divisible by f(x)

Means x+2 is a factor of f (x), so

f (-2) = 0

$(-2)^3$ + 6 $(-2)^2$ +4 (-2) + k = 0

-16 + 24 + k = 0

k= - 8

3. Question

If x-a is a factor of x^3-$3x^2a+2a^2x+b$, then the value of b is

A. 0

B. 2

C. 1

D. 3

Answer

Let f (x) = x^3 - $3x^2$a + $2a^2$x + b

Since, x - a is a factor of f(x)

So, f (a) = 0

a^3- 3 a^2 (a) + $2a^2$ (a) + b = 0

a^3 - $3a^3$ + $2a^3$ + b= 0

b = 0

4. Question

If $x^{140}+2x^{151}$ $+k$ is divisible by $x+1$, then the value of k is

A. 1

B. -3

C. 2

D. -2

Answer

Let f (x) = x^{140} + $2x^{151}$ + k

Since, x+1 is a factor of f (x)

So, f (-1) = 0

$(-1)^{140}$ + $2(-1)^{151}$ +k = 0

1 - 2 + k=0

k = 1

5. Question

If x+2and x-1 are the factors of x^3+10x^2+mx+n, then the value of m and n are respectively

A. 5 and -3

B. 17 and -8

C. 7 and -18

D. 23 and -19

Answer

Let f (x) = x^3+10x^2 +mx + n

Since, (x + 2) and (x - 1) are factor of f (x)

So, f (-2) = 0

$(-2)^3$ + 10 $(-2)^2$ + m (-2) + n

32 - 2m + n = 0 (i)

f (1) = 0

$(1)^3$ + 10 $(1)^2$ + m (1) + n = 0

11 + m + n = 0 (ii)

(2) - (1)

3m -21 = 0

m = 7 (iii)

Using (iii) and (ii), we get

11 + 7 + n= 0

n = - 18

6. Question

Let $f(x)$ be a polynomial such that $f\left(-\frac{1}{2}\right)$= 0, then a factor of $f(x)$ is

A. 2x -1

B. 2x+1

C. x-1

D. x +1

Answer

Let f(x) be a polynomial and f $(\frac{-1}{2})$ = 0

x + $\frac{1}{2}$ = 2x + 1 is a factor of f (x)

7. Question

When x^3-2x^2+ax=b is divided by x^2-2x-3, the remainder is x-6. The value of a and b respectively

A. -2, -6

B. 2 and -6

C. -2 and 6

D. 2 and 6

Answer

Let p(x) = x^3 -2(x^2) + ax - b

q (x) = x^2 -2x -3

r (x) = x - 6

Therefore,

f(x) = p (x) – r (x)

f(x) = x^3 - 2x^2 + ax - b - x - 6

= x^3- 2x^2 + (a - 1) x - (b - 6)

q(x) = x^2 - 2x - 3

= (x + 1) (x - 3)

Thus,

(x + 1) and (x - 3) are factor of f (x)

a + b = 4

f (3) = 0

3^3 - 2 $(3)^2$ + (a-1) 3 - b + 6 = 0

12 + 3a - b = 0

a = - 2, b = 6

8. Question

One factor of x^4+x^2-20 is x^2+5. The other factor is

A. x^2-4

B. x-4

C. x^2-5

D. x+2

Answer

f (x) = x^4 +x^2 -20

(x^2 + 5) (x^2 - 4)

Therefore, (x^2+5) and (x^2-4) are the factors of f(x)

9. Question

If (x-1) is a factor of polynomial $f(x)$ but not of $g(x)$, then it must be a factor of

A. $f(x)\ g(x)$

B. $-f(x)+g(x)$

C. $f(x)-g(x)$

D. $\{f(x)+g(x)\}g(x)$

Answer

Given,

(x-1) is a factor of f (x) but not of g(x).

Therefore, x-1 is also a factor of f(x) g(x).

10. Question

(x+1) is a factor of x^n+1 only if

A. n is an odd integer

B. n is an even integer

C. n is a negative integer

D. n is a positive integer

Answer

Let f (x) = x^n +1

Since, x+ 1 is a factor of f (x), so

f (-1) = 9

Thus, n is an odd integer.

11. Question

If x+2 is a factor of $x^2+mx+14$, then m =

A. 7

B. 2

C. 9

D. 14

Answer

f (x) = x^2 + mx + 14

Since, (x +2) is a factor of f (x), so

f (-2) =0

$(-2)^2$ + m (-2) + 14 = 0

18 - 2m = 0

m = 9

12. Question

If x -3 is a factor of $x^2-ax-15$, then a =

A. -2

B. 5

C. -5

D. 3

Answer

Let, f(x) = x^2 - ax - 15

Since, (x -3) is a factor of f (x), so

f (3) = 0

3^2 - a (3) -15 = 0

9 - 3a - 15 = 0

a = -2

13. Question

If x^2+x+1 is a factor of the polynomial $3x^2+8x^2+8x+3+5k$, then the value of k is

A. 0

B. 2/5

C. 5/2

D. -1

Answer

Let, p (x) = $3x^3$ + 8 $(x)^2$ + 8x + 3 + 5k

g (x) = x^2 + x + 1

Given g (x) is a factor of p (x) so remainder will be 0

Remainder= -2 + 5k

Therefore, -2 + 5k = 0

k = 2/5

14. Question

If $(3x\text{-}1)^7 = a_7x^7+a_6x^6+a_5x^5+....a_1x+a_0$, then $a_7+a_6+a_5+....+a_1+a_0=$

A. 0

B. 1

C. 128

D. 64

Answer

We have,

$(3x – 1)^7 = a_7x^7+a_6x^6+a_5x^5+....a_1x+a_0$

Putting x = 1, we get

$(3 * 1 – 1)^7 = a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0$

$(2)^7 = a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0$

$a_7+a_6+a_5+....+a_1+a_0$ = 128

15. Question

If $x^{51}+51$ is divide by $x+1$, the remainder is

A. 0

B. 1

C. 49

D. 50

Answer

Let, f(x) = x^{51} + 51

Since, x + 1 is divided by f(x) so,

f (-1)= $(-1)^{51}$ + 51

= - 1 + 51

= 50

Thus, remainder is 50

16. Question

If $x+1$ is a factor if the polynomial $2x^2+kx$, then $k=$

A. -2

B. -3

C. 4

D. 2

Answer

Let, f (x) = $2x^2$ + kx

Since, x + 1 is divided by f (x) so,

f (-1)=0

2 (-1) + k (-1) = 0

k = 2

17. Question

If $x+a$ is a factor of $x^4-a^2x^2+3x-6a$, then $a=$

A. 0

B. -1

C. 1

D. 2

Answer

Let, f (x) = x^4 $-a^2x^2$ + 3x - 6a

Since, x + a is divided by f (x) so,

f (-a) = 0

$(-a)^4$ - a^2 $(-a)^2$ + 3 (-a) - 6a = 0

- 9a = 0

a = 0

18. Question

The value of k for which x-1 is a factor of $4x^3+3x^2-4x+k$, is

A. 3

B. 1

C. -2

D. -3

Answer

Since, x-1 is a factor of f (x)

Therefore,

f (1) = 0

$4(1)^3 + 3(1)^2 - 4(1) + k = 0$

4 + 3 - 4 + k = 0

k = - 3

19. Question

If both x-2 and $x - \frac{1}{2}$ are factors of px^2+5x+r, then

A. $p = r$

B. $p + r = 0$

C. $2p + r = 0$

D. $p + 2r = 0$

Answer

Let f(x)= px^2 +5x +r

Since, x-2 and x-1/2 are factors of f (x)

f (2) = 0

4p + 10 + r = 0 (i)

f(1/2) = 0

p + 10 + 4r = 0 (ii)

(i) * (ii), we get,

4p + 40 + 16r = 0 (iii)

Subtracting (i) and (iii)

-30 - 15r = 0

r = - 2

Putting value of r in (i),

4p + 10 - 2 = 0

p = -2

Therefore, p = r

20. Question

If x^2-1 is a factor of $ax^4+bx^3+cx^2+dx+e$, then

A. $a + c + e = b + d$

B. $a + b + e = c + d$

C. $a + b + c = d + e$

D. $b + c + d = a + e$

Answer

Let f (x) = $ax^4 + bx^3 + cx^2 + dx + e$

Since, $x^2 - 1$ is a factor of f(x)

Therefore,

f (-1) = 0

a (-1) + b $(-1)^3$ + c $(-1)^2$ + d (-1) + e = 0

a + c + e = b + d