24. Measures of Central Tendency

Exercise 24.1

1. Question

If the heights of 5 persons are 140 cm, 150 cm, 152 cm, 158 cm and 161 cm respectively, find the mean height.

Answer

It is given that,

The height of 5 person are = 140 cm, 150 cm, 152 cm, 158 cm and 161 cm

Therefore, Mean height = $\frac{Sum\ of\ height}{Total\ person}$

= $\frac{140+150+152+158+161}{5}$

= $\frac{761}{5}$

= 152.2

2. Question

Find the mean of 994, 996, 998, 1002 and 1000.

Answer

Given numbers are: 994, 996, 998, 1002 and 1000

Therefore, Mean = $\frac{Sum\ of\ numbers}{Total\ numbers}$

= $\frac{994+996+998+1000+1002}{5}$

= $\frac{4990}{5}$

= 998

3. Question

Find the mean of first five natural numbers.

Answer

Given that the first five natural numbers are: 1, 2, 3, 4, 5

Therefore, Mean = $\frac{Sum\ of\ numbers}{Total\ numbers}$

$= \frac{1+2+3+4+5}{5}$

$= \frac{15}{5}$

$= 3$

4. Question

Find the mean of all factors of 10.

Answer

All factors of 10 are: 1, 2, 5, 10

Therefore, Mean $= \frac{Sum\ of\ factors}{Total\ factors}$

$= \frac{1+2+5+10}{4}$

$= \frac{18}{4}$

$= 4.5$

5. Question

Find the mean of first 10 even natural numbers.

Answer

Given that the first 10 even natural numbers be 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Mean $= \frac{Sum\ of\ all\ numbers}{Total\ numbers}$

$= \frac{2+4+6+8+10+12+14+16+18+20}{10}$

$= \frac{110}{10}$

$= 11$

6. Question

Find the mean of x, $x+2$, $x+4$, $x+6$, $x+8$.

Answer

Numbers be: x, $x+2$, $x+4$, $x+6$, $x+8$

Therefore, Mean $= \frac{Sum\ of\ numbers}{Total\ numbers}$

$= \frac{x+x+2+x+4+x+6+x+8}{5}$

$= \frac{5x+20}{5}$

= x + 4

7. Question

Find the mean of first five multiples of 3.

Answer

First five multiples of 3 are:

3, 6, 9, 12, 15

Therefore, Mean = $\frac{Sum\ of\ numbers}{Total\ numbers}$

= $\frac{3+6+9+12+15}{5}$

= $\frac{45}{5}$

= 9

8. Question

Following are the weights (in kg) of 10 new born babies in a hospital on a particular day:

3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6

Find the mean $\overline{X}$

Answer

The weight (in kg) of 10 new born babies = 3.4, 3.6, 4.2, 4.5, 3.9, 4.1, 3.8, 4.5, 4.4, 3.6

Therefore, Mean = $\frac{Sum\ of\ weight}{Total\ babies}$

= $\frac{3.4+3.6+4.2+4.5+3.9+4.1+3.8+4.5+4.4+3.6}{10}$

= $\frac{40}{10}$

= 4 kg

9. Question

The percentage of marks obtained by students of a class in mathematics are:

64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1

Find their mean.

Answer

The percentage marks obtained by students are:

64, 36, 47, 23, 0, 19, 81, 93, 72, 35, 3, 1

Therefore, Mean = $\frac{Total\ Marks}{Total\ students}$

$= \frac{64+36+47+23+0+19+81+93+22+35+3+1}{12}$

$= \frac{474}{12}$

= 39.5

10. Question

The numbers of children in 10 families of a locality are:

2, 4, 3, 4, 2, 0, 3, 5, 1, 1, 5

Find the mean number of children per family.

Answer

The number of children in 10 families:

2, 4, 3, 4, 2, 0, 3, 5, 1, 1, 5

Therefore, Mean = $\frac{Total\ children}{Total\ families}$

$= \frac{2+4+3+4+2+0+3+5+1+1+5}{10}$

$= \frac{30}{10}$

= 3

11. Question

If M is the mean of x_1, x_2, x_3, x_4, x_5 and x_6, prove that

$(x_1-M)+ (x_2-M)+ (x_3-M)+ (x_4-M)+ (x_5-M)+(x_6-M) = 0$.

Answer

Let M is the mean of x_1, x_2, x_3, x_4, x_5 and x_6

Then, M = $\frac{x1+x2+x3+x4+x5+x6}{6}$

$6M = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

To prove: $(x_1-M) + (x_2-M) + (x_3-M) + (x_4-M) + (x_5-M) + (x_6-M) = 0$

Proof: L.H.S

$= (x_1 - M) + (x_2 - M) + (x_3 + M) + (x_4 - M) + (x_5 - M) + (x_6 - M)$

$= (x_1 + x_2 + x_3 + x_4 + x_5 + x_6) - (M+ M + M + M + M + M)$

= 6M – 6M

= 0

= R.H.S

12. Question

Durations of sunshine (in hours) in Amritsar for first 10 days of August 1997 as reported by the Meteorological Department are given below:

9.6, 5.2, 3.5, 1.5, 1.6, 2.4, 2.6, 8.4, 10.3, 10.9

(i) Find the mean $\overline{X}$

(ii) Verify that $\sum_{i=1}^{10}(x_i - \overline{X}) = 0$

Answer

Duration of sunshine (in hours) for 10 days are:

9.6, 5.2, 3.5, 1.5, 1.6, 2.4, 2.6, 8.4, 10.3, 10.9

(i) Mean, $\overline{X}$

$= \frac{Sum\ of\ all\ numbers}{Total\ numbers}$

$= \frac{9.6+5.2+3.5+1.5+1.6+2.4+2.6+8.4+10.3+10.9}{10}$

$= \frac{56}{10}$

$= 5.6$

(ii) L.H.S = $\sum_{i=1}^{10}(x_i - \overline{X})$

$= (x_1 - \overline{X}) + (x_2 - \overline{X}) + (x_3 - \overline{X}) + (x_4 - \overline{X}) + \ldots\ldots\ldots + (x_{10} - \overline{X})$

$= (9.6 - 5.6) + (5.2 - 5.6) + (3.5 - 5.6) + (1.5 - 5.6) + (1.6 - 5.6) + (2.4 - 5.6)$

$= (4) + (-0.4) + (-2.1) + (-4.1) + (-4) + (-3.2) + (-3) + (2.8) + (4.7) + (5.3)$

$= 16.8 - 16.8$

$= 0$

13. Question

Explain, by taking a suitable example, how the arithmetic mean alters by (i) adding a constant *k* to each term, (ii) subtracting a constant *k* from each them, (iii) multiplying each term by a constant *k* and (iv) dividing each term by a non-zero constant *k*.

Answer

Let us say numbers are be

3, 4, 5

Therefore, Mean = $\frac{Sum\ of\ numbers}{Total\ numbers}$

$= \frac{3+4+5}{3}$

$= \frac{12}{3}$

$= 4$

(i) = 2 on each term

New numbers are = 5, 6, 7

New mean = $\frac{5+6+7}{3}$

$= \frac{18}{3}$

$= 6$

Therefore new mean will be 2 more than the original mean.

(ii) Subtracting constant terms ‘k’ = 2 in each term

New numbers are = 1, 2, 3

Therefore, new mean = $\frac{1+2+3}{3}$

$= \frac{6}{3}$

$= 2$

Therefore, new mean will be 2 less than the original mean.

(iii) = 2 in each term

New numbers are = 6, 7, 8

Therefore, new mean = $\frac{6+7+8}{3}$

$= \frac{24}{3}$

$= 8$

Therefore, new mean will be 2 times of the original mean.

(iv) Divide by constant term ‘k’ = 2 in each term

New numbers are = 1.5, 2, 2.5

Therefore, new mean = $\frac{1.5+2+2.5}{3}$

$= \frac{6}{3}$

$= 2$

Therefore, new mean will be half of the original mean.

14. Question

The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Answer

Mean marks of 100 students = 40

Sum of marks of 100 students = 100 * 40

= 4000

Correct value = 53

Incorrect value = 83

Correct sum = 4000 – 83 + 53

= 3970

Therefore, correct mean = $\frac{3970}{100}$

= 39.7

15. Question

The traffic police recorded the speed (in km/h) of 10 motorists as 47, 53, 49, 60, 39, 42, 55, 57, 52, 48. Later on an error in recording instrument was found. Find the correct overage speed of the motorists if the instrument recorded 5 km/hr. less in each case.

Answer

The speed of 10 Motorist are:

47, 53, 49, 60, 39, 42, 55, 57, 52, 48

Later on it was discovered that the instrument recorded 5 km/h less than in each case

Therefore, correct values are = 52, 58, 54, 65, 44, 47, 60, 62, 57, 53

Therefore, correct mean = $\frac{52+58+54+65+44+47+60+62+57+53}{10}$

= $\frac{552}{10}$

= 55.2 km/h

16. Question

The mean of five numbers is 27. If one number is excluded, their mean is 25. Find the excluded number.

Answer

The mean of the numbers is 27

The sum of 5 numbers = 5 * 27

= 135

If one number is excluded, then the new mean is 25

Therefore sum of new numbers = 4 * 25

= 100

Therefore, excluded number = 135 – 100

= 35

17. Question

The mean weight per student in a group of 7 students is 55 kg. The individual weights of 6 of them (in kg) are 52, 54, 55, 53, 56 and 54. Find the weight of the seventh student.

Answer

The mean weight per student in a group of 7 students = 55 kg

Weight of 6 students (in kg) = 52, 54, 55, 53, 56 and 54

Let weight of 7^{th} student = x kg

Therefore, Mean = $\frac{Sum\ of\ all\ weights}{Total\ students}$

55 = $\frac{52+54+55+53+56+54+x}{7}$

385 = 324 + x

x = 385 – 324

x = 61 kg

Therefore, weight of 7^{th} student = 61 kg

18. Question

The mean weight of 8 numbers is 15. If each number is multiplied by 2, what will be the new mean?

Answer

We have,

The mean weight of 8 numbers is 15

Then, the sum of 8 numbers = 8 * 15

= 120

If each number is multiplied by 2

Then, New mean = 120 * 2

= 240

Therefore, new mean = $\frac{240}{8}$

= 30

19. Question

The mean of 5 numbers is 18. If one number is excluded, their mean is 16. Find the excluded number.

Answer

The mean of 5 numbers is 18

Then, the sum of 5 numbers = 5 * 18

= 90

If the one number is excluded

Then, the mean of 4 numbers = 16

Therefore, sum of 4 numbers = 4 * 16

= 64

Excluded number = 90 – 64

= 26

20. Question

The mean of 200 items was 50. Later on, it was discovered that the two items were misread as 92 and 8 instead of 192 and 88. Find the correct mean.

Answer

The mean of 200 items = 50

Then the sum of 200 items = 200 * 50

= 10,000

Correct values = 192 and 88

Incorrect values = 92 and 8

Therefore, correct sum = 10,000 – 92 – 8 + 192 + 88

= 10,180

Therefore, correct mean = $\frac{10,180}{200}$

= 50.9

21. Question

Find the values of *n* and $\overline{X}$ in each of the following cases:

(i) $\sum_{i=1}^{n}(x_i - 12) = -10$ and $\sum_{i=1}^{n}(x_i - 3) = 62$

(ii) $\sum_{i=1}^{n}(x_i - 10) = 30$ and $\sum_{i=1}^{n}(x_i - 6) = 150$

Answer

(i) Given, $\sum_{i=1}^{n}(x_i - 12) = -10$

$= (x_1 - 12) + (x_2 - 12) + \ldots. + (x_n + 12) = -10$

$= (x_1 + x_2 + x_3 + x_4 + \ldots.. + x_n) + (12 + 12 + 12 + \ldots. + 12) = -10$

$= \Sigma x - 12n = -10$ (I)

And $\sum_{i=1}^{n}(x_i - 3) = 62$

$= (x_1 - 3) + (x_2 - 3) + (x_3 - 3) + \ldots. + (x_n - 3) = 62$

$= (x_1 + x_2 + x_3 + \ldots. + x_n) - (3 + 3 + 3 + \ldots. + 3) = 62$

$= \Sigma x - 3n = 62$ (II)

BY subtracting (I) from (II), we get

$\Sigma x - 3n - \Sigma x - 12n = 62 + 10$

$= 9n = 72$

$= n = \frac{72}{9}$

$= 8$

Put value of n in equation (I), we get

$\Sigma x - 12 * 8 = -10$

$= \Sigma x - 96 = -10$

$= \Sigma x = -10 + 96 = 86$

Therefore, $\overline{X} = \frac{\sum x}{n}$

$= \frac{86}{8}$

$= 10.75$

(ii) Given, $\sum_{i=1}^{n}(x_i - 10) = 30$

$= (x_1 - 10) + (x_2 - 10) + \ldots. + (x_n - 10) = 30$

$= (x_1 + x_2 + x_3 + \ldots. + x_n) - (10 + 10 + 10 + \ldots. + 10) = 30$

$= \Sigma x - 10n = 30$ (I)

And $\sum_{i=1}^{n}(x_i - 6)$ = 150

= $(x_1 - 6) + (x_2 - 6) + \ldots + (x_n - 6) = 150$

= $(x_1 + x_2 + x_3 + \ldots + x_n) - (6 + 6 + 6 + \ldots + 6) = 150$

= $\Sigma x - 6n = 150$

By subtracting (I) from (II), we get

$\Sigma x - 6n - \Sigma x - 10n = 150 - 30$

= $\Sigma x - \Sigma x + 4n = 120$

n = $\frac{120}{4}$

= 30

Put value of n in (I), we get

$\Sigma x - 10 * 30 = 30$

$\Sigma x - 300 = 30$

$\Sigma x = 30 + 300$

= 330

Therefore, $\overline{x} = \frac{\Sigma x}{n}$

= $\frac{330}{30}$

= 11

22. Question

The sums of the deviations of a set of *n* values x_1, x_2, x_3,...., $x_n$measured from 15 and -3 are -90 and 54 respectively. Find the value of *n* and mean.

Answer

Given, $\sum_{i=1}^{n}(xi + 5)$= -90

= $(x_1 - 15) + (x_2 - 15) + \ldots + (x_n - 15) = - 90$

= $(x_1 + x_2 + \ldots + x_n) - (15 + 15 + \ldots + 15) = -90$

= $\Sigma x - 15n = - 90$ (I)

And, $\sum_{i=1}^{n}(xi + 3)$ = 54

= $(x_1 + 3) + (x_2 + 3) + \ldots + (x_n + 3) = 54$

= $(x_1 + x_2 + \ldots + x_n) + (3 + 3 + \ldots + 3) = 54$

= ∑x + 3n = 54 (II)

Subtracting (I) from (II), we get

∑x + 3n - ∑x + 15n = 54 + 90

18n = 144

n = $\frac{144}{18}$

= 8

Put value of n in (I), we get

∑x – 15 * 8 = - 90

∑x – 120 = - 90

∑x = 30

Therefore, Mean = $\frac{\sum x}{n}$

= $\frac{30}{8}$

= $\frac{15}{4}$

23. Question

Find the sum of the deviations of the variable values 3, 4, 6, 7, 8, 14 from their mean.

Answer

Values are 3, 4, 6, 7, 8, 14

Therefore, Mean = $\frac{Sum\ of\ numbers}{Total\ numbers}$

= $\frac{3+4+6+7+8+14}{6}$

= 42/6

= 7

Therefore, sum of deviation of values from their mean

= (3 – 7) + (4 – 7) + (6 – 7) + (7 – 7) + (8 – 7) + (14 – 7)

= (-4) + (-3) + (-1) + (0) + (1) + (7)

= - 8 + 8

= 0

24. Question

If $\overline{X}$ is the mean of the ten natural numbers, x_1, x_2, x_3,…., x_{10}, show that

$(x_1 - \overline{X}) + (x_2 - \overline{X}) + \ldots + (x_{10} - \overline{X}) = 0$.

Answer

We have, $\overline{X} = \frac{x1+x2+\cdots.+x10}{10}$

$x_1 + x_2 + x_3 + \ldots + x_{10} = 10\overline{X}$ (i)

Now, $(x_1 - \overline{X}) + (x_2 - \overline{X}) + \ldots + (x_{10} - \overline{X})$

$= (x_1 + x_2 + \ldots + x_{10}) - (\overline{X} + \overline{X} + \ldots +$ up to 1o terms)

$= 10\ \overline{X} - 10\ \overline{X}$ [From (i)]

$= 0$

Therefore, $(x_1 - \overline{X}) + (x_2 - \overline{X}) + \ldots + (x_{10} - \overline{X}) = 0$

Hence, proved

Exercise 24.2

1. Question

Calculate the mean for the following distribution:

x :	5	6	7	8	9
f :	4	8	14	11	3

Answer

X	f	fx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	∑fx = 281

Therefore, Mean ($\overline{X}$) = $\frac{\sum fx}{N}$

= $\frac{281}{40}$

= 7.025

2. Question

Find the mean of the following data:

x:	19	21	23	25	27	29	31
f:	13	15	16	18	16	15	13

Answer

X	f	fx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	N = 106	∑fx = 2650

Therefore, Mean ($\overline{x}$) = $\frac{\sum fx}{N}$

= $\frac{2650}{106}$

= 25

3. Question

Find the mean of the following data is 20.6. Find the value of *p.*

Answer

x	f	fx
10	3	30
15	10	150
P	25	25P
25	7	175
35	5	175
	N = 50	∑fx = 25P + 530

It is given that,

Mean = 20.6

$\frac{\sum fx}{N}$ = 20.6

$\frac{25P+530}{50}$ = 20.6

$25P + 530 = 20.6\ (50)$ = 1030

$25P = 1030 - 530$

25P = 500

$P = \frac{500}{25} = 20$

Therefore, P = 20

4. Question

If the mean of the following data is 15. Find *p.*

$x:$	5	10	15	20	25
$f:$	6	p	6	10	5

Answer

X	f	fx
5	6	30
10	p	10p
15	6	90
20	10	200
25	5	125
	N = p + 27	∑fx = 10p + 445

Given, mean = 15

$\frac{\sum fx}{N}$ = 15

$\frac{10p+445}{p+27}$ = 15

$10p + 445 = 15p + 405$

15p – 10p = 445 – 405

5p = 40

p = $\frac{40}{5}$

= 8

5. Question

Find the value of p for the following distribution whose mean is 16.6.

x:	8	12	15	p	20	25	30
f:	12	16	20	24	16	8	4

Answer

x	f	fx
8	12	96
12	16	192
15	20	300
p	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	∑fx = 24p + 1225

Given, mean = 16.6

$\frac{\sum fx}{N}$ = 16.6

$\frac{24p+1228}{100}$ = 16.6

$24p = 1660 - 1220$

24p = 432

p = $\frac{432}{24}$ = 18

6. Question

Find the missing value of *p*, for the following distribution whose mean is 12.58.

x:	5	8	10	12	p	20	25
f:	2	5	8	22	7	4	2

Answer

X	f	fx
5	2	10
8	5	40
10	8	80
12	22	264
p	7	7p
20	4	80
25	2	50
	N = 50	∑fx = 7p + 524

Given, mean = 12.58

$\frac{\sum fx}{N}$ = 12.58

$\frac{7p+524}{50}$ = 12.58

$7p + 524 = 629$

7p = 629 – 524

7p = 105

p = $\frac{105}{7}$

= 15

7. Question

Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
f:	6	8	15	p	8	4

Answer

x	f	fx
3	6	18
5	8	40
7	15	105
9	P	9P
11	8	88
13	4	52
	N = P + 41	∑fx = 9P + 303

Given, mean = 7.68

$\frac{\sum fx}{N}$ = 7.68

$\frac{9P+303}{P+41}$ = 7.68

$9P + 303 = 7.68P + 314.88$

9P – 7.68P = 314.88 – 303

1.32P = 11.88

$P = \frac{11.88}{1.32}$

P = 9

8. Question

Find the value of *p*, if the mean of the following distribution is 20.

x:	15	17	19	$20+p$	23
f:	2	3	4	$5p$	6

Answer

X	f	fx
15	2	30
17	3	51
19	4	76
20 + p	5p	$100p + 5p^2$
23	6	138
	$\sum f = 15 + 5p$	$\sum fx = 295 + 100p + 5p^2$

Mean, $\overline{x} = \frac{\sum fx}{\sum f}$

$20 = \frac{29+20+p*p}{3+p}$

$60 + 20p = 59 + 20p + p^2$

$p^2 - 1 = 0$

$p = 1$

9. Question

Find the mean of the following distribution.

x:	10	12	20	25	35
f:	3	10	15	7	5

Answer

x	f	fx
10	3	30
12	10	120
20	15	300
25	7	175
35	5	175
	N = 40	∑fx = 800

Therefore, mean ($\overline{x}$) = $\frac{\sum fx}{N}$

= $\frac{800}{40}$

= 20

Therefore, $\overline{x}$ = 20

10. Question

Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	Not available	55
IV	40	50

If the average score of the candidates of all the four school is 66, find the number of candidates that appeared from school III.

Answer

Let no. of students appeared from school III = x

Schools	No. of candidates	Average score
I	60	75
II	48	80
III	x	55
IV	40	50

Given, Average score of all schools = 66

$\frac{N1\bar{x}1+N2\bar{x}2+N3\bar{x}3+N4\bar{x}4}{N1+N2+N3+N4}$ = 66

$\frac{60*75+48*80+x*55+40*50}{60+48+x+40}$ = 66

$\frac{4500+3840+55x+2000}{148+x}$ = 66

$\frac{10340+55x}{148+x}$ = 66

$10340 + 55x = 66x + 9768$

10340 – 9768 = 66x – 55x

11x = 572

x = $\frac{572}{11}$

= 52

Therefore, no. of candidates appeared from school (III) are 52

11. Question

Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in

the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Answer

No. of heads per toss (x)	No. of tosses (f)	fx
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125
	N = 1000	∑fx = 2470

Therefore, mean number of heads per toss = $\frac{\sum fx}{N}$

= $\frac{2470}{1000}$

= 2.47

12. Question

Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 50.

x:	10	30	50	70	*90*	
f :	17	f_1	32	f_2	19	Total 120.

Answer

x	f	fx
10	17	170
30	f_1	$30f_1$
50	32	1600
70	f_2	$70f_2$
90	19	1710
	N = 120	$\sum fx = 3480 + 30f_1 + 70f_2$

It is given that, mean = 50

$\frac{\sum fx}{N}$ = 50

$\frac{3480+30f1+70f2}{N}$ = 50

3480 + 30f_1 + 70f_2 = 50 (120)

30f_1 + 70f_2 = 6000 – 3480

10 (3f_1 + 7f_2) = 10 (252)

3f_1 + 7f_2 = 252 (i) [Dividing by 10]

And N = 120

17 + f_1 + 32 + f_2 + 19 = 120

68 + f_1 + f_2 = 120

f_1 + f_2 = 52

Multiplying by '3' on both sides, we get

3f_1 + 3f_2 = 156 (ii)

Subtracting (ii) from (i), we get

3f_1 + 7f_2 – 3f_1 – 3f_2 = 252 – 156

4f_2 = 96

f_2 = $\frac{96}{4}$

= 24

Put value of f_2 in (i), we get

3f_1 + 7 * 24 = 252

3f_1 = 252 – 168

3f_1 = 84

f_1 = $\frac{84}{3}$

= 28

Exercise 24.3

1. Question

Find the median of the following data

83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Answer

Given numbers are:

83, 37, 70, 29, 45, 63, 41, 70, 34, 54

Arrange the numbers in ascending order:

29, 34, 37, 41, 45, 54, 63, 70, 70, 83

n = 10 (even)

Therefore, Median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

= $\frac{\frac{10}{2}th\ value+\left(\frac{10}{2}+1\right)th\ value}{2}$

= $\frac{5th\ value+6th\ value}{2}$

= $\frac{45+54}{2}$

= $\frac{99}{2}$ = 49.5

2. Question

Find the median of the following data

133, 73, 89, 108, 94, 104, 94, 85, 100, 120

Answer

Given numbers are:

133, 73, 89, 108, 94, 104, 94, 85, 100, 120

Arrange in ascending order:

78, 85, 89, 94, 94, 100, 104, 108, 120, 133

n = 10 (even)

Therefore, median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

= $\frac{\frac{10}{2}th\ value+\left(\frac{10}{2}+1\right)th\ value}{2}$

= $\frac{5th\ value+6th\ value}{2}$

= $\frac{90+104}{2}$ = 9.7

3. Question

Find the median of the following data

31, 38, 27, 28, 36, 25, 35, 40

Answer

Given numbers are:

31, 38, 27, 28, 36, 25, 35, 40

Arranging in increasing order:

25, 27, 28, 31, 35, 36, 38, 40

n = 8 (even)

Therefore, Median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

= $\frac{\frac{8}{2}th\ value+\left(\frac{8}{2}+1\right)th\ value}{2}$

= $\frac{4th\ value+5th\ value}{2}$

= $\frac{31+35}{2}$

= $\frac{66}{2}$ = 33

4. Question

Find the median of the following data

15, 6, 16, 8, 22, 21, 9, 18, 25

Answer

Given, numbers are 15, 6, 16, 8, 22, 21, 9, 18, 25

Arrange in increasing order:

6, 8, 9, 15, 16, 18, 21, 22, 25

n = 9 (odd)

Therefore, median = $\frac{n+1}{2}$ value

= $\frac{9+1}{2}$ value

= 5^{th} value

= 16

5. Question

Find the median of the following data

41, 43, 127, 99, 71, 92, 71, 58, 57

Answer

Given numbers are:

41, 43, 127, 99, 71, 92, 71, 58, 57

Arrange in increasing order:

41, 43, 57, 58, 71, 71, 92, 99, 127

n = 9 (odd)

Therefore, median = $\frac{n+1}{2}$ value

= $\frac{9+1}{2}$ value

= 5^{th} value = 71

6. Question

Find the median of the following data

25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Answer

Given numbers are:

25, 34, 31, 23, 22, 26, 35, 29, 20, 32

Arrange in increasing order:

20, 22, 23, 25, 26, 29, 31, 32, 34, 35

n = 10 (even)

Therefore, median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

= $\frac{\frac{10}{2}th\ value+\left(\frac{10}{2}+1\right)th\ value}{2}$

= $\frac{5th\ value+6th\ value}{2}$

= $\frac{26+29}{2}$

= $\frac{55}{2}$

7. Question

Find the median of the following data

12, 17, 3, 14, 5, 8, 7, 15

Answer

Given, numbers are:

12, 17, 3, 14, 5, 8, 7, 15

Arrange in increasing order:

3, 5, 7, 8, 12, 14, 15, 17

n = 8 (even)

Therefore, median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

$= \frac{\frac{8}{2}th\ value+\left(\frac{8}{2}+1\right)th\ value}{2}$

$= \frac{4th\ value+5th\ value}{2}$

$= \frac{8+12}{2} = 10$

8. Question

Find the median of the following data

92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Answer

Given, numbers are:

92, 35, 67, 85, 72, 81, 56, 51, 42, 69

Arrange in increasing order:

35, 42, 51, 56, 67, 69, 72, 81, 85, 92

n = 10 (even)

Therefore, median = $\frac{\frac{n}{2}th\ value+\left(\frac{n}{2}+1\right)th\ value}{2}$

$= \frac{\frac{10}{2}th\ value+\left(\frac{10}{2}+1\right)th\ value}{2}$

$= \frac{5th\ value+6th\ value}{2}$

$= \frac{67+69}{2} = \frac{136}{2}$

= 68

9. Question

Numbers 50, 42, 35, 2x+10, 2x-8, 12, 11, 8 are written in descending order and their median is 25, find x.

Answer

Given, no. of observations, n = 8

Median = $\frac{\frac{n}{2}th\ observation+\left(\frac{n}{2}+1\right)th\ observation}{2}$

= $\frac{2x+10+2x-8}{2}$

= 2x + 1

Given, median = 25

2x + 1 = 25

2x = 24

x = 12

10. Question

Find the median of the following observations: 46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33. If 92 is replaced by 99 and 41 by 43 in the above data, find the new median?

Answer

Given, numbers are:

46, 64, 87, 41, 58, 77, 35, 90, 55, 92, 33

Arrange in increasing order:

33, 35, 41, 46, 55, 58, 64, 77, 87, 90, 92

n = 11 (odd)

Therefore, median = $\frac{n+1}{2}$

= $\frac{11+1}{2}$

= $\frac{12}{2}$ = 6^{th} value

= 58

If 92 is replaced by 99 and 41 by 43, then the new values are:

33, 35, 43, 46, 55, 58, 64, 77, 81, 90, 99

Therefore, n = 11 (odd)

New median = $\frac{n+1}{2}$ value

= $\frac{11+1}{2}$

= 6^{th} value = 58

11. Question

Find the median of the following data: 41, 43, 127, 99, 61, 92, 71, 58, 57 if 58 replaced by 85, what will be the new median.

Answer

Given, numbers are:

41, 43, 127, 99, 61, 92, 71, 58, 57

Arrange in ascending order:

41, 43, 57, 58, 61, 71, 92, 99, 127

n = 9 (odd)

Therefore, median = $\frac{n+1}{2}$ value

= $\frac{9+1}{2}$

= 5^{th} value = 61

If 58 is replaced by 85

Then the new value be in order:

41, 43, 57, 61, 71, 85, 92, 99, 127

n = 9 (odd)

Therefore, median = $\frac{n+1}{2}$

= $\frac{9+1}{2}$

= 5^{th} value = 71

12. Question

The weights (in kg) of 15 students are: 31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30. Find the median. If the weight 44 kg is replaced by 46 kg and 27 kg by 25 kg, find the new median.

Answer

Given, numbers are:

31, 35, 27, 29, 32, 43, 37, 41, 34, 28, 36, 44, 45, 42, 30

Arrange in increasing order:

27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 44, 45

n = 15 (odd)

Therefore, median = $\frac{n+1}{2}$ value

= $\frac{15+1}{2}$

= 8^{th} value = 35 kg

If the weight 44kg is replaced by 46 kg and 27 kg is replaced by 25 kg

Then new values in order be

25, 28, 29, 30, 31, 32, 34, 35, 36, 37, 41, 42, 43, 45, 46

n = 15 (odd)

Therefore, new median = $\frac{n+1}{2}$

= $\frac{15+1}{2}$

= 8^{th} value = 35kg

13. Question

The following observations have been arranged in ascending order. If the median of the data is 63, find the value of *x*.

29, 32, 48, 50, *x*, *x*+2, 72, 78, 84, 95

Answer

Total number of observation in the given data is 10 (even number). So median of this data will be mean of $\frac{10}{2}$ i.e., 5^{th} and $\frac{10}{2}$ + 1 i.e. 6^{th} observations.

So, median of data = $\frac{5th\ observations+6th\ observations}{2}$

63 = $\frac{x+x+2}{2}$

63 = $\frac{2x+2}{2}$

63 = x + 1

x = 62

Exercise 24.4

1. Question

Find out the mode of the following marks obtained by 15 students in a class:

Marks: 4, 6, 5, 7, 9, 8, 10, 4, 7, 6, 5, 9, 8, 7, 7.

Answer

Marks	4	5	6	7	8	9	10
No. of students	2	2	2	4	2	2	1

Since, the maximum frequency corresponds to the value 7,then mode = 7 marks.

2. Question

Find the mode from the following data:

125, 175, 225, 125, 225, 175, 325, 125, 375, 225, 125

Answer

Values	125	175	225	325	375
Frequency	4	2	3	1	1

Since, maximum frequency 4 corresponds to 125, then mode = 125

3. Question

Find the mode for the following serried:

7.5, 7.3m 7.2m 7.2, 7.4, 7.7, 7.7, 7.5, 7.3, 7.2, 7.6, 7.2

Answer

Values	7.2	7.3	7.4	7.5	7.6	7.7
Frequency	4	2	1	2	1	2

Since, maximum frequency 4 corresponds to value 7.2, then mode = 7.2

4. Question

Find the mode of the following data in each case:

(i) 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18

(ii) 7, 9, 12, 13, 7, 12, 15, 7, 12, 7, 25, 18, 7

Answer

(i) Arranging the data in ascending order:

14, 14, 14, 14, 17, 18, 18, 18, 22, 23, 25, 28

Here observation 14 is having the highest frequency.

So, mode = 14

(ii)

Values	7	9	12	13	15	18	25
Frequency	5	1	3	1	1	1	1

Since, maximum frequency 5 corresponds to value 7.

Then, the mode = 7

5. Question

The demand of different shirt sizes, as obtained by a survey, is given below:

Size	38	39	40	41	42	43	44
Number of persons (wearing it) :	26	39	20	15	13	7	5
Total = 125							

Find the modal shirt sizes, as observed from the survey.

Answer

Size	38	39	40	41	42	43	44	Total
No. of persons	26	39	20	15	13	7	5	125

Since, maximum frequency 39 corresponds to value 39.

Then, mode = 39

CCE - Formative Assessment

1. Question

If the ratio of mean and median of a certain data is 2:3, then find the ratio of its mode and mean.

Answer

Using empirical formula

Mode = 3 median - 2 mean

Mode = 3 * (3x) - 2(2x)

Mode = 9x - 4x = 5x

Mode: Mean = 5x: 2x = 5: 2

2. Question

If the ratio of mode and median of a certain data is 6 : 5, then find the ratio of its mean and median.

Answer

Mode = 3 Median – 2 Mean

$\frac{Mode}{Median} = \frac{6}{5}$

Mode = $\frac{6\ median}{5}$

$\frac{6\ median}{5}$ = 3 Median - 2 Mean

2 Mean = 3 Median - $\frac{6\ median}{5}$

2 Mean = $\frac{9}{5\ median}$

Mean = $\frac{9}{10\ median}$

Therefore, $\frac{Mean}{Median}$ = 9: 10

3. Question

If the mean of $x+2$, $2x+3$, $3x+4$, $4x+5$ is $x+2$, find x.

Answer

Given that,

$x + 2 = \frac{x+2+2x+3+3x+4+4x+5+x+2}{5}$

4x + 8 = 10x + 14

-6x = 6

x= -1

4. Question

The arithmetic mean and mode of a data are 24 and 12 respectively, then find the median of the data.

Answer

3 Median = Mode + 2 Mean

3 Median = 12 + 2 (24)

3 Median = 12 + 48

3 Median = 60

Median = 20

5. Question

If the difference of mode and median of a data is 24, then find the difference of median and mean.

Answer

Mode - Median = 24

Mode = Median + 24 (1)

Since, by the formula

Mode = 3 Median - 2 Mean (2)

From (1)

Median + 24 = 3 Median - 2 Mean

2 Median - 2 Mean = 24

Dividing the whole equation by 2, we get

Median - Mean = 12

6. Question

If the median of scores $\frac{x}{2}, \frac{x}{3}, \frac{x}{4}, \frac{x}{5}$ and $\frac{x}{6}$ (where $x>0$) is 6, then find the value of $\frac{x}{6}$.

Answer

We know,

Median = $\frac{n+1}{2}$ term

= $\frac{5+1}{2}$ term

= 3rd term

Therefore, $\frac{x}{4}$ = 6

x = 24

Hence, $\frac{x}{6} = \frac{24}{6}$

= 4

7. Question

If the mean of 2, 4, 6, 8, x, y is 5, then find the value of $x + y$.

Answer

Mean = $\frac{Sum\ of\ terms}{Total\ terms}$

5 = $\frac{(2+4+6+8+x+y)}{6}$

30 = 20 + x+ y

x + y = 10

8. Question

If the mode of scores 3, 4, 3, 5, 4, 6, 6, x is 4, find the value of x.

Answer

Since, the value of mode is 4.

Hence, x = 4

9. Question

If the median of 33m 28, 20, 25, 34, x is 29, find the maximum possible value of x.

Answer

There are 6 observations i.e. even no. of observations

So, median will be:

$\frac{(3rd\ observation+4th\ observation)}{2}$

Arranging in increasing order:

20, 25, 28, x, 33, 34

If we put any other number at 4^{th} position the n, we will not get the median = 29

So, $\frac{28+x}{2}$ = 29

28 + x = 58

x = 58 – 28

= 30

10. Question

If the median of the scores 1, 2, *x*, 4, 5 (where 1<2<*x*<4<5) is 3, then find the mean of the scores.

Answer

Since, the number of terms are 5

Hence, the Median will be the 6th term which is x

x = 3

Mean = $\frac{(1+2+3+4+5)}{5}$

= $\frac{15}{5}$

= 3

1. Question

Which one of the following is not a measure of central value?

A. Mean

B. Range

C. Median

D. Mode

Answer

Since range is the difference between the lowest and highest values.

2. Question

The mean of *n* observations is $\overline{X}$. If *k* is added to each observation, then the new mean is

A. $\overline{X}$

B. $\overline{X}$ +*k*

C. $\overline{X}$ -*k*

D. *k* $\overline{X}$

Answer

Mean = (x_1 + x_2 + ...+ n terms)/N

= x_1 + k + x_2 + k +...+n terms/N

= Mean + k

= $\overline{X}$ + k

3. Question

The mean of *n* observations is $\overline{X}$. If each observation is multiplied by *k*, the mean of new observations is

A. *k* $\overline{X}$

B. $\frac{\overline{X}}{k}$

C. $\overline{X}$ +*k*

D. $\overline{X}$ -*k*

Answer

Mean = $\frac{(x1+x2+\cdots.+n\ terms)}{N}$

= $\frac{kx1+kx2+\cdots.+n\ terms}{N}$

= k$\frac{(x1+x2+\cdots.+n\ terms)}{N}$

= k (Mean)

= *k* $\overline{X}$

4. Question

The mean of a set of seven numbers is 81. If one of the numbers is discarded, the mean of the remaining numbers is 78. The value if discarded number is

A. 98

B. 99

C. 100

D. 101

Answer

Let the discarded number be x,

$\frac{6\ (78)+x}{7}$ = 81

(468 + x) = 567

x = 567 - 468

= 99

5. Question

For which set of number do the mean, median and mode all have the same value?

A. 2, 2, 2, 2, 4

B. 1, 3, 3, 3, 5

C. 1, 1, 2, 5, 6

D. 1, 1, 1, 2, 5

Answer

1, 3, 3, 3, 5

Mean = $\frac{(1+3+3+3+5)}{5}$

= $\frac{15}{5}$

= 3

Median = 3rd term

= 3

Mode = 3 (The highest occurring number)

6. Question

For the set of numbers 2, 2, 4, 5 and 12, which of the following statements is true?

A. Mean = Median

B. Mean>Mode

C. Mean<Mode

D. Mode = Median

Answer

Mean = $\frac{(2+2+4+5+12)}{5}$

= $\frac{25}{5}$

= 5

Mode = 2

Hence, the statement ‘Mean>Mode’ is correct.

7. Question

If the arithmetic mean of 7, 5, 13, *x* and 9 is 10, then the value of *x* is

A. 10

B. 12

C. 14

D. 16

Answer

Mean = $\frac{(7+5+13+x+9)}{5}$

10 = $\frac{(34+x)}{5}$

50 - 34 = x

x = 16

8. Question

If the mean of five observations *x*, *x*+2, *x*+4, *x*+6, *x*+8, is 11, then the mean of first three observations is

A. 9

B. 11

C. 13

D. none of these

Answer

First, you need to solve for x:

$\frac{(x+x+2+x+4+x+6+x+8)}{5}$ = 11

x + x + 2 + x + 4 + x + 6 + x + 8 = 55

5x + 20 = 55

5x = 35

x = 7

Now, substitute for x in each of the last three terms:

x + 4 + x + 6 + x + 8

(7 + 4) + (7 + 6) + (7 + 8)

(11) + (13) + (15)

$\frac{39}{3}$ = 13

The mean is 13

9. Question

Mode is

A. Least frequent value

B. Middle most value

C. Most frequent value

D. None of these

Answer

Mode is the value that occurs most frequently in a given set of data.

10. Question

The following is the data of wages per day: 5, 4, 7, 5, 8, 8, 8, 5, 7, 9, 5, 7, 9, 10, 8. The mode of the data is

A. 7

B. 5

C. 8

D. 10

Answer

The mode of the data is 8 as it occurs the maximum times.

11. Question

The empirical relation between mean, mode and median is

A. Mode = 3 Median – 2 Mean

B. Mode = 2 Median – 3 Mean

C. Median = 3 Mode – 2 Mean

D. Mean = 3 Median – 2 Mode

Answer

This is an approximate relation that holds when the distribution is symmetrical or moderately skewed. It does not hold when the distribution is too skewed. When the distribution is symmetric, this relation holds exactly because in that case, mean = median = mode

12. Question

The median of the following data: 0, 2, 2, 2, -3, 5, -1, 5, 5, -3, 6, 6, 5, 6 is

A. 0

B. -1.5

C. 2

D. 3.5

Answer

D) 3.5First of all we arrange the given data in increasing order.-3, -3, -1, 0, 2, 2, 2, 5, 5, 5, 5, 6, 6, 6Now we count the no. of observations i.e. 14 which is even.So applying Median formula for even no. of observation we get,median = $\frac{(\frac{n}{2})^{th} + (\frac{n}{2}+1)^{th}}{2}$

where, x^{th} represent x^{th} term.

median = $\frac{\frac{14}{2}^{th} + (\frac{14}{2}+1)^{th}}{2}$

median = $\frac{7^{th}+8^{th}}{2} = \frac{2+5}{2}$

median = 3.5

13. Question

The mean of a, b, c, d and e is 28. If the mean of a, c and e is 24, what is the mean of b and d?

A. 31

B. 32

C. 33

D. 34

Answer

Given: The mean of a, b, c, d, and e is 28 and the mean of a, c and e are 24.

To find: the mean of b and d.

Solution: Mean = $\frac{(a+b+c+d+e)}{5}$ = 28

⇒ a + b + c + d + e = 28 × 5 = 140

Also,$\frac{(a+c+e)}{3}$ = 24

⇒ a + c + e = 72

⇒ a + b + c + d + e – a – c – e = 140 - 72

⇒ b + d = 68

Therefore, mean = $\frac{(b+d)}{2} = \frac{68}{2} = 34$

14. Question

The algebraic sum of the deviations of a set of *n* values from their mean is

A. 0

B. *n*-1

C. *n*

D. *n*+1

Answer

Mean = $\bar{x} = \frac{Sum\ of\ all\ quantities}{Total}$

$\bar{x} = \frac{x1+x2+x3+\cdots.+xn}{n}$

$x_1 + x_2 + x_3 + \ldots. + x_n = n\bar{x}$ (i)

Algebraic sum of the deviation from the mean is:

Sum = $(x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + \ldots. + (x_n - \bar{x})$

= $x_1 + x_2 + x_3 + \ldots. + x_n - (\bar{x} + \bar{x} + \bar{x} + \ldots.$ n times)

= $x_1 + x_2 + x_3 + \ldots. + x_n - n\bar{x}$

Using (i), we get

Sum = $n\bar{x} - n\bar{x}$

= 0

Hence, algebraic sum of the deviations of a set of n values from mean is 0.

15. Question

A, B, C are three sets of values of *x*:

A: 2, 3, 7, 1, 3, 2, 3

B: 7, 5, 9, 12, 5, 3, 8

C: 4, 4, 11, 7, 2, 3, 4

Which one of the following statements is correct?

A. Mean of *A* =Mode of *C*

B. Mean of *C* = Median of *B*

C. Median of *B* = Mode of *A*

D. Mean, Median and Mode of *A* are equal.

Answer

A: 2, 3, 7, 1, 3, 2, 3

A: 1, 2, 2, 3, 3, 3, 7

Mode = 3 (occurs maximum number of times)

Median = 3 (the middle term)

Mean = $\frac{(1+2+2+3+3+3+7)}{7}$

$= \frac{21}{7}$

= 3

Hence, option (d) is true.