21. Surface Area and Volume of a Sphere

Exercise 21.1

1. Question

Find the Find the surface area of a sphere of radius :

(i) 10.5 cm

(ii) 5.6 cm

(iii) 14 cm

Answer

Surface area of a shere = $4\pi r^2$, where r is radius

(i) r is 10.5 cm

$\Rightarrow$ surface area = $4 \times (22/7) \times (10.5)^2 = 1386\ cm^2$

(ii) r is 5.6 cm

$\Rightarrow$ surface area = $4 \times (22/7) \times 5.6^2 = 394.24\ cm^2$

(iii) r is 14 cm

$\Rightarrow$ surface area = $4 \times (22/7) \times 14^2 = 2464\ cm^2$

2. Question

Find the surface area of a sphere of diameter :

(i) 14 cm

(ii) 21 cm

(iii) 3.5 cm

Answer

Surface area of a sphere of diameter ‘d’ = πd^2

(i) d is 14 cm

$\Rightarrow$ surface area = $(22/7) \times (14)^2 = 616\ cm^2$

(ii) d is 21 cm

$\Rightarrow$ surface area = $(22/7) \times (21)^2 = 1386\ cm^2$

(iii) d is 3.5 cm

$\Rightarrow$ surface area = $(22/7) \times (3.5)^2 = 38.5 cm^2$

3. Question

Find the total surface area of a hemisphere and a solid hemisphere each of radius 10 cm. (Use π = 3.14)

Answer

We have,

Radius = 10 cm

Total surface area of a hemisphere = $2\pi r^2$

$\Rightarrow$ Total surface area of a hemisphere = $2 \times 3.14 \times 10 \times 10 = 628\ cm^2$

Total surface area of a solid hemisphere = $3\pi r^2$

⇒ Total surface area of a solid hemisphere = 3 × 3.14 × 10 × 10 = 942 cm^2

4. Question

The surface area of a sphere is 5544 cm^2, find its diameter.

Answer

Let the radius of the sphere be *r* cm.

We know that, surface area of a sphere = $4\pi r^2$

Given, surface area of a sphere is 5544 cm^2

⇒ $4\pi r^2 = 5544$

⇒ $4 \times \frac{22}{7} \times r^2 = 5544$

⇒ $r^2 = 441$

⇒ r = 21 cm

Thus diameter = 42 cm

5. Question

A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of the plating it on the inside at the rate of Rs. 4 per 100 cm^2.

Answer

Surface area of a hemisphere = $2\pi r^2$

Given, diameter of the hemisphere bowl is 10.5 cm

Surface area of the bowl = 2 × (22/7) × $(10.5/2)^2$ = 173.25 cm^2

Given, cost of the plating it on the inside at the rate of Rs. 4 per 100 cm^2

Cost of plating the hemisphere bowl $= \frac{4}{100} \times 173.25 = Rs.\ 6.93$

6. Question

The dome of a building is in the form of a hemisphere. Its radius is 63 dm. Find the cost of painting it at the rate of Rs. 2 per sq. m.

Answer

Surface area of a hemisphere = $2\pi r^2$

Given, dome of a building is in the form of a hemisphere. Its radius is 63 dm.

1 dm = 0.1m

Thus, 63 dm = 6.3 m

Surface area of the dome = 2 × (22/7) × $(6.3)^2$ = 249.48 m^2

Cost of painiting it at Rs. 2 per sq. m. = 249.48 × 2 = Rs. 498.96

7. Question

Assuming the earth to be a sphere of radius 6370 km, how many square kilometres is area of the land, if three-fourth of the earth's surface is covered by water?

Answer

Surface area of a spere = $4\pi r^2$

Given, earth is a sphere of radius 6370 km.

Surface area of earth = $4 \times (22/7) \times 6370^2$

$\Rightarrow$ Surface area of earth = $510109600\ km^2$

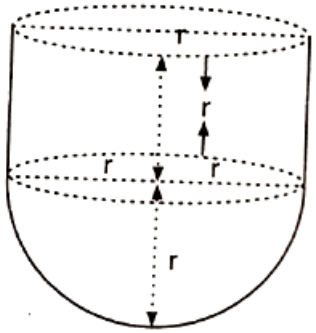
Now, three-fourth of the earth's surface is covered by water

Area covered by land = $1/4 \times 510109600 = 127527400\ km^2$

8. Question

A cylinder of same height and radius is placed on the top of a hemisphere. Find the curved surface area of the shape if the length of the shape is 7 cm.

Answer



Given, cylinder of same height and radius is placed on the top of a hemisphere.

Also, length of the shape is 7 cm

$\Rightarrow r + r = 7$

$\Rightarrow r = 3.5$ cm

Curved surface area of a hemisphere = $2\pi r^2$

Curved surface area of a cylinder = $2\pi r^2$

Total surface area of the shape = $4\pi r^2$

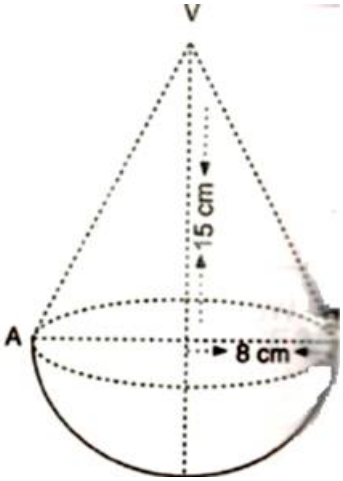
$\Rightarrow$ Total surface area of the shape = $4 \times (22/7) \times (3.5)^2$

$\Rightarrow$ Total surface area of the shape = $154\ cm^2$

9. Question

A wooden toy is in the form of a cone surmounted on a hemisphere. The diameter of the base of the cone is 16 cm and its height is 15 cm. Find the cost of painting the toy at Rs. 7 per 100 cm^2.

Answer



From the figure,

Radius of the hemisphere = 8 cm

Height of the cone = 15 cm

Lateral length of the cone = $\sqrt{(h^2 + r^2)} = \sqrt{(15^2 + 8^2)} = 17$ cm

Curved surface area of hemisphere = $2\pi r^2$

Curved surface area of the cone = πrl

Total curved surface area of the toy = $2\pi r^2 + \pi rl$

$\Rightarrow$ Total curved surface area of the toy = $2 \times (22/7) \times 8^2 + (22/7) \times 8 \times 17$

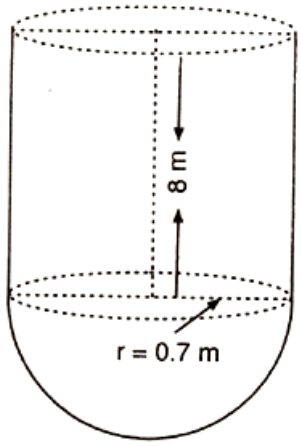
$\Rightarrow$ Total curved surface area of the toy = 829.714 cm^2

Cost of painting the toy at Rs. 7 per 100 $cm^2 = \frac{829.714}{100} \times 7 = Rs.\ 58.08$

10. Question

A storage tank consists of a circular cylinder with a hemisphere adjoined on either end. If the external diameter of the cylinder be 1.4 m and its length be 8 m, find the cost of painting it on the outside at the rate of Rs. 10 per m^2.

Answer



Curved surface area of a cylinder = $2\pi rh$

Curved surface area of hemisphere = $2\pi r^2$

Given, external diameter of the cylinder be 1.4 m and its length be 8 m

Thus r = 0.7m and h = 8m

Total curved surface area = $2\pi rh + 2\pi r^2$

$\Rightarrow$ Total curved surface area = $2 \times (22/7) \times 0.7 \times 8 + 2 \times (22/7) \times 0.7^2 = 38.28\ m^2$

Cost of painting it on the outside at the rate of Rs. 10 per m^2 = 38.28 × 10 = Rs. 382.80

11. Question

The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Answer

Surface area of a shephe = $4\pi r^2$

Ratio of surface areas of spheres = square of ratio of their radius

Given, diameter of the moon is approximately one fourth of the diameter of the earth

$\therefore r_m = 1/4 \times r_e$

$\Rightarrow r_m : r_e = 1 : 4$

Ratio of their surface area = 1 : 16

12. Question

A hemi-spherical dome of a building needs to be painted. IF the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is Rs. 5 per 100 cm^2.

Answer

Circumference of a circle = $2\pi r$

Surface area of a hemisphere = $2\pi r^2$

Given, base of the dome is 17.16 m

$\Rightarrow 2 \times (22/7) \times r = 17.6$

$\Rightarrow r = 2.8$ m

Surface area of the hemisphere = $2 \times (22/7) \times 2.8^2 = 49.28\ m^2 = 492800\ cm^2$

Cost of painting is Rs. 5 per 100 cm^2,

Cost of painting the dome = $(492800/100) \times 5$ = Rs. 24640

13. Question

The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in Fig. 21.11. Eight such spheres are used for this purpose, and are to be painted silver. Each support is a cylinder of a radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint cost 5 paise per cm^2.

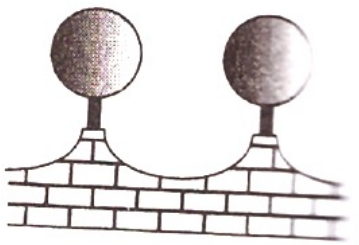


Fig. 21.11

Answer

Surface area of a sphere = $4\pi r^2$

Curved surface area of a cylinder = $2\pi rh$

Base area of a cylinder = πr^2

Given, front compound wall of a house is decorated by wooden spheres of diameter 21 cm placed on small supports.

Each support is a cylinder of a radius 1.5 cm and height 7 cm and is to be painted black

Thus, the surface area of the sphere would be reduced by the cylindrical supports base.

Total surface area of the spherers = $8 \times (4 \times (22/7) \times (21/2)^2 - (22/7) \times 1.5^2) = 11031.43\ cm^2$

Total curved surface area of the spheres = $8 \times 2 \times (22/7) \times 1.5 \times 7 = 528\ cm^2$

Silver paint costs 25 paise per cm^2 and black paint cost 5 paise per cm^2

Total cost of painting = $11031.43 \times 0.25 + 528 \times 0.05$ = Rs. 2784.26

Exercise 21.2

1. Question

Find the volume of a sphere whose radius is :

(i) 2 cm

(ii) 3.5 cm

(iii) 10.5 cm

Answer

Volume of a sphere = $(4/3)\pi r^3$

(i) radius is 2 cm

⇒ Volume of the sphere = $(4/3) \times (22/7) \times 2^3 = 33.52\ cm^3$

(ii) radius is 3.5 cm

⇒ Volume of the sphere = $(4/3) \times (22/7) \times 3.5^3 = 179.67\ cm^3$

(iii) radius is 10.5 cm

⇒ Volume of the sphere = $(4/3) \times (22/7) \times (10.5)^3 = 4851\ cm^3$

2. Question

Find the volume of a sphere whose diameter is:

(i) 14 cm

(ii) 3.5 dm

(iii) 2.1 m

Answer

Volume of sphere = $(1/6)\pi d^3$

(i) Diamter is 14 cm

⇒ Volume of the sphere = $(1/6) \times (22/7) \times 14^3 = 1437.33\ cm^3$

(ii) Diamter is 3.5 dm = 35 cm

⇒ Volume of the sphere = $(1/6) \times (22/7) \times 35^3 = 22.46\ dm^3$

(iii) Diameter is 2.1 m

⇒ Volume of the sphere = $(1/6) \times (22/7) \times 2.1^3 = 4.851\ m^3$

3. Question

A hemispherical tank has inner radius of 2.8 m. Find its capacity in litres.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Given, hemispherical tank has inner radius of 2.8 m

⇒ Volume of the tank = $(2/3) \times (22/7) \times 2.8^3$

⇒ Volume of the tank = $45.976\ m^3$ = 45976 litres

4. Question

A hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm. Find the volume of steel used in making the bowl.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Given, hemispherical bowl is made of steel 0.25 cm thick. The inside radius of the bowl is 5 cm.

Outer radius = 5 + 0.25 cm = 5.25 cm

Volume of steel used in the making of the bowl $= \frac{2}{3} \times \frac{22}{7} \times (5.25^3 - 5^3)\ cm^3$

⇒ Volume of steel used in the making of the bowl = 41.28 cm^3

5. Question

How many bullet can be made out of a cube of lead, whose edge measures 22 cm, each bullet being 2 cm in diameter?

Answer

Volume of a cube = $side^3$

Volume of a sphere = $(4/3)\pi r^3$

Given, edge of a cube is 22 cm and the bullet are of 2 cm diameter

Thus, volume of the cube = 22 × 22 × 22 = 10648 cm^3

Volume of each bullet = (4/3) × (22/7) × 1^3 = 4.19 cm^3

No. of bullet that can be made out of the cube of lead = 10648/4.19 = 2541

6. Question

A shopkeeper has one laddoo of radius 5 cm. With the same material, how many laddoos of radius 2.5 cm can be made.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Given, shopkeeper has one laddoo of radius 5 cm

Volume of the laddoo = (4/3) × (22/7) × 5^3

Now, from this one laddoo, laddoos of radius 2.5 cm are to be made.

Volume of laddoo of radius 2.5cm = (4/3) × (22/7) × 2.5^3

∴ No. of laddoos of radius 2.5 cm that can be made $= \frac{\left(\frac{4}{3} \times \frac{22}{7} \times 5^3\right)}{\frac{4}{3} \times \frac{22}{7} \times 2.5^3} = 8$

7. Question

A spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls. If the diameters of two balls be $\frac{3}{2}$cm and 2 cm, find the diameter of the third ball.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Total volume remains same during recasting.

Given, spherical ball of lead 3 cm in diameter is melted and recast into three spherical balls and the diameters of two balls are $\frac{3}{2}$cm and 2 cm.

Let the diameter of the third ball be ‘a’ cm.

$$\therefore \frac{4}{3} \times \pi \times \left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi \times \left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi \times 1^3 + \frac{4}{3}\pi \times \left(\frac{a}{2}\right)^3$$

⇒ 27/8 = (27/64) + 1 + $(a^3/8)$

⇒ $a^3/8 = 125/64$

⇒ $a^3 = 125/8$

⇒ a = 5/2 cm

8. Question

A sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$cm. Find the radius of the cylinder.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Let the radius of the cylinder be *r* cm.

Given, sphere of radius 5 cm is immersed in water filled in a cylinder, the level of water rises $\frac{5}{3}$cm

Volume of the sphere = Volume of the water in the cylinder.

⇒ $\frac{4}{3}\pi \times 5^3 = \pi \times r^2 \times \frac{5}{3}$

⇒ $r^2 = 4 \times 5^2$

⇒ *r* = 10 cm

9. Question

If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere?

Answer

Volume of a sphere = $(4/3)\pi r^3$

Let the radius of first sphere be'r'.

Radius of 2nd sphere = 2r

Ratio of the volume of the first sphere to that of the second sphere $= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi (2r)^3}$ = 1 : 8

10. Question

A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a cone = $(1/3)\pi r^2 h$

Given, cone and a hemisphere have equal bases which implies they have the same radius.

Height of the hemisphere is its radius.

Let the base radius be 'r' and the height of cone be 'h'.

Given, cone and hemisphere have equal volume.

$(2/3)\pi r^3 = (1/3)\pi r^2 h$

⇒ h : r = 2 : 1

11. Question

A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, a vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively.

$$\Rightarrow \frac{2}{3}\pi(3.5)^3 = \pi \times 7^2 \times h$$

⇒ h = 7/12 cm

12. Question

A cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, cylinder whose height is two thirds of its diameter, has the same volume as a sphere of radius 4 cm.

⇒ h = (2/3) × 2r = 4r/3

Thus, $\pi \times r^2 \times \frac{4r}{3} = \frac{4}{3} \times \pi \times 4^3$

⇒ r = 4 cm

13. Question

A vessel in the form of a hemispherical bowl is full of water. The contents are emptired into a cylinder. The internal radii of the bowl and cylinder are respectively 6 cm and 4 cm. Find the height of water in the cylinder.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, internal radii of the bowl and cylinder are respectively 6 cm and 4 cm.

$$\Rightarrow \frac{2}{3} \times \pi \times 6^3 = \pi \times 4^2 \times h$$

⇒ h = 9 cm

14. Question

A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 9 cm. Find the radius of the ball. (Use π =22/7).

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, a cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 9 cm.

Volume of water displaced = Volume of the iron ball

$\Rightarrow \frac{4}{3}\pi r^3 = \pi \times 16^2 \times 9$

$\Rightarrow r^3 = 1728$

$\Rightarrow r = 12$ cm

15. Question

A cylinder of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm. Find the radius of the ball (Use π= 22/7)

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, a cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical iron ball is dropped into the cylinder and thus the level of water is raised by 6.75 cm.

Volume of water displaced = Volume of the iron ball

$\Rightarrow \frac{4}{3}\pi r^3 = \pi \times 12^2 \times 6.75$

$\Rightarrow r^3 = 729$

$\Rightarrow r = 9$ cm

16. Question

The diameter of a copper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. If the length of the wire is 108 m, find its diameter.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, diameter of a copper sphere is 18 cm. The sphere is melted and is drawn into a long wire of uniform circular cross-section. The length of the wire is 108 m.

Long wire can be assumed to be a cylinder.

$\Rightarrow \frac{4}{3}\pi \times 9^3 = \pi \times r^2 \times 10800$

$\Rightarrow r = 0.6$ cm

17. Question

A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimetres?

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil.

Level of the oil has to rise by 2 cm.

Let the number of spheres required be ‘n’.

$\Rightarrow n \times \frac{4}{3} \times \pi \times 1.5^3 = \pi \times 6^2 \times 2$

⇒ n = 16

18. Question

A measuring jar of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely. What will be the change in the level of water in the jar?

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Amount of water displaced = Volume of the spheres

Given, measuring jar of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely.

Let the rise in level of water be 'h' cm.

$$\Rightarrow \pi \times 5^2 \times h = 4 \times \frac{4}{3} \pi \times 1^3$$

⇒ h = 16/75 cm

19. Question

The diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm. Find the length of the wire.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, diameter of a sphere is 6 cm. It is melted and drawn into a wire of diameter 0.2 cm.

Long wire can be assumed to be a cylinder.

$$\Rightarrow \frac{4}{3} \pi \times 3^3 = \pi \times 0.1^2 \times l$$

⇒ l = 3600 cm = 36 m

20. Question

The radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. If it is melted and recast into a solid cylinder of height $2\frac{2}{3}$cm. Find the diameter of the cylinder.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, radius of the internal and external surfaces of a hollow spherical shell are 3 cm and 5 cm respectively. It is melted and recast into a solid cylinder of height $2\frac{2}{3}$cm.

Volume of material = Volume of the solid cylinder

$$\Rightarrow \frac{4}{3} \pi (5^3 - 3^3) = \pi \times r^2 \times \frac{8}{3}$$

⇒ $r^2 = 49$

⇒ r = 7 cm

Diameter = 14 cm

21. Question

A hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm. Find the radius of the base.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Given, hemisphere of lead of radius 7 cm is cast into a right circular cone of height 49 cm

$$\Rightarrow \frac{2}{3}\pi\times 7^3 = \frac{1}{3}\times\pi\times 49\times r^2$$

$\Rightarrow r^2 = 14$

$\Rightarrow r = 3.74$ cm

22. Question

A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cone = $(1/3)\pi r^2 h$

Given, radius of the internal and external surfaces of a hollow spherical shell are 2 cm and 4 cm respectively. It is melted into a cone of base radius 4 cm.

$\Rightarrow$ Volume of material in sphere = Volume of the cone

$$\Rightarrow \frac{4}{3}\times\pi\times(4^3 - 2^3) = \frac{1}{3}\pi\times 4^2\times h$$

$\Rightarrow h = 14$ cm

$L^2 = h^2 + r^2$

$\Rightarrow l = \sqrt{(14^2 + 4^2)}$

$\Rightarrow l = \sqrt{212} = 14.56$ cm

23. Question

A metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm. Find how many cones are obtained.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cone = $(1/3)\pi r^2 h$

Given, metallic sphere of radius 10.5 cm is melted and thus recast into small cones, each of radius 3.5 cm and height 3 cm.

Let the number of cones be ‘n’.

$\Rightarrow n \times (1/3)\pi \times 3.5^2 \times 3 = (4/3) \times \pi \times 10.5^3$

$\Rightarrow n = 126$

24. Question

A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Given, cone and a hemisphere have equal bases and equal volume

Height of a hemisphere is the radius and equal bases implies equal base radius.

$(2/3)\pi r^3 = (1/3)\pi r^2 h$

$\Rightarrow r : h = 1 : 2$

25. Question

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1 : 2 : 3.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Volume of a cylinder = $\pi r^2 h$

Given, a cone, a hemisphere and a cylinder stand on equal bases and have the same height.

Height of a hemisphere is the radius and equal bases implies equal base radius.

Thus, height of cone = height of cylinder = base radius = r

Ratio of volumes = $(1/3)\pi r^2 h : (2/3)\pi r^3 : \pi r^2 h$

$\Rightarrow$ Ratio of volumes = $r^3 : 2r^3 : 3r^3 = 1 : 2 : 3$

26. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75 cm. What is the radius of the ball?

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Volume of water displaced = Volume of the iron ball

Given, cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball is dropped into the tub and thus the level of water is raised by 6.75 cm

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi \times 12^2 \times 6.75$$

$\Rightarrow r^3 = 729$

$\Rightarrow r = 9$ cm

27. Question

The largest sphere is carved out of a cube of side 10.5 cm. Find the ratio of their volumes.

Answer

Largest sphere that can be carved out of a cube will have its diameter as the side of the cube.

Radius of the largest sphere carved out of a cube of side 10.5 cm = 10.5/2 = 5.25 cm

Volume of a cube = $side^3$

Volume of a sphere = $(4/3)\pi r^3$

Ratio of their volumes $= \frac{\frac{4}{3}\pi \times 5.25^3}{(10.5)^3} = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{8} = 11:21$

28. Question

A sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere. Find the ratio of their volumes.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Volume of a cylinder = $\pi r^2 h$

Given, sphere, a cylinder and a cone have the same diameter. The height of the cylinder and also the cone are equal to the diameter of the sphere

Height of cone and cylinder = 2r

Ratio of their volumes = $(4/3)\pi r^3 : \pi r^2 h : (1/3)\pi r^2 h$

⇒ Ratio of their volumes = $4r^3 : 6r^3 : 2r^3 = 2 : 3 : 1$

29. Question

A cube of side 4 cm contains a sphere touching its side. Find the volume of the gap in between.

Answer

Volume of a cube = $side^3$

Volume of a sphere = $(4/3)\pi r^3$

Given, cube of side 4 cm contains a sphere touching its side

Radius of the sphere = 4/2 = 2 cm

Volume of the gap in between = $4^3 - (4/3)\pi \times 2^3$

⇒ Volume of the gap in between = 30.48 cm^3

30. Question

A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of material = $(2/3)\pi \times (r_0^3 - r_i^3)$

Given, hemispherical tank is made up of an iron sheet 1 cm thick and the inner radius is 1 m

⇒ r_0 = 1 + 0.01 = 1.01 m

Volume of iron used $= \frac{2}{3}\pi(1.01^3 - 1) = 0.0634\, m^2$

31. Question

A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to full this capsule?

Answer

Volume of a sphere = $(4/3)\pi r^3$

Given, capsule of medicine is in the shape of a sphere of diameter 3.5 mm.

Radius = 3.5/2 = 1.75 mm

Volume of medicine filled inside = (4/3) × π × 1.75^3 = 22.458 mm^3

32. Question

The diameter of the moon is approximately one fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Answer

Volume of a sphere = (4/3)πr^3

Given, diameter of the moon is approximately one fourth of the diameter of the earth

Radius of moon = 1/4 × radius of the earth.

Ratio of their volume $= \frac{\frac{4}{3}\pi\left(\frac{r}{4}\right)^3}{\frac{4}{3}\pi r^3} = 1:64$

Volume of the moon is $1/64^{th}$ times the volume of the earth.

CCE - Formative Assessment

1. Question

Find the surface area of a sphere of radius 14 cm.

Answer

Surface area of a sphere = $4\pi r^2$

Surface area of a sphere of radius 14 cm = 4 × (22/7) × 14^2 = 2464 cm^2

2. Question

Find the total surface area of a hemisphere of radius 10 cm.

Answer

Total surface area of a hemisphere = $3\pi r^2$

Total surface area of a hemisphere of radius 10 cm = 3 × π × 10^2 = 942 cm^2

3. Question

Find the radius of a sphere whose surface area is 154 cm^2.

Answer

Surface area of a sphere = $4\pi r^2$

Given, surface area is 154 cm^2

⇒ 4 × (22/7) × r^2 = 154

⇒ r^2 = 49/4

⇒ r = 3.5 cm

4. Question

The hollow sphere, in which the circus motor cyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Answer

Surface area of a sphere = $4\pi r^2$

Given, hollow sphere, in which the circus motor cyclist performs his stunts, has a diameter of 7 m

Area available to the motorcyclist for riding = $4 \times (22/7) \times 3.5^2 = 154\ m^2$

5. Question

Find the volume of a sphere whose surface area is 154 cm^2.

Answer

Surface area of a sphere = $4\pi r^2$

Given, surface area is 154 cm^2

$\Rightarrow 4 \times (22/7) \times r^2 = 154$

$\Rightarrow r^2 = 49/4$

$\Rightarrow r = 3.5$ cm

Volume of a sphere = $(4/3)\pi r^3$

$\Rightarrow$ Volume of the given sphere = $(4/3)\ \pi \times 3.5^3 = 179.66\ cm^3$

6. Question

How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter?

Answer

Volume of a cube = $side^3$

Volume of a sphere = $(4/3)\pi r^3$

Given, spherical bullets are to be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter.

Let the number of bullets be ‘a’.

$\Rightarrow 44^3 = a \times (4/3) \times (22/7) \times 2^3$

$\Rightarrow a = 2541$

7. Question

If a sphere of radius $2r$ has the same volume as that of a cone with circular base of radius r, then find the height of the cone.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Given, a sphere of radius $2r$ has the same volume as that of a cone with circular base of radius r

$$\Rightarrow \left(\frac{4}{3}\right)\pi(2r)^3 = \frac{1}{3}\pi r^2 \times h$$

$\Rightarrow h = 32r$

8. Question

If a hollow sphere of internal and external diamaters 4 cm and 8 cm respectively melted into a cone of base diameter 8 cm, then find the height of the cone.

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cone = $(1/3)\pi r^2 h$

Given, diameter of the internal and external surfaces of a hollow spherical shell are 4 cm and 8 cm respectively. It is melted into a cone of base diameter 8 cm.

⇒ Volume of material in sphere = Volume of the cone

$$\Rightarrow \frac{4}{3}\times\pi\times(4^3-2^3)=\frac{1}{3}\pi\times4^2\times h$$

⇒ h = 14 cm

9. Question

The surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm. Find the height of the cone.

Answer

Surface area of sphere = $4\pi r^2$

Curved syrface area of a cone = πrl

Given, surface area of a sphere of radius 5 cm is five times the area of the curved surface of a cone of radius 4 cm

⇒ $4 \times \pi \times 5^2 = 5 \times \pi \times 4 \times l$

⇒ l = 5 cm

$L^2 = h^2 + r^2$

⇒ $5^2 = h^2 + 4^2$

⇒ $h^2 = 9$

⇒ h = 3 cm

10. Question

If a sphere is inscribed in a cube, find the ratio of the volume of cube to the volume of the sphere.

Answer

Volume of a cube = $side^3$

Volume of a sphere = $(4/3)\pi r^3$

Given, sphere is inscribed in a cube

Diameter of sphere = side of the cube

Side of cube = 2r

Ratio of the volume of cube to the volume of the sphere$=\frac{(2r)^3}{\frac{4}{3}\pi\times r^3}=6:\pi$

1. Question

In a cone the number of faces is

A. 1

B. 2

C. 3

D. 4

Answer

Faces are the flat surface on a 3D figure.

Thus, cone has one flat surface, i.e., the base.

Number of faces of a cone = 1

2. Question

The total surface area of a hemisphere of radius *r* is

A. πr^2

B. $2\pi r^2$

C. $3\pi r^2$

D. $4\pi r^2$

Answer

Total surface area of a hemisphere = curved surface area + base area

⇒ total surface area of a hemisphere of radius *r* is = $2\pi r^2 + \pi r^2 = 3\pi r^2$

3. Question

The ratio of the total surface area of a sphere and a hemisphere of same radius is

A. 2 : 1

B. 3 : 2

C. 4 : 1

D. 4 : 3

Answer

Total surface area of a sphere = $4\pi r^2$

Total surface area of a hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Ratio of the total surface area of a sphere and a hemisphere of same radius = 4 : 3

4. Question

A sphere and a cube are of the same height. The ratio of their volumes is

A. 3 : 4

B. 21: 11

C. 4 : 3

D. 11 : 21

Answer

Volume of a cube = side^3

Volume of a sphere = $(4/3)\pi r^3$

Given, sphere and a cube are of the same height.

Side = diameter = 2r

Ratio of their volumes $= \frac{\frac{4}{3}\times\frac{22}{7}\times r^3}{(2r)^3} = 11 : 21$

5. Question

The largest sphere is cut off from a cube of side 6 cm. The volume of the sphere will be

A. 27π cm^3

B. 36π cm^3

C. $108\pi\ cm^3$

D. $12\pi\ cm^3$

Answer

Largest sphere that can be cut out of a cube will have its diameter as the side of the cube.

Radius of the largest sphere cut out of a cube of side 6 cm = 6/2 = 3 cm

Volume of a sphere = $(4/3)\pi r^3$

⇒ Volume of the largest sphere that is cut off from a cube of side 6 cm $= \frac{4}{3} \times \pi \times 3^3 = 36\pi\ cm^3$

6. Question

A cylindrical rod whose height is 8 times of its radius is melted and recast into spherical balls of same radius. The number of balls will be

A. 4

B. 3

C. 6

D. 8

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, cylindrical rod whose height is 8 times of its radius is melted and recast into spherical balls of same radius.

Let the number of such balls be ‘a’.

⇒ $\pi \times r^2 \times 8r = a \times (4/3)\pi \times r^3$

⇒ a = 6

7. Question

If the ratio of volumes of two spheres is 1: 8, then the ratio of their surface areas is

A. 1 : 2

B. 1 : 4

C. 1 : 8

D. 1 : 16

Answer

Ratio of volume of spheres = $(\text{ratio of radius})^3$

Given, ratio of volumes of two spheres is 1: 8

⇒ $(\text{ratio of radius})^3 = 1 : 8$

⇒ ratio of radius = 1 : 2

Ratio of surface area = $(\text{ratio of radius})^2$

⇒ Ratio of surface area = 1 : 4

8. Question

If the surface area of a sphere is $144\pi m^2$, then its volume (in m^3) is

A. 288π

B. 316π

C. 300π

D. 188π

Answer

Surface area of a sphere = $4\pi r^2$

Given, surface area of a sphere is $144\pi m^2$

$\Rightarrow 4\pi r^2 = 144\pi$

$\Rightarrow r = 6$ m

Volume of the sphere = $(4/3) \times \pi \times 6^3$

$\Rightarrow$ Volume of the sphere = 288π m^3

9. Question

If a solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius, then the surface area of each ball (in sq. cm) is

A. 100π

B. 75π

C. 60π

D. 50π

Answer

Volume of sphere = $(4/3)\pi r^3$

Given, solid sphere of radius 10 cm is moulded into 8 spherical solid balls of equal radius

$\Rightarrow (4/3)\pi \times 10^3 = 8 \times (4/3)\pi \times r^3$

$\Rightarrow r= 10/2 = 5$cm

Surface area of a sphere = $4\pi r^2$

Thus, the surface area of each sphere = $4 \times \pi \times 5^2 = 100\pi$

10. Question

The ratio between the volume of a sphere and volume of a circumscribing right circular cylinder is

A. 2 : 1

B. 1 : 1

C. 2 : 3

D. 1 : 2

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

If a cylinder circumscrimbes a sphere of radius r , then its base radius is ‘r’ and height is diameter = 2r

Ratio between the volume of a sphere and volume of a circumscribing right circular cylinder

$$= \frac{\frac{4}{3}\pi r^3}{\pi \times r^2 \times 2r} = 2:3$$

11. Question

If a sphere is inscribed in a cube, then the ratio of the volume of the sphere to the volume of the cube is

A. π : 2

B. π : 3

C. π : 4

D. π : 6

Answer

Volume of a cube = side^3

Volume of a sphere = (4/3)πr^3

Given, sphere is inscribed in a cube

Diameter of sphere = side of the cube

Side of cube = 2r

Ratio of the volume of the sphere to the volume of cube $= \frac{\frac{4}{3}\pi r^3}{(2r)^3} = \pi : 6$

12. Question

If a solid sphere of radius *r* is melted and cast into the shape of a solid cone of height *r*, then the radius of the base of the cone is

A. 2*r*

B. 3*r*

C. *r*

D. 4*r*

Answer

Volume of a sphere = (4/3)πr^3

Volume of a solid cone = (1/3)πr^{2}h

Given, solid sphere of radius *r* is melted and cast into the shape of a solid cone of height *r*

Let the base radius be A.

⇒ (4/3)πr^3 = (1/3)π × A^2 × r

⇒ A = 2r

13. Question

A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is *r,* then the volume of the cylinder is

A. 4π*r*^{3}

B. $\frac{8}{3}\pi r^3$

C. 2π*r*^{3}

D. 8π*r*^{3}

Answer

Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r*

Thus, height of the cylinder = diameter = 2r and base radius = r

Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$

14. Question

A cone and a hemisphere have equal bases and equal volumes the ratio of their heights is

A. 1 : 2

B. 2 : 1

C. 4 : 1

D. $\sqrt{2}$: 1

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Given, cone and a hemisphere have equal bases and equal volume

Height of a hemisphere is the radius and equal bases implies equal base radius.

$(2/3)\pi r^3 = (1/3)\pi r^2 h$

$\Rightarrow$ r : h = 1 : 2

15. Question

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is

A. 1 : 2 : 3

B. 2 : 1 : 3

C. 2 : 3 : 1

D. 3 : 2 : 1

Answer

Volume of a hemisphere = $(2/3)\pi r^3$

Volume of a right circular cone = $(1/3)\pi r^2 h$

Volume of a cylinder = $\pi r^2 h$

Given, a cone, a hemisphere and a cylinder stand on equal bases and have the same height.

Height of a hemisphere is the radius and equal bases implies equal base radius.

Thus, height of cone = height of cylinder = base radius = r

Ratio of volumes = $(1/3)\pi r^2 h : (2/3)\pi r^3 : \pi r^2 h$

$\Rightarrow$ Ratio of volumes = $r^3 : 2r^3 : 3r^3 = 1 : 2 : 3$