

14. Quadrilaterals

Exercise 14.1

1. Question

Three angles of a quadrilateral are respectively equal to 110° , 50° and 40° . Find its fourth angles.

Answer

Given,

Three angles of quadrilateral = 110° , 50° , 40°

Let fourth angle be = X°

As we know sum of all angles of a quadrilateral = 360°

So,

$$110^\circ + 50^\circ + 40^\circ + X^\circ = 360^\circ$$

$$X^\circ = 360^\circ - 200^\circ = 160^\circ$$

2. Question

In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 4 : 5. Find the measure of each angles of the quadrilateral.

Answer

Given,

In quadrilateral ABCD,

$$A:B:C:D = 1:2:3:4:5$$

Let angles A, B, C, D = x , $2x$, $4x$, $5x$

So,

$$x + 2x + 4x + 5x = 360^\circ$$

$$12x = 360^\circ$$

$$x = \frac{360^\circ}{12} = 30^\circ$$

So,

$$\angle A = x = 30^\circ$$

$$\angle B = 2x = 2 \times 30^\circ = 60^\circ$$

$$\angle C = 4x = 4 \times 30^\circ = 120^\circ$$

$$\angle D = 5x = 5 \times 30^\circ = 150^\circ$$

3. Question

In a quadrilateral ABCD, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$.

Answer

Given,

In quadrilateral ABCD,

CO is the bisector of $\angle C$

DO is the bisector of $\angle D$

In $\triangle COD$

$$\frac{1}{2}\angle C + \frac{1}{2}\angle D + \angle COD = 180^\circ$$

$$\Rightarrow \angle COD = 180^\circ - \frac{1}{2}(\angle D + \angle C)$$

$$\Rightarrow \angle D + \angle C = 360 - (\angle A + \angle B)$$

So,

$$\Rightarrow \angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$$

$$\Rightarrow \angle COD = \frac{1}{2}(\angle A + \angle B) \text{ Proved.}$$

4. Question

The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer

Given,

Ratio of angles of quadrilateral = 3:5:9:13

Let angles are = $3x, 5x, 9x, 13x$

So,

$$3x + 5x + 9x + 13x = 360^\circ$$

$$30x = 360^\circ$$

$$x = \frac{360^\circ}{30} = 12^\circ$$

Angles would be =

$$3x = 3 \times 12 = 36$$

$$5x = 5 \times 12 = 60^\circ$$

$$9x = 9 \times 12 = 108^\circ$$

$$13x = 13 \times 12 = 156^\circ$$

Exercise 14.2

1. Question

Two opposite angles of a parallelogram are $(3x-2)^\circ$ and $(50-x)^\circ$. Find the measure of each angle of the parallelogram.

Answer

Given,

Two opposite angles of a parallelogram $(3x - 2)$ and $(50 - x)$

Opposite angles of a parallelogram are equal,

$$3x - 2 = 50 - x$$

$$4x = 52^\circ$$

$$x = 13^\circ$$

So, angles are

$$3x - 2 = 3 \times 13 - 2 = 37^\circ$$

$$50 - x = 50 - 13 = 37^\circ$$

$$\text{Sum of other two angles} = 360 - 2 \times 37 = 360 - 74 = 286^\circ$$

$$\text{So each angle will be} = \frac{286}{2} = 143^\circ$$

2. Question

If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Answer

Given,

In a parallelogram

$$\text{one angle} = \frac{2}{3} \text{ adjacent angle}$$

Let angle = x

$$\text{So adjacent angle} = \frac{2}{3}x$$

We know that opposite angles of parallelogram are equal

$$\text{So, four angles will be} = x, x, \frac{2}{3}x, \frac{2}{3}x$$

$$\text{And } x + x + \frac{2}{3}x + \frac{2}{3}x = 360^\circ$$

$$\frac{10}{3}x = 360^\circ = 10x = 360^\circ \times 3$$

$$x = \frac{360 \times 3}{10} = 108^\circ$$

$$\text{So, } x = 108^\circ$$

$$\text{Adjacent angles will be} = \frac{2}{3}x = \frac{2}{3} \times 108 = 72^\circ$$

$$\text{Angles are} = 108^\circ, 108^\circ, 72^\circ \text{ and } 72^\circ$$

3. Question

Find the measure of all the angles of a parallelogram, if one angle is 24° less than twice the smallest angle.

Answer

Given,

Let the smallest angle = x°

Then the other angle = $(2x - 24)^\circ$

So,

$$x^\circ + x^\circ + 2x - 24 + 2x - 24 = 360^\circ$$

$$6x = 360 + 48 = 408$$

$$x = \frac{408^\circ}{6} = 68^\circ$$

$$\text{Other angles} = 2x - 24 = 2 \times 68 - 24 = 136 - 24 = 112^\circ$$

$$\text{Angles are} = 68^\circ, 68^\circ, 112^\circ, 112^\circ$$

4. Question

The perimeter of a parallelogram is 22 cm. If the longer side measures 6.5 cm. what is the measure of the shorter side?

Answer

Given,

Perimeter of parallelogram = 22cm

Longer side = 6.5cm

Let shorter side be = x cm

So,

$$2(6.5 + x) = 22\text{cm}$$

$$13.0 + 2x = 22\text{cm}$$

$$2x = 22 - 13 = 9\text{cm}$$

$$x = \frac{9}{2} = 4.5\text{cm}$$

5. Question

In a parallelogram ABCD, $\angle D = 135^\circ$, determine the measures of $\angle A$ and $\angle B$.

Answer

Given,

In a parallelogram ABCD

$$\angle D = 135^\circ$$

$$\angle C + \angle D = 180^\circ \text{.. (supplementary angles)}$$

$$\angle C = 180^\circ - 135^\circ = 45^\circ$$

$$\angle C = \angle A \text{ and } \angle D = \angle B \text{.. (Opposite angles of parallelogram)}$$

$$\angle A = 45^\circ$$

$$\angle B = 135^\circ$$

6. Question

ABCD is a parallelogram in which $\angle A = 70^\circ$. Compute $\angle B$, $\angle C$ and $\angle D$.

Answer

Given,

In a parallelogram ABCD,

$$\angle A = 70^\circ$$

$$\angle A + \angle B = 180^\circ \text{.. (supplementary angles)}$$

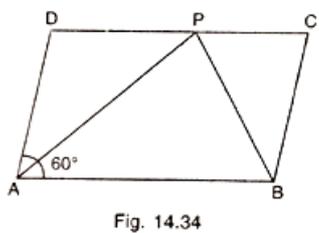
$$\angle B = 180^\circ - 70^\circ = 110^\circ$$

$$\angle A = \angle C \text{ and } \angle B = \angle D \text{.. (Opposite angles of parallelogram)}$$

$$\angle A = 70, \angle B = 110, \angle C = 70, \angle D = 110^\circ$$

7. Question

In Fig. 14.34, ABCD is a parallelogram in which $\angle A = 60^\circ$. If the bisectors of $\angle A$ and $\angle B$ meet at P, prove that $AD = DP$, $PC = BC$ and $DC = 2AD$.



Answer

Given,

In a parallelogram ABCD,

$$\angle A = 60^\circ$$

So,

$$\angle B = 180 - 60 = 120 \text{ (supplementary angles)}$$

$$\angle ABP = \angle PCB = \frac{120}{2} = 60^\circ$$

$$\angle DPA = \angle BAP = 30^\circ \text{ (AB parallel to DC and AP intersects them)}$$

$$\angle DPA = \angle DAP = 30^\circ$$

$$AD = DP \dots \text{(i) (proved)}$$

Similarly,

$$\angle BPC = \angle ABP = 60^\circ$$

$$\text{In triangle BPC, } \angle BPC = \angle PCB = 60^\circ$$

$$BC = CP = AD \dots \text{(ii) (proved)}$$

$$BC = AD$$

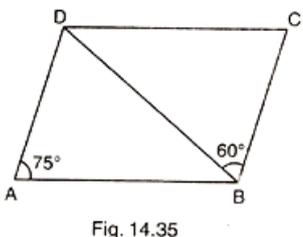
From equation (i) and (ii),

$$CP = DP$$

$$DC = 2AD \text{ (proved)}$$

8. Question

In Fig. 14.35, ABCD is a parallelogram in which $\angle DAB = 75^\circ$ and $\angle DBC = 60^\circ$. Compute $\angle CDB$ and $\angle ADB$.



Answer

Given,

In a parallelogram ABCD,

$$\angle DAB = 75^\circ$$

$$\angle DBC = 60^\circ$$

$$\angle A + \angle B = 180^\circ \text{ (supplementary angles)}$$

$$\angle B = 180^\circ - 75^\circ = 105^\circ$$

$$\angle DBA + \angle DBC = 105^\circ$$

$$\angle DBA = 105^\circ - 60^\circ = 45^\circ$$

In a triangle ABD

$$\angle DAB + \angle DBA + \angle ADB = 180^\circ$$

$$75^\circ + 45^\circ + \angle ADB = 180^\circ$$

$$\angle ADB = 180^\circ - 120^\circ = 60^\circ$$

$$\angle A = \angle C = 75^\circ \text{ (opposite angles of parallelogram)}$$

$$\angle C + \angle D = 180^\circ \text{ (supplementary angles of parallelogram)}$$

$$\angle D = 180^\circ - 75^\circ = 105^\circ$$

$$\angle ADB + \angle CDB = 105^\circ$$

$$\angle CDB = 105^\circ - \angle ADB$$

$$\angle CDB = 105^\circ - 60^\circ = 45^\circ$$

9. Question

In Fig. 14.36, ABCD is a parallelogram and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that $AF=2AB$.

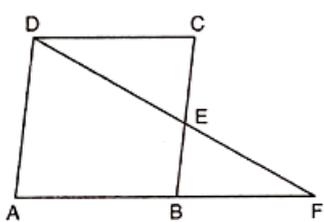


Fig. 14.36

Answer

Given,

In a parallelogram ABCD,

E = mid point of side BC

$$AD \parallel BC$$

$$AD \parallel BE$$

E is mid point of BC

So, in $\triangle DEC$ and $\triangle BEF$

$$BE = EC \text{.. (E is the mid point)}$$

$$\angle DEC = \angle BEF$$

$$\angle DCB = \angle FBE \text{ (vertically opposite angles)}$$

$$\text{So, } \triangle DEC \cong \triangle BEF$$

$$DC = FB$$

$$= AB + DC = FB + AB$$

$$= 2AB = AF \text{ (proved)}$$

10. Question

Which of the following statements are true (T) and which are false (F)?

- (i) In a parallelogram, the diagonals are equal.
- (ii) In a parallelogram, the diagonals bisect each other.
- (iii) In a parallelogram, the diagonals intersect each other at right angles.
- (iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.
- (v) If all the angles of a quadrilateral are equal, it is a parallelogram.
- (vi) If three sides of a quadrilateral are equal, it is a parallelogram.
- (vii) If three angles of a quadrilateral are equal, it is a parallelogram.
- (viii) If all the sides of a quadrilateral are equal it is a parallelogram.

Answer

(i) False

Reason: Imagine a parallelogram and draw its diagonals. Now the areas of the two triangles on one of the bases is equal. But by Heron's formula, the areas are not equal and if the areas are not equal how can be the diagonals because area can only be equal if both the triangles have equal diagonals.

(ii) True

Proof:

Let's take a parallelogram ABCD,

The diagonals AC and BD intersect each other at O,

$$AO = OC \text{ and } BO = OD$$

In $\triangle AOB$ and $\triangle COD$,

We have

$$\angle BAO = \angle OCD \text{ (alternate interior angles)}$$

$$\angle AOB = \angle CDO \text{ (alternate interior angles)}$$

$$AB = DC \text{ (opposite sides)}$$

$$\triangle AOB \cong \triangle COD \text{ (by ASA)}$$

$$AO = OC \text{ and } DO = OB$$

(iii) False

According to the definition a parallelogram is a simple (non-self-intersecting) quadrilateral with two pairs of parallel sides. The opposite or facing sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.

It turns out that a parallelogram has its diagonals meeting at right angles if and only if the parallelogram is a rhombus (all sides equal).

(iv) True

Reason: Let's take ABCD is a quadrilateral,

$$\text{Where } AB = CD \text{ \& } AD = BC$$

We have to prove : ABCD is a parallelogram

Join AC, it's a diagonal

In $\triangle ABC$ and $\triangle CDA$

$$AB = CD$$

$$BC = DA$$

$$AC = CA$$

$$\triangle ABC \cong \triangle CDA$$

Hence,

$$\angle BAC = \angle DCA$$

For lines AB and CD with transversal AC,

$\angle BAC$ & $\angle DCA$ are alternate angles and are equal.

So, AB and CD lines are parallel.

$$\angle BCA = \angle DAC$$

For lines AD and BC with transversal AC,

$\angle BCA$ & $\angle DAC$ are alternate angles and are equal.

So, AD and BC lines are parallel.

Thus in ABCD,

Both pairs of opposite sides are parallel,

So, ABCD is a parallelogram.

(v) False

Reason: If all the angles of a quadrilateral are equal, then it's a rectangle.

(vi) False

Reason:

As we know that the opposite or facing sides of a parallelogram are of equal length and the opposite angles of a parallelogram are of equal measure.

In case of square and rhombus all sides are equal, but if three sides are equal then it doesn't satisfy the property of a parallelogram.

(vii) False

Reason:

Same reason as for the sides, it does not satisfy the property of a parallelogram.

(viii) True

Yes if all sides of a parallelogram then it's a square or rhombus.

Exercise 14.3

1. Question

In a parallelogram ABCD, determine the sum of angles $\angle C$ and $\angle D$.

Answer

Given,

In a parallelogram ABCD,

$\angle C$ and $\angle D$ are supplementary angles of a parallelogram

$$\text{So, } \angle C + \angle D = 180^\circ$$

2. Question

In a parallelogram ABCD, if $\angle B = 135^\circ$, determine the measures of its other angles.

Answer

Given,

$$\angle B = 135^\circ$$

$$\angle A + \angle B = 180^\circ \text{ (supplementary angles of parallelogram)}$$

$$\angle A = 180^\circ - 135^\circ = 45^\circ$$

$$\angle C = \angle A = 45^\circ \text{ (opposite angles of parallelogram)}$$

$$\angle D = \angle B = 135^\circ$$

3. Question

ABCD is a square. AC and BD intersect at O. State the measure of $\angle AOB$.

Answer

Given,

ABCD is a square

AC and BD intersect at O

$$\angle AOB = 90^\circ \text{ (diagonals of square bisect each other at } 90^\circ)$$

4. Question

ABCD is a rectangle with $\angle ABD = 40^\circ$. Determine $\angle DBC$.

Answer

Given,

ABCD is a rectangle

$$\angle ABD = 40^\circ$$

$$\angle ABD + \angle DBC = 90^\circ \text{ (angles of rectangle)}$$

$$\angle DBC = 90^\circ - 40^\circ = 50^\circ$$

5. Question

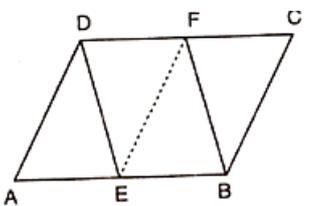
The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Answer

Given,

In a parallelogram ABCD

AB & CD bisect at E & F



$$AB \parallel CD$$

$$AB = DC$$

$$EB \parallel DF$$

$$\text{So, } EB = DF$$

$$= \frac{1}{2}AB = \frac{1}{2}DC$$

So, EBFD is a parallelogram.

6. Question

P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Proves also that AC bisects PQ.

Answer

Given,

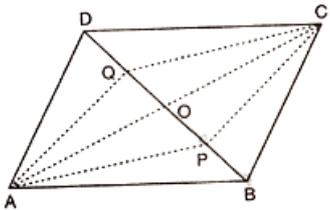
In a parallelogram ABCD

Since diagonal of parallelogram bisects each other

$$OA = OC \text{ and } OB = OD$$

Since P&Q are the point of trisection of BD

$$BP = PQ = QD$$



Now, $OB = OD$ and $BP = QD$

$$OB - BP = OD - QD$$

$$OP = OQ$$

Diagonals of quadrilateral bisect each other

Hence, APCQ is a parallelogram.

7. Question

ABCD is a square E, F, G and H are points on AB, BC, CD and DA respectively, such that $AE = BF = CG = DH$. Prove that EFGH is a square.

Answer

Given,

ABCD is a square

E,F,G,H are the points of AB, BC, CD, DA

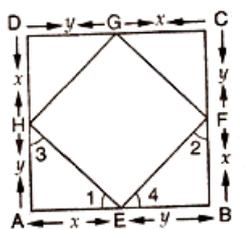
Such that $AE = BF = CG = DH$

In $\triangle AEH$ & $\triangle BFE$

Let,

$$AE = BF = CG = DH = x$$

$$BE = CF = DG = AF = y$$



In $\triangle AEH$ & $\triangle BFE$

$$AE = BF \text{ (given)}$$

$$\angle A = \angle B \text{ (each equal)}$$

$$AH = BE$$

So, by SAS congruency

$$\triangle AEH \cong \triangle BFE$$

$$\angle 1 = \angle 2 \text{ \& } \angle 3 = \angle 4$$

$$\angle 1 + \angle 3 = 90^\circ$$

$$\angle 2 + \angle 4 = 90^\circ$$

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$$\angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^\circ$$

$$2(\angle 1 + \angle 4) = 180^\circ$$

$$\angle 1 + \angle 4 = \frac{180}{2} = 90^\circ$$

So, $\angle HEF = 90^\circ$

Similarly we have,

$$\angle F = \angle G = \angle H = 90^\circ$$

Hence, EFGH is a square.

8. Question

ABCD is a rhombus, EABF is a straight line such that $EA = AB = BF$. Prove that ED and FC when produced meet at right angles.

Answer

Given,

ABCD is a rhombus,

EABF is a straight line

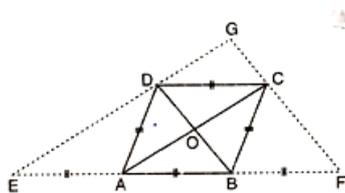
$$EA = AB = BF$$

$$OA = OC$$

$OB = OD$ (diagonals of rhombus are perpendicular bisector of each other)

$$\angle AOD = \angle COD = 90^\circ$$

$$\angle AOB = \angle COB = 90^\circ$$



In $\triangle BDE$,

A and O are mid points of BE and BD

$$OA \parallel DE$$

$$OC \parallel DG$$

In $\triangle CFA$

B and O are the mid points of AF and AC

$$OB \parallel CF \text{ and}$$

OD \parallel GC

Thus in quadrilateral DOCG

OC \parallel DG and

OD \parallel GC

DOCG is a parallelogram

$\angle DGC = \angle DOC$

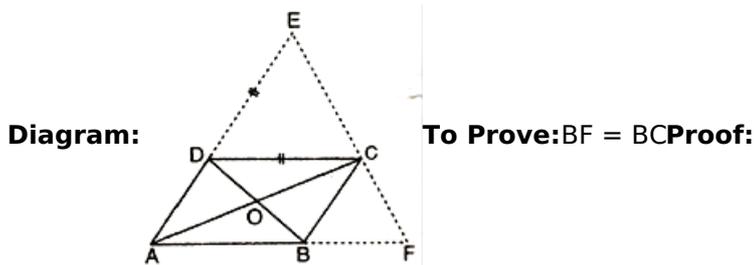
$\angle DGC = 90^\circ$ (proved)

9. Question

ABCD is a parallelogram, AD is produced to E so that $DE = DC$ and EC produced meets AB produced in F. Prove that $BF = BC$.

Answer

Given: ABCD is a parallelogram,



In $\triangle ACE$

D and O are the mid points of AE and AC

DO \parallel EC

OB \parallel CF and $AB = BF \rightarrow$ (i)

$DC = BF$ ($AB = DC$ as ABCD is a parallelogram and opposite sides of a parallelogram are equal and parallel))

In $\triangle EDC$ and $\triangle CBF$

$DC = BC$

$\angle EDC = \angle CBF$

$\angle ECD = \angle CFB$

So, by ASA congruency

$\triangle EDC \cong \triangle CBF$

$DE = BC$

$DC = BC$

$AB = BC$

$BF = BC$ ($AB = BF$ from (i))

Exercise 14.4

1. Question

In a $\triangle ABC$, D, E and F are, respectively, the mid points of BC, CA and AB. If the lengths of side AB, BC and CA are 7 cm, 8 cm and 9 cm, respectively, find the perimeter of $\triangle DEF$.

Answer

Given,

In $\triangle ABC$

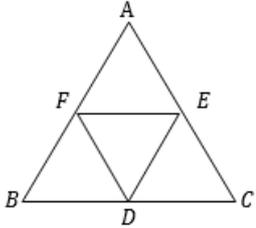
D, E, F are the mid points of BC, CA and AB

$$AB = 7\text{cm}$$

$$BC = 8\text{cm}$$

$$CA = 9\text{cm}$$

We need to find out the perimeter of DEF



So,

$$DE = \frac{1}{2}BC = \frac{1}{2} \times 8 = 4\text{cm}$$

$$EF = \frac{1}{2}AB = \frac{1}{2} \times 7 = 3.5\text{cm}$$

$$DF = \frac{1}{2}AC = \frac{1}{2} \times 9 = 4.5\text{cm}$$

So, perimeter of DEF

$$DE+EF+DF = 4+3.5+4.5 = 12\text{cm}$$

2. Question

In a triangle $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 60^\circ$ and $\angle C = 70^\circ$. Find the measure of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Answer

Given,

In a $\triangle ABC$

$$\angle A = 50^\circ$$

$$\angle B = 60^\circ$$

$$\angle C = 70^\circ$$

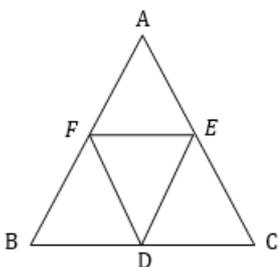
Let DEF are mid point of $\triangle ABC$

BDEF and CDFE are parallelogram

$$\angle B = \angle E \text{ \& \ } \angle C = \angle F$$

$$\angle E = 60^\circ$$

$$\angle F = 70^\circ$$



Similarly

$$\angle D = \angle A$$

$$\angle D = 50^\circ$$

3. Question

In a triangle, P, Q and R are the mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

Answer

Given,

In $\triangle ABC$

P, Q and R are the mid-points of sides BC, CA and AB

$$AC = 21\text{cm}$$

$$BC = 29\text{cm}$$

$$AB = 30\text{cm}$$

Since ARPQ is a parallelogram

$$\text{Perimeter of parallelogram ARPQ} = 2(AP+AQ)$$

$$AB+AC = 30+21 = 51\text{cm}$$

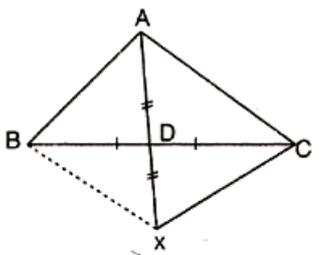
4. Question

In a $\triangle ABC$, median AD is produced to X such that $AD=DX$. Prove that ABXC is a parallelogram.

Answer

Given,

In $\triangle ABC$,



AD is produced to X

$$AD=DX$$

In quadrilateral ABXC

$$AD=DX \text{ (Given)}$$

$$BD = DC \text{ (given)}$$

So diagonal AX and BC bisect each other

Therefore ABXC is a parallelogram.

5. Question

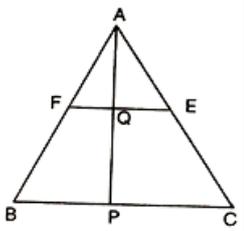
In a $\triangle ABC$, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects EF at Q. Prove that $AQ=QP$.

Answer

Given,

In $\triangle ABC$,

E & F are the mid-points of AC and AB



$EF \parallel BC$

SO, $FQ \parallel BP$

Q is the mid point of AP

$AQ = QP$ (Proved)

6. Question

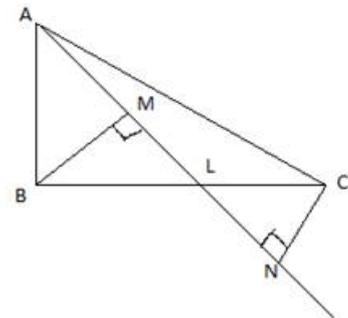
In a $\triangle ABC$, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that $ML = NL$.

Answer

Given,

In $\triangle ABC$,

BM & CN are perpendiculars from B & C



In $\triangle BLM$ and $\triangle CLN$

$\angle BML = \angle CNL = 90^\circ$

$BL = CL$ [L is mid point of BC]

$\angle MLB = \angle NLC$ [vertically opposite angles]

$\therefore \triangle BLM = \triangle CLN$

$\therefore LM = LN$ (corresponding sides of congruent triangles)

7. Question

In Fig. 14.95, triangle ABC is right-angled at B. Given that $AB = 9$ cm, $AC = 15$ cm and D, E are the mid-points of the sides AB and AC respectively, calculate

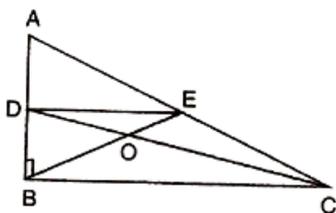


Fig. 14.95

- (i) The length of BC
- (ii) The area of $\triangle ADE$.

Answer

Given,

$\triangle ABC$ is right angled at B

$$AB = 9\text{cm}$$

$$AC = 15\text{cm}$$

D, E are mid points of side AB and AC

(i) by using pythagores theorem

$$BC = \sqrt{AC^2 - AB^2}$$

$$BC = \sqrt{15^2 - 9^2}$$

$$= \sqrt{144} = 12\text{cm}$$

(ii) Area of $\triangle ADE$

$$= \frac{1}{2} \times DE \times AD = \frac{1}{2} \left\{ \left(\frac{1}{2} BC \times \frac{1}{2} AB \right) \right\}$$

$$= \frac{1}{2} (6 \times 4.5) = 13.5\text{cm}^2$$

8. Question

In Fig. 14.96, M, N and P are mid points of AB, AC and BC respectively. If $MN=3$ cm, $NP=3.5$ cm and $MP = 2.5$ cm, calculate BC, AB and AC.

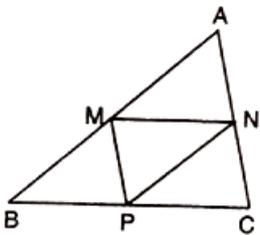


Fig. 14.96

Answer

Given,

M, N, P are mid points AB, AC and BC

$$MN = 3\text{cm}$$

$$NP = 3.5\text{cm}$$

$$MP = 2.5\text{cm}$$

$$MN = \frac{1}{2} BC$$

$$BC = 2MN = 2 \times 3 = 6\text{cm}$$

$$MP = \frac{1}{2} AC$$

$$AC = 2MP = 2 \times 2.5 = 5\text{cm}$$

$$AP = \frac{1}{2} AB$$

$$AB = 2AP = 2 \times 3.5\text{cm}$$

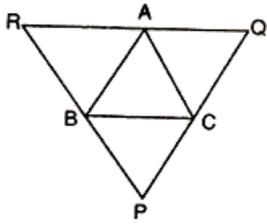
9. Question

ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC .

Answer

Given,

In ΔABC



ABCQ and ARBC are parallelograms,

$$BC = AQ \text{ \& }$$

$$BC = AR$$

$$AQ = AR \text{ (A is mid point of QR)}$$

Similarly B and C are the mid point of PR and PQ

$$AB = \frac{1}{2}PQ$$

$$BC = \frac{1}{2}QR$$

$$AC = \frac{1}{2}PR$$

$$PQ = 2AB$$

$$QR = 2BC$$

$$PR = 2CA$$

$$PQ+QR+RP = 2(AB+BC+CA)$$

$$\text{Perimeter of PQR} = 2(\text{Perimetre of } \Delta ABC)$$

10. Question

In Fig. 14.97, $BE \perp AC$. AD is any line from A to BC intersecting BE in H, P, Q and R are respectively the mid points of AH, AB and BC. Prove that $\angle PQR = 90^\circ$

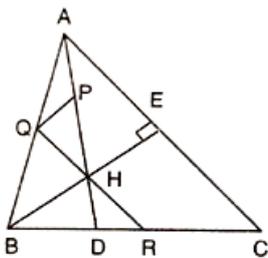


Fig. 14.97

Answer

Given,

In ΔABC ,

Q, R are the mid points of AB and AC

QR \parallel AC.. (i)

In $\triangle ABH$,

Q, P are the mid points of AB and AH respectively,

QP \parallel BH

QP \parallel BE.. (ii)

But $AC \perp BE$ therefore from (i) and (ii)

QP \perp QR

$\angle PQR = 90^\circ$

11. Question

In Fig. 14.98, $AB=AC$ and $CP \parallel BA$ and AP is the bisector of exterior $\angle CAD$ of $\triangle ABC$. Prove that (i) $\angle PAC = \angle BCA$
(ii) ABCP is a parallelogram.

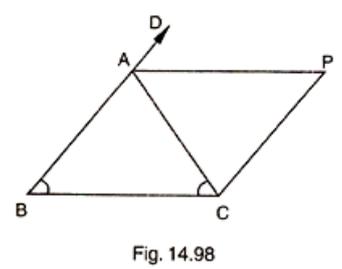


Fig. 14.98

Answer

Given,

In figure 14.98

$AB = AC$

$CP \parallel BA$

AP is bisector of exterior angle $\angle CAD$

$AB = AC$

$\angle C = \angle B$

NOW,

$\angle CAD = \angle B + \angle C$

$2\angle CAP = 2\angle C$

$\angle CAP = \angle C$

$AP \parallel BC$

But, $AB \parallel CP$ (given)

Hence ABCP is a parallelogram

12. Question

ABCD is a kite having $AB=AD$ and $BC=CD$. Prove that the figure formed by joining the mid-points of the sides, in order, is a rectangle.

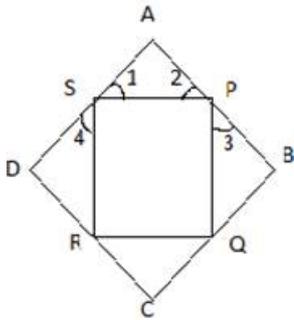
Answer

Given,

ABCD is a kite, in which

$AB=AD$ and $BC=CD$

P,Q,R,S are mid points of side AB,BC,CD &DA



In $\triangle ABC$, P&Q are mid points of AB & BC

$$\therefore PQ \parallel AC , PQ = \frac{1}{2}AC \dots \dots \dots (i)$$

In $\triangle ADC$, R & S are mid points of CD & AD

$$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2} AC \dots \dots \dots (ii)$$

From (i) and(ii) we have

$$PQ \parallel RS , PQ=RS$$

Since $AB=AD$

$$= \frac{1}{2} AB = \frac{1}{2} AD$$

$$AP=AS \dots \dots \dots (iii)$$

$$= \angle 1 = \angle 2 \dots \dots \dots (iv)$$

Now in $\triangle PBQ$ and $\triangle SDR$

$$PB = SD \because AD=AB = \frac{1}{2} AD = \frac{1}{2} AB$$

$$BQ = DR \because PB=SD$$

And $PQ = SR \because PQRS$ is a parallelogram

So by sss congruency

$$\triangle PBQ \cong \triangle SDR$$

$$\angle 3 = \angle 4$$

$$\text{Now, } \angle 3 + \angle SPQ + \angle 2 = 180^\circ$$

$$\angle 1 + \angle PSR + \angle 4 = 180^\circ$$

$$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$$

$$\angle SPQ = \angle PSR \text{ (} \angle 2 = \angle 1 \text{ and } \angle 3 = \angle 4 \text{)}$$

$$\therefore \angle SPQ + \angle PSR = 180^\circ$$

$$= 2\angle SPQ = 180^\circ = \angle SPQ = 90^\circ$$

Hence , PQRS is a parallelogram.

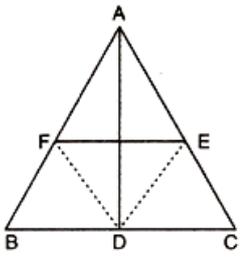
13. Question

Let ABC be an isosceles triangle in which $AB=AC$. If D, E, F be the mid-points of the sides BC, CA and AB respectively, show that the segment AD and EF bisect each other at right angles.

Answer

Given,

ABC is an isosceles triangle, D, E, F are mid points of side BC, CA, AB



$\therefore AB \parallel DF$ and $AC \parallel FE$

ABDF is a parallelogram

$AF = DE$ and $AE = DF$

$$= \frac{1}{2} AB = DE \text{ and } \frac{1}{2} AC = DF$$

$DE = DF$ ($\because AB = AC$)

$AE = AF = DE = DF$

ABDF is a rhombus

$\therefore AD$ and FE bisect each other at right angle.

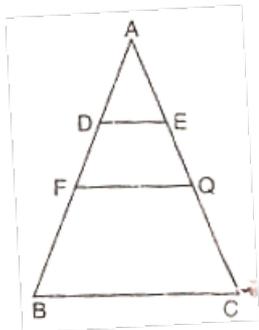
14. Question

ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4} AB$ and E is a point on AC such that $AE = \frac{1}{4} AC$. Prove that $DE = \frac{1}{4} BC$.

Answer

Given,

Let P and Q be the mid points of AB and AC respectively



Then, $PQ \parallel BC$ and $PQ = \frac{1}{2} BC$(i)

In ΔAPQ , D and E are mid points of AP and AQ resp.

$\therefore DE \parallel PQ$, $DE = \frac{1}{2} PQ$(ii)

From (i) and(ii)

$$= DE = \frac{1}{2} PQ = \frac{1}{2} \left(\frac{1}{2} BC \right)$$

$$= DE = \frac{1}{4} BC \text{ proved}$$

15. Question

In Fig. 14.99, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that $CQ = \frac{1}{4} AC$. If PQ produced meets BC at R, Prove that R is a mid-point of BC.

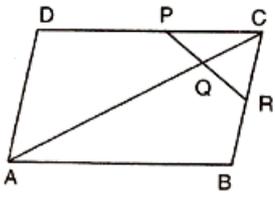
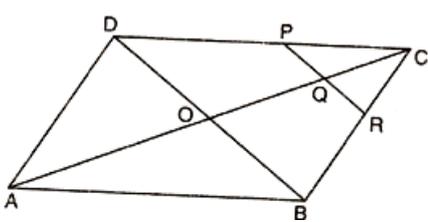


Fig. 14.99

Answer

Given,

Join B and D suppose AC and BD cut at D



Then, $OC = \frac{1}{2}AC$

Now, $CQ = \frac{1}{4}AC$

$= CQ = \frac{1}{2} \left(\frac{1}{2}AC \right) = \frac{1}{2} \times OC$

In ΔDCO , P & Q are midpoints of DC & OC

$\therefore PQ \parallel PO$

Also in ΔCOB , Q is mid point of OC and $QR \parallel OB$

$\therefore R$ is mid point of BC

16. Question

In Fig. 14.100, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

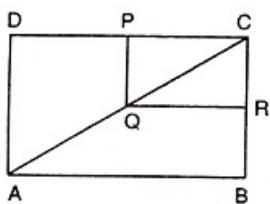


Fig. 14.100

(i) $DP = PC$ (ii) $PR = \frac{1}{2}AC$.

Answer

(i) In ΔADC , Q is mid point of AC such that

$PQ \parallel AD$

$\therefore P$ is mid point of DC

$= DP = DC$ (converse of mid point theorem)

(ii) Similarly, R is the mid point of BC

$= PR = \frac{1}{2}BD$

$PR = \frac{1}{2}AC \because$ diagonals of rectangle are equal

Proved.

17. Question

ABCD is a parallelogram, E and F are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP=PH.

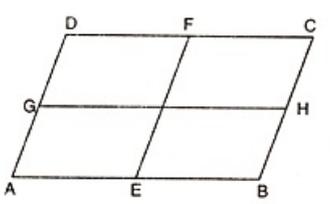
Answer

Since E and F are midpoints of AB and CD

$$\therefore AE = BE = \frac{1}{2} AB$$

And $CF = DF = \frac{1}{2} CD$

$$\because AB = CD$$



$$\therefore \frac{1}{2} AB = \frac{1}{2} CD$$

$$= BE = CF$$

\therefore BEFC is a parallelogram

$$= BE \parallel EF \text{ and } BF = PH \text{-----(i)}$$

Now, $BC \parallel EF$

$$= AD \parallel EF \because BC \parallel AD \text{ as } ABCD \text{ is a parallelogram}$$

= AEFD is a parallelogram

$$= AE = GP$$

But E is the midpoint of AB

$$\therefore AE = BE$$

$$= GP = PH \text{ proved}$$

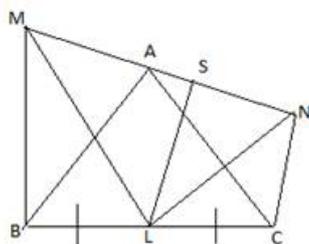
18. Question

BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM=LN.

Answer

draw LS perpendicular to line MN

\therefore the line BM, LS, CN being the same perpendiculars, on line MN are parallel to each other



We have, $BL = LC$ (L is mid point of BC)

Using intercept theorem,

$$MS = SN \dots\dots\dots(i)$$

Now in $\triangle MSL$ and $\triangle LSN$

$$MS = SN$$

$\angle LSM = \angle LSN = 90^\circ$ (LS perpendicular to MN) and $SL = LS$ is common

$\therefore \triangle MSL \cong \triangle LSN$ (SAS congruency)

$\therefore LM = LN$ proved

19. Question

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

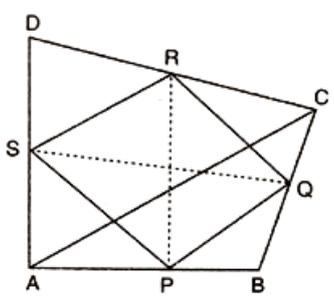
Answer

$ABCD$ is a quadrilateral

P, Q, R, S are mid points of sides AB, BC, CD and DA

In $\triangle ABC$, P and Q are the mid points of AB and AC respectively

So, by using mid point theorem,



$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \dots\dots\dots(i)$$

Similarly in $\triangle BCD$

$$RS \parallel AC \text{ and } RS = \frac{1}{2} AC \dots\dots\dots(ii)$$

From equation (i) and (ii)

$$PQ \parallel RS \text{ and } PQ = RS$$

Similarly, we have

$$PS \parallel QR \text{ and } PS = QR$$

Hence , $PQRS$ is a parallelogram.

Since, diagonals of a parallelogram bisect each other

Hence, PR and QS bisect each other proved

20. Question

Fill in the blanks to make the following statements correct:

- (i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is.....
- (ii) The triangle formed by joining the mid-points of the sides of a right triangle is.....
- (iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is.....

Answer

- (i) isosceles.
- (ii) Right triangle.

(iii) Parallelogram.

CCE - Formative Assessment

1. Question

In a parallelogram ABCD, write the sum of angles A and B.

Answer

Given,

In parallelogram ABCD

$\angle A + \angle B = 180^\circ$ (Adjacent angles of a parallelogram are supplementary)

2. Question

In a parallelogram ABCD, if $\angle D = 115^\circ$, then write the measure of $\angle A$.

Answer

Given,

In parallelogram ABCD

$\angle D = 115^\circ$ Given

$\therefore \angle A$ & $\angle D$ are adjacent angles of parallelogram

$\therefore \angle A + \angle D = 180^\circ$

$\angle A = 180^\circ - 115^\circ = 65^\circ$

3. Question

PQRS is a square such that PR and SQ intersect at O. State the measure of $\angle POQ$.

Answer

Given,

In square PQRS

We know that diagonals of a square bisect each other at 90°

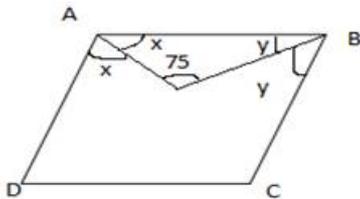
Hence, $\angle POQ = 90^\circ$

4. Question

In a quadrilateral ABCD, bisectors of angles A and B intersect at O such that $\angle AOB = 75^\circ$, then write the value of $\angle C + \angle D$.

Answer

Given,



In quadrilateral ABCD

$\angle x + \angle y + 75^\circ = 180^\circ$ (angle sum property of triangle)

$= \angle x + \angle y = 180^\circ - 75^\circ = 105^\circ$

$= 2(\angle x + \angle y) = 2 \times 105^\circ = 210^\circ$

$\therefore \angle C + \angle D = 360^\circ - 210^\circ = 150^\circ$

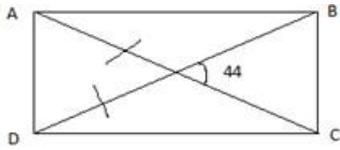
5. Question

The diagonals of a rectangle ABCD meet at O. If $\angle BOC = 44^\circ$, find $\angle OAD$.

Answer

Given,

ABCD is a rectangle



Diagonals meet at O, $\angle BOC = 44^\circ$

$\therefore \angle AOD = \angle BOC$ (vertically opposite angles)

$$= \angle AOD = 44^\circ$$

$$AO = OD$$

$\angle OAD = \angle ODA$ (Angles facing same side)

$$\text{So, } \angle OAD = \frac{180^\circ - 44^\circ}{2} = \frac{136^\circ}{2} = 68^\circ$$

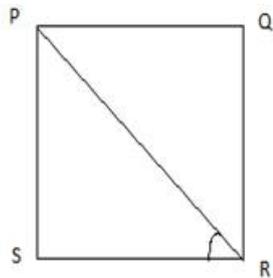
6. Question

If PQRS is a square, then write the measure of $\angle SRP$.

Answer

Given,

PQRS is a square.



We know that each angle of a square is 90° and diagonals of square bisect the angles

$$\text{Hence, } \angle SRP = \frac{90^\circ}{2} = 45^\circ$$

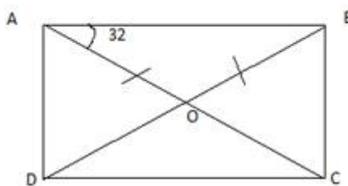
7. Question

If ABCD is a rectangle with $\angle BAC = 32^\circ$, find the measure of $\angle DBC$.

Answer

Given,

ABCD is a rectangle



$$\angle BAC = 32^\circ$$

∴ we know that diagonals of a rectangle bisect each other

Hence , $AO = BO$

And, $\angle DBA = \angle BAC = 32^\circ$ (angles in front of same sides)

∴ $\angle DBC + \angle DBA = 90^\circ$

∴ $\angle DBC = 90^\circ - 32^\circ = 58^\circ$

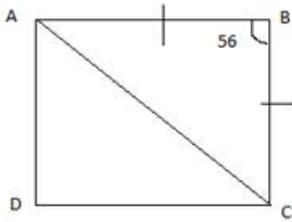
8. Question

If ABCD is a rhombus with $\angle ABC = 56^\circ$, find the measure of $\angle ACD$.

Answer

Given,

ABCD is a rhombus



$\angle ABC = 56^\circ$

Since , $AB = BC$

∴ $\angle BAC = \angle BCA = \frac{1}{2} (180^\circ - 56^\circ) = \frac{124^\circ}{2} = 62^\circ$

∴ $AB = BC$

$\angle ACD = \angle BAC = 62^\circ$

∴ $AB \parallel BC$

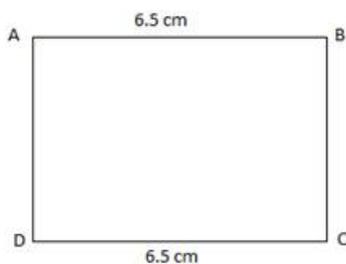
9. Question

The perimeter of a parallelogram is 22 cm. If the longer side measure 6.5 cm, what is the measure of shorter side?

Answer

Given,

Perimeter of a parallelogram = 22cm



Longer side = 6.5 cm

∴ $AB + BC + CD + DA = 22\text{cm}$

Let length of shorter side = x cm

= $2x + 2 \times 6.5 = 22$

= $2x = 22 - 13 = 9$

= $X = \frac{9}{2} = 4.5 \text{ cm}$

10. Question

If the angles of a quadrilateral are in the ratio 3:5:9:13, then find the measure of the smallest angle.

Answer

Given,

Ratio of angles of quadrilateral = 3:5:9:13

Let sides are = $3x, 5x, 9x, 13x$

= $3x+5x+9x+13x = 360^\circ$ (angle sum property of quadrilateral)

= $30x = 360^\circ$

= $x = \frac{360^\circ}{30} = 12$

So, length of smallest angle = $3x = 3 \times 12 = 36^\circ$

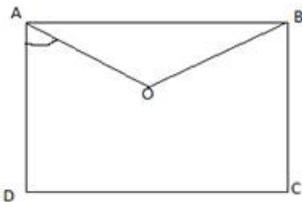
11. Question

If the bisectors of two adjacent angles A and B of a quadrilateral ABCD intersect at a point O such that $\angle C + \angle D = k\angle AOB$, then find the value of k.

Answer

Given ,

ABCD is a quadrilateral



$\angle C + \angle D = k \angle AOB$

$\therefore \angle D + \angle C = k$

= $\angle A + \angle B = 360^\circ - k$

= $\angle OAB + \angle OBA = \frac{360^\circ - k}{2}$

= $360^\circ - \frac{360^\circ - k}{2} = \frac{k}{2} = k = 2$

12. Question

In a parallelogram ABCD, if $\angle A = (3x-20)^\circ$, $\angle B = (y+15)^\circ$ and $\angle C = (x+40)^\circ$, then find the values of x and y.

Answer

Given,

In parallelogram ABCD

Opposite Angles are equal.

$\therefore \angle A = \angle C$

$\Rightarrow 3x-20 = x + 40$

$\Rightarrow 2x = 60$

$\Rightarrow x = 30$

Consecutive angles are supplementary ($A + B = 180^\circ$).

$\Rightarrow 3x - 20 + y + 15 = 180$

$$\Rightarrow 3x + y = 185$$

$$\Rightarrow 3 \times 30 + y = 185$$

$$\Rightarrow y = 185 - 90 = 95$$

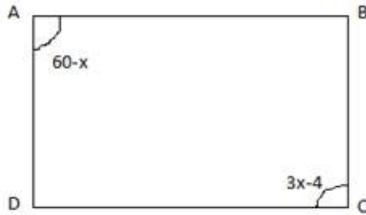
13. Question

If measures opposite angles of a parallelogram are $(60-x)^\circ$ and $(3x-4)^\circ$, then find the measures of angles of the parallelogram.

Answer

Given,

Measure of opposite angles of parallelogram = $(60-x)^\circ$, $(3x-4)^\circ$



$$= 60-x^\circ = 3x-4^\circ \text{ (opposite angles of parallelogram)}$$

$$= 4x = 56^\circ$$

$$= x = \frac{56^\circ}{4} = 14$$

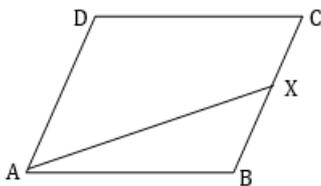
$$\text{Hence angles of parallelograms} = 60 - 14^\circ = 46^\circ$$

$$\text{Other two angles} = 180 - 46 = 134^\circ$$

14. Question

In a parallelogram ABCD, the bisector of $\angle A$ also bisects BC at X. Find AB : AD.

Answer



Given,

In parallelogram ABCD ,

Bisector of $\angle A$ bisects BC at X

$\therefore AD \parallel BC$ and AX cuts them so

$$\angle DAX = \angle AXB \text{ (alternate angles)}$$

$$\angle DAX = \angle XAB \text{ (AX is bisector of } \angle A)$$

$$\therefore \angle AXB = \angle XAB$$

$$AB = BX \text{ (sides opposite of equal angles)}$$

$$\text{Now, } \frac{AB}{AD} = \frac{BX}{BC}$$

$$= \frac{AB}{AD} = \frac{BX}{2BX} \text{ (}\because X \text{ is mid point of BC)}$$

$$= \frac{AB}{AD} = \frac{1}{2} = AB:AD = 1:2$$

15. Question

In Fig. 14.111, PQRS is an isosceles trapezium. Find x and y.

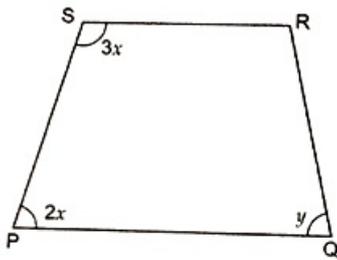


Fig. 14.111

Answer

Given,

PQRS is an isosceles trapezium

$PS = RQ$ (Given)

$= \angle P + \angle S = 180^\circ$ (sum of adjacent angles are supplementary)

$= 2x + 3x = 180^\circ$

$= 5x = 180^\circ$

$= x = \frac{180^\circ}{5} = 36^\circ$

$\because \angle P = \angle Q \because PS = RQ$

$= 2x = y$

$= 2 \times 36 = y$

$= y = 72^\circ$

16. Question

In Fig. 14.112, ABCD is a trapezium. Find the values of x and y.

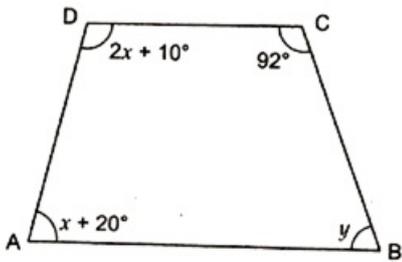


Fig. 14.112

Answer

Given,

ABCD is a trapezium

$= \angle A + \angle D = 180^\circ$

$= 2x + 10^\circ + x + 20^\circ = 180^\circ$

$= 3x + 30^\circ = 180^\circ$

$= x = \frac{150^\circ}{3} = 50^\circ$

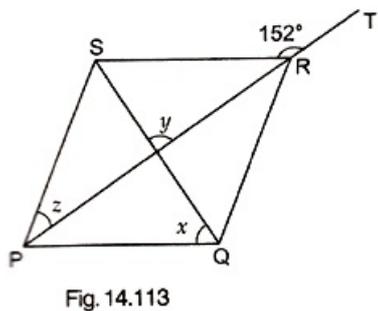
$\angle B + \angle C = 180^\circ$

$$= y + 92^\circ = 180^\circ$$

$$= y = 180^\circ - 92^\circ = 88^\circ$$

17. Question

In Fig. 14.113, PQRS is a rhombus in which the diagonal PR is produced to T. If $\angle SRT = 152^\circ$, find x, y and z.



Answer

Given,

PQRS is a rhombus

$$\angle SRT = 152^\circ$$

We know that diagonals of a rhombus bisect each other at 90°

$$\text{Hence, } \angle SOR = 90^\circ = y = 90^\circ$$

$$\angle SRT + \angle SRO = 180^\circ \text{ (linear pair of angles)}$$

$$= 152^\circ + \angle SRO = 180^\circ$$

$$\angle SRO = 180^\circ - 152^\circ = 28^\circ$$

$$\text{NOW, } \angle SRO = z \because SR = SP$$

$$= z = 28^\circ$$

$$\text{And, } \angle RSO + \angle SRO + y = 180^\circ \text{ (angle sum property of triangle)}$$

$$= \angle RSO + 28^\circ + 90^\circ = 180^\circ$$

$$= \angle RSO = 180^\circ - 118^\circ = 62^\circ$$

$$= \angle X = \angle RSO = 62^\circ \text{ (alternate angles)}$$

18. Question

In Fig. 14.114, ABCD is a rectangle in which diagonal AC is produced to E. If $\angle ECD = 146^\circ$, find $\angle AOB$.

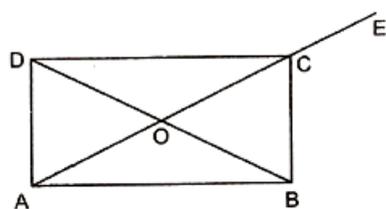


Fig. 14.114

Answer

Given,

ABCD is a rectangle

$$\angle ECD = 146^\circ$$

$$= \angle ECD + \angle DCO = 180^\circ \text{ (Linear pair of angles)}$$

$$= 146^\circ + \angle DCO = 180^\circ$$

$$= \angle DCO = 180^\circ - 146^\circ = 34^\circ$$

AND, $\angle BOC = 180^\circ - \angle OCB - \angle OBC \because \angle OCB = \angle OBC$

$$= \angle BOC = 180^\circ - 34^\circ - 34^\circ = 112^\circ$$

$\angle BOC + \angle AOB = 180^\circ$ (Linear pair)

$$= 112^\circ + \angle AOB = 180^\circ$$

$$= \angle AOB = 180^\circ - 112^\circ = 68^\circ$$

19. Question

In Fig. 14.115, ANCD and AEFH are two parallelograms. If $\angle C = 58^\circ$, find $\angle F$.

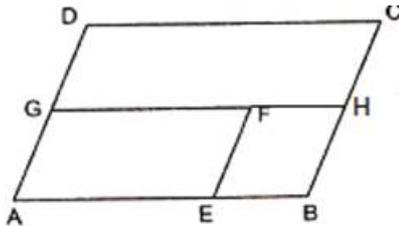


Fig. 14.115

Answer

Given,

ABCD & AEFH are two parallelograms

$$\angle C = 58^\circ$$

$\because AB \parallel CD, AE \parallel FH, BC \parallel AD, EF \parallel AG$

GF produced to H so $GH \parallel AB$

$$= \angle C = \angle H \text{ (corresponding angles)}$$

$$= \angle H = 58^\circ$$

$AD \parallel BC$ and GH cuts them

Hence, $\angle F = \angle H$ (corresponding angles)

$$\angle F = 58^\circ.$$

20. Question

Complete each of the following statements by means of one of those given in brackets against each:

(i) If one pair of opposite sides are equal and parallel, then the figure is (parallelogram, rectangle, trapezium)

(ii) If in a quadrilateral only one pair of opposite sides are parallel, the quadrilateral is (square, rectangle, trapezium)

(iii) A line drawn from the mid-point of one side of a triangle..... another side intersects the third side at its mid-point. (perpendicular to, parallel to, to meet)

(iv) If one angle of a parallelogram is a right angle, then it is necessarily a (rectangle, square, rhombus)

(v) Consecutive angles of a parallelogram are (Supplementary, complementary)

(vii) If opposite angles of a quadrilateral are equal, then it is necessarily a (parallelogram, rhombus, rectangle)

(vi) If both pairs of opposite sides of a quadrilateral are equal, then it is necessarily a (rectangle, parallelogram, rhombus)

(viii) If consecutive sides of a parallelogram are equal, then it is necessarily a (kite, rhombus, square)

Answer

- (i) parallelogram
- (ii) trapezium
- (iii) parallel to WW
- (iv) rectangle
- (v) supplementary
- (vi) parallelogram
- (vii) parallelogram
- (viii) rhombus

1. Question

The opposite sides of a quadrilateral have

- A. no common points
- B. one common point
- C. two common points
- D. infinitely many common points

Answer

no common points

2. Question

Two consecutive sides of a quadrilateral have

- A. no common points
- B. one common point
- C. two common points
- D. infinitely many common points

Answer

one common point

3. Question

PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases, PQRS is a parallelogram?

- A. $\angle P=100^\circ$, $\angle Q=80^\circ$, $\angle R=100^\circ$
- B. $\angle P=85^\circ$, $\angle Q=85^\circ$, $\angle R=95^\circ$
- C. $PQ=7$ cm, $QR=7$ cm, $RS=8$ cm, $SP=8$ cm
- D. $OP=6.5$ cm, $OQ=6.5$ cm, $OR=5.2$ cm, $OS=5.2$ cm

Answer

$\angle P=100^\circ$, $\angle Q=80^\circ$, $\angle R=100^\circ$

4. Question

Which of the following quadrilateral is not a rhombus?

- A. All four sides are equal

- B. Diagonals bisect each other
- C. Diagonals bisect opposite angles
- D. One angle between the diagonals is 60°

Answer

one angle between the diagonals is 60°

5. Question

Diagonals necessarily bisect opposite angles in a

- A. rectangle
- B. parallelogram
- C. isosceles trapezium
- D. square

Answer

square

6. Question

The two diagonals are equal in a

- A. parallelogram
- B. rhombus
- C. rectangle
- D. trapezium

Answer

rectangle

7. Question

We get a rhombus by joining the mid-points of the sides of a

- A. parallelogram
- B. rhombus
- C. rectangle
- D. triangle

Answer

rectangle

8. Question

The bisectors of any two adjacent angles of a parallelogram intersect at

- A. 30°
- B. 45°
- C. 60°
- D. 90°

Answer

90°

9. Question

The bisectors of the angle of a parallelogram enclose a

- A. parallelogram
- B. rhombus
- C. rectangle
- D. square

Answer

rectangle

10. Question

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a

- A. parallelogram
- B. rectangle
- C. square
- D. rhombus

Answer

parallelogram

11. Question

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a

- A. square
- B. rhombus
- C. trapezium
- D. none of these

Answer

rhombus

12. Question

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

- A. square
- B. rectangle
- C. trapezium
- D. none of these

Answer

rectangle

13. Question

The figure formed by joining the mid-points of the adjacent sides of a square is a

- A. rhombus
- B. square
- C. rectangle

D. parallelogram

Answer

square

14. Question

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

A. rectangle

B. parallelogram

C. rhombus

D. square

Answer

parallelogram

15. Question

If one side of a parallelogram is 24° less than twice the smallest angle, then the measure of the largest angle of the parallelogram is

A. 176°

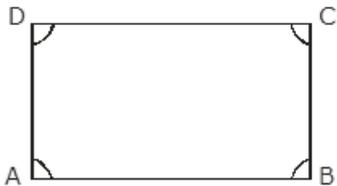
B. 68°

C. 112°

D. 102°

Answer

Let angles of parallelogram are $\angle A$, $\angle B$, $\angle C$, $\angle D$



Let smallest angle = $\angle A$

Let largest angle = $\angle B$

$$= \angle B = 2\angle A - 24^\circ \dots\dots\dots(i)$$

$$\angle A + \angle B = 180^\circ \text{ [adjacent angle of parallelogram]}$$

$$\text{So, } \angle A + 2\angle A - 24^\circ = 180^\circ$$

$$= 3\angle A = 180^\circ + 24^\circ = 204^\circ$$

$$= \angle A = \frac{204^\circ}{3} = 68^\circ$$

$$= \angle B = 2 \times 68^\circ - 24^\circ = 112^\circ$$

16. Question

In a parallelogram ABCD, if $\angle DAB = 75^\circ$ and $\angle DBC = 60^\circ$, then $\angle BDC =$

A. 75°

B. 60°

C. 45°

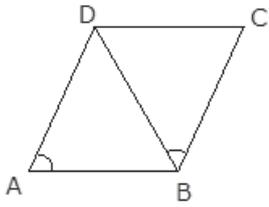
D. 55°

Answer

Given ,

In parallelogram ABCD,

$$\angle DAB = 75^\circ$$



$$\angle DBC = 60^\circ$$

$\therefore \angle DAB = \angle BCD$ [opposite angles of parallelogram]

$$\angle BCD = 75^\circ$$

Now in $\triangle BDC$

$$= \angle DBC + \angle BDC + \angle BCD = 180^\circ$$

$$= 60^\circ + \angle BDC + 75^\circ = 180^\circ$$

$$= \angle BDC = 180^\circ - 135^\circ$$

$$= \angle BDC = 45^\circ.$$

17. Question

ABCD is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF=

- A. AE
- B. BE
- C. CE
- D. DE

Answer

Given,

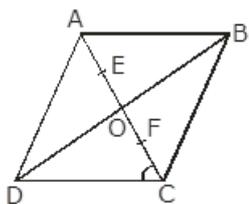
ABCD is a parallelogram

E & F are centroids of $\triangle ABD$ & $\triangle BCD$ respectively

We know that diagonals of parallelogram bisect each other & centroid of a median divides it in 2:1

So, in $\triangle ABD$,

$$= \frac{AE}{EO} = \frac{2}{1} \text{ OR } EO = \frac{1}{3}AO \text{ \& } AE = \frac{2}{3} AO \dots\dots\dots (I)$$



$$\text{Similarly, } FO = \frac{1}{3}CO \text{ \& } CF = \frac{2}{3}CO \dots\dots\dots (II)$$

$$= AO = CO$$

$$= \frac{1}{3}AO = \frac{1}{3}CO$$

From equations (i) & (ii)

$$EO = FO$$

$$EF = 2 FO \dots\dots\dots(III)$$

$$AE = CF \dots\dots(IV)$$

From equation (i)

$$= AE = \frac{2}{3}AO = \frac{2}{3}CO$$

$$= AE = \frac{2}{3}(CF + FO)$$

$$= AE = \frac{2}{3}(AE + FO) \text{ from equation (iv)}$$

$$= AE = \frac{2}{3}AE + \frac{2}{3}FO$$

$$= AE - \frac{2}{3}AE = \frac{2}{3}FO$$

$$= \frac{1}{3}AE = \frac{2}{3}FO$$

$$= \frac{1}{3}AE = \frac{1}{3}EF \text{ from equation (iii)}$$

$$= AE = EF$$

18. Question

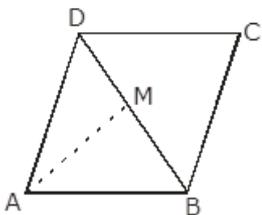
ABCD is a parallelogram M is the mid-point of BD and BM bisects $\angle B$. Then, $\angle AMB =$

- A. 45°
- B. 60°
- C. 90°
- D. 75°

Answer

Given,

ABCD is a parallelogram,



M is mid point of BD

BM bisects $\angle B$

$\therefore \angle A + \angle B = 180^\circ$ [adjacent angles of parallelogram are supplementary]

$$\text{In } \triangle AMB, \angle AMB + \frac{1}{2}\angle A + \frac{1}{2}\angle B = 180^\circ$$

$$= \angle AMB + \frac{1}{2}(\angle A + \angle B) = 180^\circ$$

$$= \angle AMB + \frac{1}{2} \times 180^\circ = 180^\circ$$

$$= \angle AMB = 180^\circ - 90^\circ = 90^\circ$$

19. Question

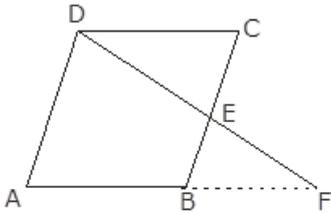
ABCD is a parallelogram and E is the mid point of BC. DE and AB when produced meet at F. Then, AF=

- A. $\frac{3}{2}$ AB
- B. 2 AB
- C. 3 AB
- D. $\frac{5}{4}$ AB

Answer

Given,

ABCD is a parallelogram



E is mid point of BC

DE & AB after producing meet at F

In $\triangle ECD$ & $\triangle BEF$,

$\angle BEF = \angle CED$ [vertically opposite angles]

$BE = EC$

$\angle EDC = \angle EFB$ [alternate angles]

$\therefore \triangle ECD \cong \triangle BEF$

So, $CD = BF$

$\therefore AB = CD$

Thus, $AF = AB + BF$

$= AF = AB + CD$

$= AF = AB + AB$

$= AF = 2AB$

20. Question

If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is

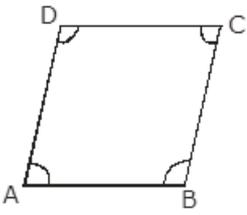
- A. 108°
- B. 54°
- C. 72°
- D. 81°

Answer

Given,

In parallelogram ABCD

$\angle A + \angle B = 180^\circ$ [adjacent angles of parallelogram are supplementary]



$$\angle A = \frac{3}{2} \angle B$$

$$\text{So, } \frac{3}{2} \angle B + \angle B = 180^\circ$$

$$= \frac{5}{2} \angle B = 180^\circ$$

$$= \angle B = \frac{180^\circ \times 2}{5} = 72^\circ$$

$$\angle A = \frac{3}{2} \times 72^\circ = 108^\circ$$

Thus smallest angle of parallelogram is 72°

21. Question

If the degree measures of the angles of quadrilateral are $4x$, $7x$, $9x$ and $10x$, what is the sum of the measures of the smallest angle and largest angle?

- A. 140°
- B. 150°
- C. 168°
- D. 180°

Answer

Given,

ABCD is a parallelogram,

Angles of quadrilateral $4x$, $7x$, $9x$, $10x$

$$= 4x + 7x + 9x + 10x = 360^\circ \text{ [angle sum property of quadrilateral]}$$

$$= 30x = 360^\circ$$

$$= x = \frac{360^\circ}{30} = 12^\circ$$

$$\text{Hence, sum of smallest and largest angles} = 4x + 10x = 4 \times 12 + 10 \times 12$$

$$= 48^\circ + 120^\circ = 168^\circ$$

22. Question

In a quadrilateral ABCD, $\angle A + \angle C$ is 2 times $\angle B + \angle D$. If $\angle A = 140^\circ$ and $\angle D = 60^\circ$, then $\angle B =$

- A. 60°
- B. 80°
- C. 120°
- D. none of these

Answer

Given,

ABCD is a parallelogram

$$\angle A + \angle C = 2(\angle B + \angle D)$$

$$\angle A = 40^\circ$$

$$\because \angle A + \angle B + \angle C + \angle D = 360^\circ \text{ [angle sum property of quadrilateral]}$$

$$= \angle A + \angle C + \angle B + \angle D = 360^\circ$$

$$= 2(\angle B + \angle D) + \angle B + \angle D = 360^\circ$$

$$= 3(\angle B + \angle D) = 360^\circ$$

$$= \angle B + \angle D = \frac{360^\circ}{3} = 120^\circ$$

$$\because \angle D = 60^\circ \text{ [given]}$$

$$\therefore \angle B = 120^\circ - 60^\circ = 60^\circ$$

23. Question

If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to

A. 16 cm

B. 15 cm

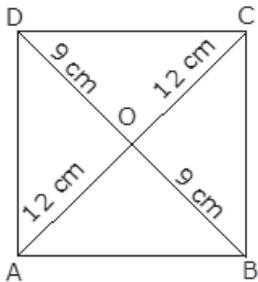
C. 20 cm

D. 17 cm

Answer

Given,

ABCD is a rhombus



$$AC = 24 \text{ cm}, BD = 18 \text{ cm}$$

$$= AB = BC = CD = DA \text{ [side of rhombus]}$$

We know that diagonals of rhombus bisect each other at 90°

In right $\triangle AOB$

$$AB^2 = BO^2 + AO^2$$

$$AB^2 = 12^2 + 9^2 = 144 + 81 = 225$$

$$AB = \sqrt{225} = 15 \text{ cm}$$

Side of rhombus = 15 cm

24. Question

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If $\angle ABD = 50^\circ$, then $\angle DPC =$

A. 70°

B. 90°

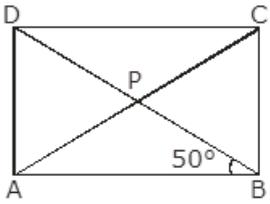
C. 80°

D. 100°

Answer

Given,

ABCD is a rectangle



Diagonals AC & BD intersect each other at P

$$\angle ABD = 50^\circ$$

∴ diagonals of rectangle bisect each other and are equal in length

$$= \angle ABD = \angle PDC \text{ [alternate angles]}$$

$$\angle PDC = \angle PCD = 50^\circ$$

In $\triangle DPC$

$$= \angle DPC + \angle PCD + \angle PDC = 180^\circ$$

$$= \angle DPC + 50^\circ + 50^\circ = 180^\circ$$

$$= \angle DPC = 180^\circ - 100^\circ = 80^\circ$$

25. Question

ABCD is a parallelogram in which diagonal AC bisects $\angle BAD$. If $\angle BAC = 35^\circ$, then $\angle ABC =$

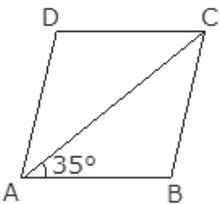
- A. 70°
- B. 110°
- C. 90°
- D. 120°

Answer

Given,

ABCD is a parallelogram

Diagonal AC bisects $\angle BAD$



$$\angle BAC = 35^\circ$$

∴ $\angle A + \angle B = 180^\circ$ (i) [angle sum property of quadrilateral]

$$\angle A = 2\angle BAC = 2 \times 35^\circ = 70^\circ$$

Putting value of $\angle A$ in equation (i)

$$= 70^\circ + \angle B = 180^\circ$$

$$= \angle B = 180^\circ - 70^\circ = 110^\circ$$

$$\angle ABC = 110^\circ$$

26. Question

In a rhombus ABCD, if $\angle ACB = 40^\circ$, then $\angle ADB =$

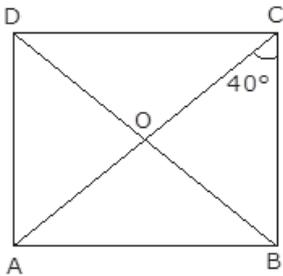
- A. 70°
- B. 45°
- C. 50°
- D. 60°

Answer

Given,

ABCD is a rhombus

$$\angle ACB = 40^\circ$$



$$\therefore \angle DAC = \angle ACB = 40^\circ \text{ [alternate angles]}$$

$$\angle AOD = 90^\circ \text{ [diagonals of rhombus are perpendicular to each other]}$$

In $\triangle AOD$

$$\angle AOD + \angle ADO + \angle DAO = 180^\circ$$

$$\angle ADO + 90^\circ + 40^\circ = 180^\circ$$

$$= \angle ADO = 180^\circ - 130^\circ = 50^\circ$$

$$= \angle ADO = \angle ADB = 50^\circ$$

27. Question

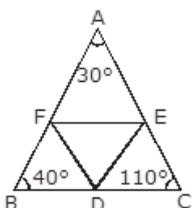
In $\triangle ABC$, $\angle A = 30^\circ$, $\angle B = 40^\circ$ and $\angle C = 110^\circ$. The angles of the triangle formed by joining the mid-points of the sides of this triangle are

- A. $70^\circ, 70^\circ, 40^\circ$
- B. $60^\circ, 40^\circ, 80^\circ$
- C. $30^\circ, 40^\circ, 110^\circ$
- D. $60^\circ, 70^\circ, 50^\circ$

Answer

Given,

In $\triangle ABC$



$$\angle A = 30^\circ, \angle B = 40^\circ, \angle C = 110^\circ$$

In figure, BDEF, DCEF, DEAF are parallelograms so,

$$\angle B = \angle E = 40^\circ [\because \angle B \& \angle E \text{ are opposite angles of parallelogram BDEF}]$$

$$\angle C = \angle F = 110^\circ [\because \angle C \& \angle F \text{ are opposite angles of parallelogram DCEF}]$$

$$\angle A = \angle D = 30^\circ [\because \angle A \& \angle D \text{ are opposite angles of parallelogram DEAF}]$$

Hence, $\angle D = 30^\circ$

$$\angle E = 40^\circ$$

$$\angle F = 110^\circ$$

28. Question

The diagonals of a parallelogram ABCD intersect at O. If $\angle BOC = 90^\circ$ and $\angle BDC = 50^\circ$, then $\angle OAB =$

A. 40°

B. 50°

C. 10°

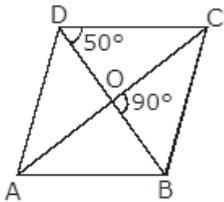
D. 90°

Answer

Given,

ABCD is a parallelogram

Diagonals of parallelogram intersect at O



$$\angle BOC = 90^\circ, \angle BDC = 50^\circ$$

$$\because \angle BOC + \angle AOB = 180^\circ \text{ [Linear pair of angles]}$$

$$90^\circ + \angle AOB = 180^\circ$$

$$= \angle AOB = 180^\circ - 90^\circ = 90^\circ$$

$$\angle BDC = \angle OBA = 50^\circ \text{ [Alternate angles]}$$

In $\triangle AOB$

$$\angle AOB + \angle OBA + \angle OAB = 180^\circ$$

$$= 90^\circ + 50^\circ + \angle OAB = 180^\circ$$

$$= \angle OAB = 180^\circ - 140^\circ = 40^\circ$$

29. Question

ABCD is a trapezium in which $AB \parallel DC$. M and N are the mid-points of AD and BC respectively, If $AB = 12\text{cm}$, $MN = 14\text{cm}$, then $CD =$

A. 10 cm

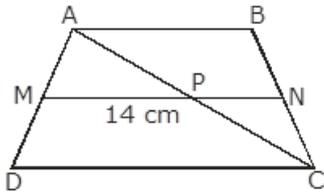
B. 12 cm

C. 14 cm

D. 16 cm

Answer

Given,



ABCD is a trapezium

$AB \parallel DC$

M, N are mid points of AD & BC

$AB = 12\text{ cm}$, $MN = 14\text{ cm}$

$\therefore AB \parallel MN \parallel CD$ [M, N are mid points of AD & BC]

$MP = NP$

By mid point theorem,

$MP = \frac{1}{2}CD$ AND $NP = \frac{1}{2}AB$

$\therefore MN = \frac{1}{2}(AB + CD)$

$= 14 = \frac{1}{2}(12 + CD) \Rightarrow CD = 28 - 12 = 16\text{ cm}$

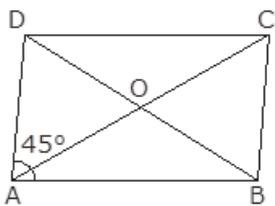
30. Question

Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 45^\circ$, then $\angle B =$

- A. 115°
- B. 120°
- C. 125°
- D. 135°

Answer

Given,



ABCD is a quadrilateral

$\angle A = 45^\circ$,

\therefore diagonals of quadrilateral bisect each other hence ABCD is a parallelogram,

$\angle A + \angle B = 180^\circ$

$= 45^\circ + \angle B = 180^\circ$

$\Rightarrow \angle B = 180^\circ - 45^\circ = 135^\circ$

31. Question

P is the mid point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. If $AD = 10\text{ cm}$, then $CD =$

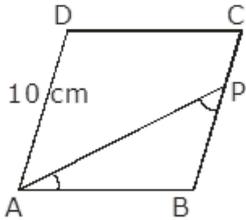
- A. 5 cm

- B. 6 cm
- C. 8 cm
- D. 10 cm

Answer

Given,

ABCD is a parallelogram,



P is mid point of side BC

$$\angle BAP = \angle DAP$$

$$AD = 10\text{cm}$$

$$\therefore AD \parallel BC$$

$$\angle DAP = \angle APB \text{ [alternate angles]}$$

$$\angle DAP = \angle BAP$$

$$AB = BP \text{ (side opposite to equal angles)}$$

$$= BP = \frac{1}{2}BC$$

$$\therefore AB = \frac{1}{2}BC = \frac{1}{2} \times 10 = 5\text{cm}$$

$$AB = CD = 5\text{cm (sides of parallelogram)}$$

$$\text{Hence , } CD = 5 \text{ cm}$$

32. Question

In $\triangle ABC$, E is the mid-point of median AD such that BE produced meets AC at F. If $AC = 10.5 \text{ cm}$, then $AF =$

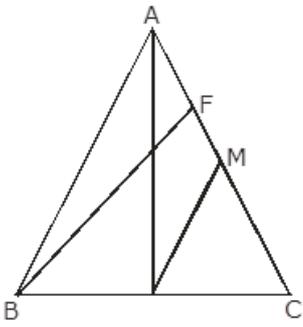
- A. 3 cm
- B. 3.5 cm
- C. 2.5 cm
- D. 5 cm

Answer

Given,

In $\triangle ABC$

E is mid point of median AD



$AC = 10.5 \text{ cm}$

Draw $DM \parallel EF$

$\therefore E$ is mid point of AD so F is mid point of AM

$AF = FM \dots\dots\dots(i)$

In ΔBFC

$EF \parallel DM$

SO, $FM = MC \dots\dots\dots(ii)$

From (i) & (ii)

$AF = MC \dots\dots\dots(iii)$

$AC = AF + MC + FM$

$= AC = AF + AF + AF$ From (i) (ii) & (iii)

$AC = 3AF$

$AF = \frac{1}{3}AC$

$AF = \frac{1}{3} \times 10.5 = 3.5 \text{ cm}$