

10. Congruent Triangles

Exercise 10.1

1. Question

In Fig. 10.22, the sides BA and CA have been produced such that $BA = AD$ and $CA = AE$. Prove that segment $DE \parallel BC$.

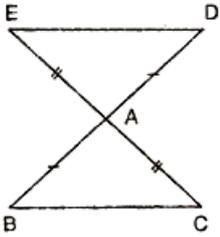


Fig. 10.22

Answer

Given,

The sides BA and CA have been produced, such that:

$$BA = AD$$

$$\text{And, } CA = AE$$

We have to prove that,

$$DE \parallel BC$$

Consider $\triangle BAC$ and $\triangle DAE$, we have

$$BA = AD \text{ and } CA = AE \text{ (Given)}$$

$$\angle BAC = \angle DAE \text{ (Vertically opposite angle)}$$

So, by SAS congruence rule we have:

$$\triangle BAC \cong \triangle DAE$$

Therefore, $BC = DE$ and

$$\angle DEA = \angle BCA,$$

$$\angle EDA = \angle CBA \text{ (By c.p.c.t)}$$

Now, DE and BC are two lines intersected by a transversal DB such that,

$$\angle DEA = \angle BCA,$$

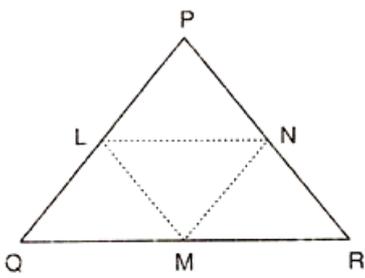
i.e., Alternate angles are equal

Therefore, $DE \parallel BC$

2. Question

In a $\triangle PQR$, if $PQ = QR$ and L , M and N are the mid points of the sides PQ , QR and RP respectively, Prove that $LN = MN$.

Answer



Given that in ΔPQR ,

$$PQ = QR$$

And, L, M, N are the mid points of the sides PQ, QR and RP respectively.

We have to prove that,

$$LN = MN$$

Here, we can observe that PQR is an isosceles triangle

$$PQ = QR$$

$$\text{And, } \angle QPR = \angle QRP \text{ (i)}$$

And also, L and M are the mid points of PQ and QR respectively

$$PL = LQ = \frac{PQ}{2}$$

$$QM = MR = \frac{QR}{2}$$

And, $PQ = QR$

$$PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2} \text{ (ii)}$$

Now, in ΔLPN and ΔMRN

$$LP = MR \text{ (From ii)}$$

$$\angle LPN = \angle MRN \text{ (From i)}$$

$$PN = NR \text{ (N is the mid-point of PR)}$$

Hence, By SAS theorem

$$\Delta LPN \cong \Delta MRN$$

Therefore, $LN = MN$ (By c.p.c.t)

3. Question

In Fig. 10.23, PQRS is a square and SRT is an equilateral triangle. Prove that

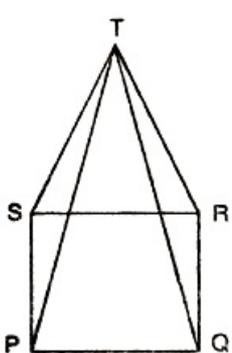


Fig. 10.23

$$\text{(i) } PT = QT \text{ (ii) } \angle TQR = 15^\circ$$

Answer

Given,

PQRS is a square and SRT is an equilateral triangle

To prove: (i) $PT = QT$

(ii) $\angle TQR = 15^\circ$

Proof: $PQ = QR = RS = SP$ (As PQRS is a square, all sides will be equal) (i)

And, $\angle SPQ = \angle PQR = \angle QRS = \angle RSP = 90^\circ$

And also,

SRT is an equilateral triangle

$SR = RT = TS$ (ii)

And, $\angle TSR = \angle SRT = \angle RTS = 60^\circ$

From (i) and (ii)

$PQ = QR = SP = SR = RT = TS$ (iii)

$\angle TSP = \angle TSR + \angle RSP$

$= 60^\circ + 90^\circ = 150^\circ$

$\angle TRQ = \angle TRS + \angle SRQ$

$= 60^\circ + 90^\circ = 150^\circ$

Therefore, $\angle TSP = \angle TRQ = 150^\circ$ (iv)

Now, in $\triangle TSP$ and $\triangle TRQ$, we have

$TS = TR$ (From iii)

$\angle TSP = \angle TRQ$ (From iv)

$SP = RQ$ (From iii)

Therefore, By SAS theorem,

$\triangle TSP \cong \triangle TRQ$

$PT = QT$ (BY c.p.c.t)

In $\triangle TQR$

$QR = TR$ (From iii)

Hence, $\triangle TQR$ is an isosceles triangle.

Therefore, $\angle QTR = \angle TQR$ (Angles opposite to equal sides)

Now,

Sum of angles in a triangle is 180°

$\angle QTR + \angle TQR + \angle TRQ = 180^\circ$

$2\angle TQR + 150^\circ = 180^\circ$ (From iv)

$2\angle TQR = 30^\circ$

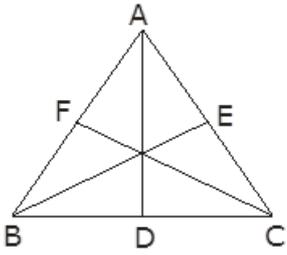
$\angle TQR = 15^\circ$

Hence, proved

4. Question

Prove that the medians of an equilateral triangle are equal.

Answer



To prove: The medians of an equilateral triangle are equal.

Median = The line joining the vertex and mid-points of opposite sides.

Proof: Let $\triangle ABC$ be an equilateral triangle

AD , BE and CF are its medians.

Let,

$$AB = AC = BC = x$$

In $\triangle BFC$ and $\triangle CEB$, we have

$$AB = AC \text{ (Sides of equilateral triangle)}$$

$$\frac{1}{2}AB = \frac{1}{2}AC$$

$$BF = CE$$

$$\angle ABC = \angle ACB \text{ (Angles of equilateral triangle)}$$

$$BC = BC \text{ (Common)}$$

Hence, by SAS theorem, we have

$$\triangle BFC \cong \triangle CEB$$

$$BE = CF \text{ (By c.p.c.t)}$$

Similarly, $AB = BE$

$$\text{Therefore, } AD = BE = CF$$

Hence, proved

5. Question

In a $\triangle ABC$, if $\angle A = 120^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Answer

Given,

$$\angle A = 120^\circ$$

$$AB = AC$$

We have to find $\angle B$ and $\angle C$:

We can observe that $\triangle ABC$ is an isosceles triangle since $AB = AC$

$$\angle B = \angle C \text{ (Angle opposite to equal sides are equal) [i]}$$

We know that,

Sum of angles in a triangle is equal to 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + \angle B = 180^\circ$$

$$\angle A + 2\angle B = 180^\circ$$

$$120^\circ + 2\angle B = 180^\circ$$

$$2\angle B = 180^\circ - 120^\circ$$

$$2\angle B = 60^\circ$$

$$\angle B = 30^\circ$$

$$\angle B = \angle C = 30^\circ$$

6. Question

In a $\triangle ABC$, if $AB = BC$ 120° and $\angle B = 70^\circ$, Find $\angle A$.

Answer

Consider $\triangle ABC$,

We have,

$$\angle B = 70^\circ$$

And, $AB = AC$

Therefore, $\triangle ABC$ is an isosceles triangle.

$\angle B = \angle C$ (Angle opposite to equal sides are equal)

$$\angle B = \angle C = 70^\circ$$

And $\angle A + \angle B + \angle C = 180^\circ$ (Angles of triangle)

$$\angle A + 70^\circ + 70^\circ = 180^\circ$$

$$\angle A = 40^\circ$$

7. Question

The vertical angle of an isosceles triangle is 100° . Find its base angles.

Answer

Consider an isosceles triangle ABC ,

Such that:

$$AB = AC$$

Given,

Vertical $\angle A$ is 100°

To find: Base angle

Since, $\triangle ABC$ is isosceles,

$\angle B = \angle C$ (Angle equal to opposite sides)

And,

$\angle A + \angle B + \angle C = 180^\circ$ (Angles of triangle)

$$100^\circ + \angle B + \angle B = 180^\circ$$

$$\angle B = 40^\circ$$

$$\angle B = \angle C = 40^\circ$$

8. Question

In Fig. 10.24, $AB = AC$ and $\angle ACD = 105^\circ$, find $\angle BAC$.

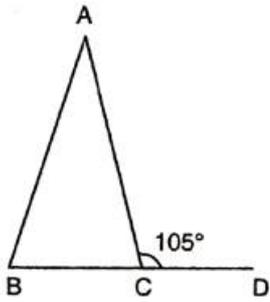


Fig. 10.24

Answer

Given,

$$AB = AC$$

$$\angle ACD = 105^\circ$$

Since, $\angle BCD = 180^\circ$ (Straight angle)

$$\angle BCA + \angle ACD = 180^\circ$$

$$\angle BCA + 105^\circ = 180^\circ$$

$$\angle BCA = 75^\circ \text{ (i)}$$

Now,

$\triangle ABC$ is an isosceles triangle

$$\angle ABC = \angle ACB \text{ (Angle opposite to equal sides)}$$

From (i), we have

$$\angle ACB = 75^\circ$$

$$\angle ABC = \angle ACB = 75^\circ$$

Sum of interior angle of triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A = 180^\circ - 75^\circ - 75^\circ$$

$$= 30^\circ$$

Therefore, $\angle BAC = 30^\circ$

9. Question

Find the measure of each exterior angle of an equilateral triangle.

Answer

To Find: Measure of each exterior angle of an equilateral triangle

Consider an equilateral triangle ABC.

We know that, for an equilateral triangle

$$AB = AC = CA$$

$$\text{And, } \angle ABC = \angle BCA = \angle CAB = \frac{180}{3}$$

$$= 60^\circ \text{ (i)}$$

Now, Extend side BC to D,

CA to E and AB to F

Here,

BCD is a straight line segment

$$\angle BCD = \text{Straight line segment} = 180^\circ$$

$$\angle BCA + \angle ACD = 180^\circ$$

$$60^\circ + \angle ACD = 180^\circ \text{ (From i)}$$

$$\angle ACD = 120^\circ$$

Similarly, we can find $\angle EAB$ and $\angle FBC$ also as 120° because ABC is an equilateral triangle.

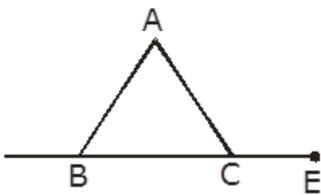
$$\text{Therefore, } \angle ACD = \angle EAB = \angle FBC = 120^\circ$$

Hence, the measure of each exterior angle of an equilateral triangle is 120°

10. Question

If the base of an isosceles triangle is produced on both sides, prove that the exterior angles so formed are equal to each other.

Answer



To prove: the exterior angles formed are equal to each other

$$\text{i.e., } \angle ADB = \angle ACE$$

Proof: Let ABC be an isosceles triangle

Where BC is the base of the triangle and AB and AC are its equal sides.

$$\angle ABC = \angle ACB$$

$$\angle B = \angle C \text{ (Angle opposite to equal sides)}$$

Now,

$$\angle ADB + \angle ABC = 180^\circ$$

$$\angle ACB + \angle ACE = 180^\circ$$

$$\angle ADB = 180^\circ - \angle B$$

And

$$\angle ACE = 180^\circ - \angle C$$

$$\angle ADB = 180^\circ - \angle B$$

And

$$\angle ACE = 180^\circ - \angle C$$

$$\angle ADB = \angle ACE$$

Hence, proved

11. Question

In Fig. 10.25, $AB = AC$ and $DB = DC$, find the ratio $\angle ABD : \angle ACD$.

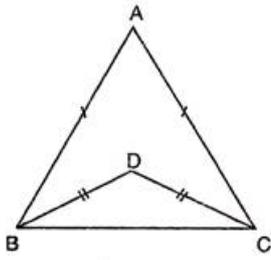


Fig. 10.25

Answer

Consider the figure,

Given,

$$AB = AC$$

$$DB = DC$$

To find: Ratio $\angle ABD = \angle ACD$

Now, $\triangle ABC$ and $\triangle DBC$ are isosceles triangles

Since, $AB = AC$

And,

$$DB = DC$$

Therefore, $\angle ABC = \angle ACB$ and,

$\angle DBC = \angle DCB$ (Angle opposite equal sides)

Now, consider $\angle ABD : \angle ACD$

$$(\angle ABC - \angle DBC) : (\angle ACB - \angle DCB)$$

$$(\angle ABC - \angle DBC) : (\angle ABC - \angle DBC) \text{ [Since, } \angle ABC = \angle ACB \text{ and } \angle DBC = \angle DCB]$$

$$1 : 1$$

Therefore, $\angle ABD : \angle ACD = 1 : 1$

12. Question

Determine the measure of each of the equal angles of a right angled isosceles triangle.

OR

ABC is a right-angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Answer

Given,

ABC is a right-angled triangle

$$\angle A = 90^\circ$$

And,

$$AB = AC$$

To find: $\angle B$ and $\angle C$

Since, $AB = AC$

Therefore, $\angle B = \angle C$

And, Sum of angles in a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$90^\circ + 2\angle B = 180^\circ$$

$$2\angle B = 90^\circ$$

$$\angle B = 45^\circ$$

Hence, the measure of each angle of the equal angles of a right angle isosceles triangle is 45° .

13. Question

AB is a line segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (See Fig. 10.26). Show that the line PQ is perpendicular bisector of AB .

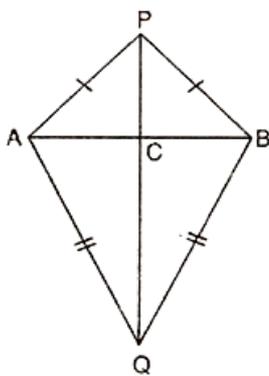


Fig. 10.26

Answer

Consider the figure,

We have AB is a line segment

P, Q are the points on opposite sides on AB

Such that,

$$AP = BP \text{ (i)}$$

$$AQ = BQ \text{ (ii)}$$

To prove: PQ is perpendicular bisector of AB

Proof: Now, consider $\triangle PAQ$ and $\triangle PBQ$,

$$AP = BP \text{ (From i)}$$

$$AQ = BQ \text{ (From ii)}$$

$$PQ = PQ \text{ (Common)}$$

Therefore, By SSS theorem

$$\triangle PAQ \cong \triangle PBQ \text{ (iii)}$$

Now, we can observe that $\triangle APB$ and $\triangle AQB$ are isosceles triangles [From (i) and (ii)]

$$\angle PAB = \angle PBA$$

And,

$$\angle QAB = \angle QBA$$

Consider, $\triangle PAC$ and $\triangle PBC$

C is the point of intersection of AB and PQ

PA = PB [From (i)]

$\angle APC = \angle BPC$ [From (iii)]

PC = PC (Common)

By SAS theorem,

$\triangle PAC \cong \triangle PBC$

AC = CB

And, $\angle PCA = \angle PCB$ (By c.p.c.t) (iv)

And also,

ACB is line segment

$\angle ACP + \angle BCP = 180^\circ$

But, $\angle ACP = \angle PCB$

$\angle ACP = \angle PCB = 90^\circ$ (v)

We have,

AC = CB

C is the mid-point of AB

From (iv) and (v), we conclude that

PC is the perpendicular bisector of AB

Since, C is the point on line PQ, we can say that PQ is the perpendicular bisector of AB.

Exercise 10.2

1. Question

In Fig. 10.40, it is given that $RT = TS$, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$. Prove that $\triangle RBT \cong \triangle SAT$.

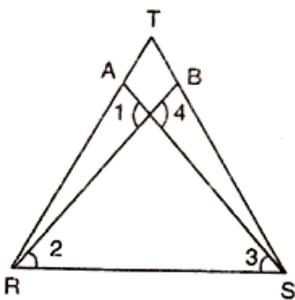


Fig. 10.40

Answer

In the figure, given that:

$RT = TS$ (i)

$\angle 1 = 2\angle 2$ (ii)

And,

$\angle 4 = 2\angle 3$ (iii)

To prove: $\triangle RBT \cong \triangle SAT$

Let the point of intersection of RB and SA be denoted by O.

Since, RB and SA intersect at O.

$$\angle AOR = \angle BOS \text{ (Vertically opposite angle)}$$

$$\angle 1 = \angle 4$$

$$2\angle 2 = 2\angle 3 \text{ [From (ii) and (iii)]}$$

$$\angle 2 = \angle 3 \text{ (iv)}$$

Now, we have in $\triangle TRS$

$$RT = TS$$

$\triangle TRS$ is an isosceles triangle

$$\text{Therefore, } \angle TRS = \angle TSI \text{ (v)}$$

But, we have

$$\angle TRS = \angle TRB + \angle 2 \text{ (vi)}$$

$$\angle TSR = \angle TSA + \angle 3 \text{ (vii)}$$

Putting (vi) and (vii) in (v), we get

$$\angle TRB + \angle 2 = \angle TSA + \angle 3$$

$$\angle TRB = \angle TSA \text{ [From (iv)]}$$

Now, in $\triangle RBT$ and $\triangle SAT$

$$RT = ST \text{ [From (i)]}$$

$$\angle TRB = \angle TSA \text{ [From (iv)]}$$

$$\angle RTB = \angle STA \text{ (Common angle)}$$

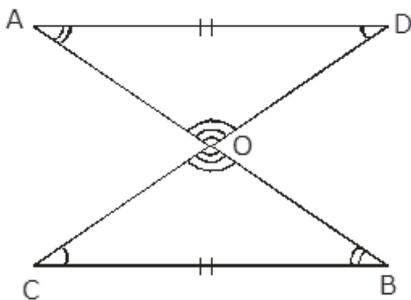
By ASA theorem,

$$\triangle RBT \cong \triangle SAT$$

2. Question

Two lines AB and CD intersect at O such that BC is equal and parallel to AD . Prove that the lines AB and CD bisect at O .

Answer



Given that,

Lines AB and CD intersect at O such that:

$$BC \parallel AD$$

$$\text{And, } BC = AD \text{ (i)}$$

To prove: AB and CD bisect at O

Proof: In $\triangle AOD$ and $\triangle BOC$

$$AD = BC \text{ [From (i)]}$$

$$\angle OBC = \angle OAD \text{ (} AD \parallel BC \text{ and } AB \text{ is transversal)}$$

$$\angle OCB = \angle ODA \text{ (} AD \parallel BC \text{ and } CD \text{ is transversal)}$$

Therefore, by ASA theorem:

$$\Delta AOD \cong \Delta BOC$$

$$OA = OB \text{ (By c.p.c.t)}$$

And,

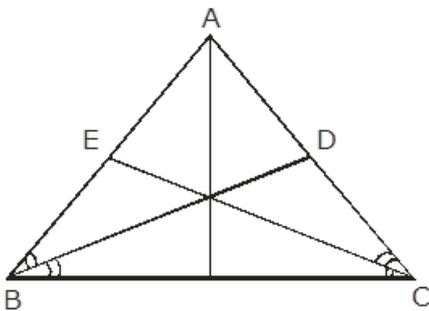
$$OD = OC \text{ (By c.p.c.t)}$$

Hence, AB and CD bisect each other at O .

3. Question

BD and CE are bisectors of $\angle B$ and $\angle C$ of an isosceles ΔABC with $AB = BC$. Prove that $BD = CE$.

Answer



Given,

In isosceles ΔABC ,

BD and CE are bisectors of $\angle B$ and $\angle C$

And,

$$AB = AC$$

To prove: $BD = CE$

Proof: In ΔBEC and ΔCDB , we have

$$\angle B = \angle C \text{ (Angles opposite to equal sides)}$$

$$BC = BC \text{ (Common)}$$

$$\angle BCE = \angle CBD \text{ (Since, } \angle C = \angle B \frac{1}{2} \angle C = \frac{1}{2} \angle B \angle BCE = \angle CBD)$$

By ASA theorem, we have

$$\Delta BEC \cong \Delta CDB$$

$$EC = BD \text{ (By c.p.c.t)}$$

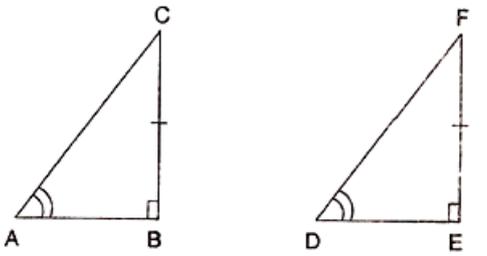
Hence, proved

Exercise 10.3

1. Question

In two triangles one side and acute angle of one are equal to the corresponding side and angle of the other. Prove that the triangles are congruent.

Answer



Given that in two right angle triangles one side and acute angle of one are equal to the corresponding side and angle of the other.

We have to prove that the triangles are congruent.

Let us consider two right angle triangles. Such that,

$$\angle B = \angle E = 90^\circ \text{ (i)}$$

$$AB = DE \text{ (ii)}$$

$$\angle C = \angle F \text{ (iii)}$$

Now, observe the two triangles ABC and DEF

$$\angle C = \angle F \text{ (iv)}$$

$$\angle B = \angle E \text{ [From (i)]}$$

$$AB = DE \text{ [From (ii)]}$$

So, by AAS theorem, we have

$$\triangle ABC \cong \triangle DEF$$

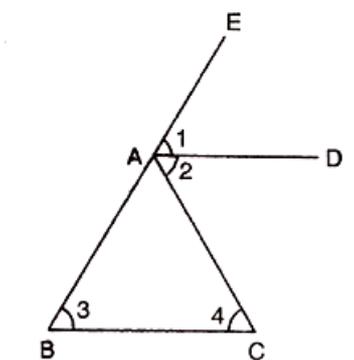
Therefore, the two triangles are congruent

Hence, proved

2. Question

If the bisector of the exterior vertical angle of a triangle be parallel to the base. Show that the triangle is isosceles.

Answer



Given that the bisector of exterior vertical angle of a triangle is parallel to the base and we have to prove that the triangle is isosceles.

Let, ABC be a triangle such that AD is the angular bisector of exterior vertical angle EAC and $AD \parallel BC$

Let, $\angle EAD = 1$

$$\angle DAC = 2$$

$$\angle ABC = 3$$

$$\angle ACB = 4$$

We have,

$1 = 2$ (Therefore, AD is the bisector of $\angle EAC$)

$1 = 3$ (Corresponding angles)

And,

$2 = 4$ (Alternate angles)

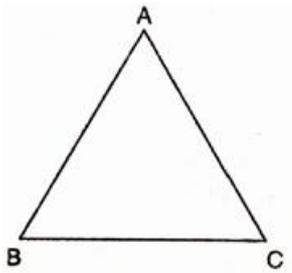
$3 = 4 = AB = AC$

Since, in $\triangle ABC$, two sides AB and AC are equal we can say that $\triangle ABC$ is isosceles.

3. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, calculate the angles of the triangle.

Answer



Let $\triangle ABC$ be isosceles

Such that,

$AB = BC$

$\angle B = \angle C$

Given, that vertex angle A is twice the sum of the base angles B and C.

i.e., $\angle A = 2(\angle B + \angle C)$

$\angle A = 2(\angle B + \angle B)$

$\angle A = 2(2\angle B)$

$\angle A = 4\angle B$

Now,

We know that the sum of all angles of triangle = 180°

$\angle A + \angle B + \angle C = 180^\circ$

$4\angle B + \angle B + \angle B = 180^\circ$ (Therefore, $\angle A = 4\angle B$, $\angle C = \angle B$)

$6\angle B = 180^\circ$

$\angle B = \frac{180}{6}$

$= 30^\circ$

Since, $\angle B = \angle C = 30^\circ$

And, $\angle A = 4\angle B$

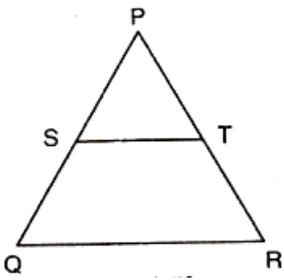
$= 4 * 30^\circ = 120^\circ$

Therefore, the angles of the triangle are $120^\circ, 30^\circ, 30^\circ$.

4. Question

PQR is a triangle in which $PQ = PR$ and S is any point on the side PQ . Through S , a line is drawn parallel to QR and intersecting PR at T . Prove that $PS = PT$.

Answer



Given that PQR is a triangle

Such that,

$$PQ = PR$$

And, S is any point on side PQ and $ST \parallel QR$

We have to prove $PS = PT$

Since,

$$PQ = PR$$

PQR is isosceles

$$\angle Q = \angle R$$

$$\text{Or, } \angle PQR = \angle PRQ$$

Now,

$$\angle PST = \angle PQR$$

And,

$$\angle PTS = \angle PRQ \text{ (Corresponding angles as } ST \parallel QR)$$

Since,

$$\angle PQR = \angle PRQ$$

$$\angle PST = \angle PTS$$

Now, in $\triangle PST$

$$\angle PST = \angle PTS$$

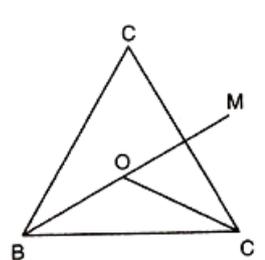
Therefore, $\triangle PST$ is an isosceles triangle

$$PS = PT$$

5. Question

In a $\triangle ABC$, it is given that $AB = AC$ and the bisectors of $\angle B$ and $\angle C$ intersect at O . If M is a point on BO produced, prove that $\angle MOC = \angle ABC$.

Answer



Given that in $\triangle ABC$,

$AB = AC$ and the bisectors of $\angle B$ and $\angle C$ intersect at O and M is a point on BO produced.

We have to prove $\angle MOC = \angle ABC$

Since,

$$AB = AC$$

$\triangle ABC$ is isosceles

$$\angle B = \angle C$$

Or,

$$\angle ABC = \angle ACB$$

Now,

BO and CO are bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$$\angle ABO = \angle OBC = \angle ACO = \angle OCB = \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB \quad (i)$$

We have, in $\triangle OBC$

$$\angle OCB + \angle OBC + \angle BOC = 180^\circ \quad (ii)$$

And also,

$$\angle BOC + \angle COM = 180^\circ \quad (iii) \text{ [Straight angle]}$$

Equating (ii) and (iii), we get

$$\angle OCB + \angle OBC + \angle BOC = \angle BOC + \angle COM$$

$$\angle OBC + \angle OBC = \angle MOC$$

$$2\angle OBC = \angle MOC$$

$$2\left(\frac{1}{2}\angle ABC\right) = \angle MOC \text{ [From (i)]}$$

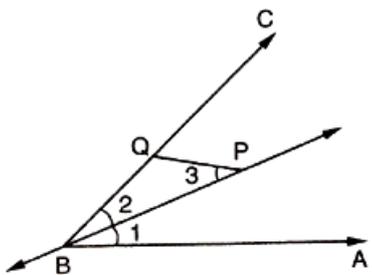
$$\angle ABC = \angle MOC$$

Therefore, $\angle MOC = \angle ABC$

6. Question

P is a point on the bisector of an angle $\angle ABC$. If the line through P parallel to AB meets BC at Q , prove that the triangle BPQ is isosceles.

Answer



Given that P is the point on the bisector of an angle $\angle ABC$, and $PQ \parallel AB$

We have to prove that BPQ is isosceles

Since,

$$BP \text{ is the bisector of } \angle ABC = \angle ABP = \angle PBC \quad (i)$$

Now,

$PQ \parallel AB$

$\angle BPQ = \angle ABP$ (ii) [Alternate angles]

From (i) and (ii), we get

$\angle BPQ = \angle PBC$

Or,

$\angle BPQ = \angle PBQ$

Now, in $\triangle BPQ$

$\angle BPQ = \angle PBQ$

$\triangle BPQ$ is an isosceles triangle

Hence, proved

7. Question

Prove that each angle of an equilateral triangle is 60° .

Answer

Given to prove that each angle of the equilateral triangle is 60°

Let us consider an equilateral triangle ABC

Such that,

$AB = BC = CA$

Now,

$AB = BC$

$\angle A = \angle C$ [i] (Opposite angles to equal sides are equal)

$BC = AC$

$\angle B = \angle A$ [ii] (Opposite angles to equal sides are equal)

From [i] and [ii], we get

$\angle A = \angle B = \angle C$ [iii]

We know that,

Sum of all angles of triangles = 180°

$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle A + \angle A = 180^\circ$

$3\angle A = 180^\circ$

$\angle A = \frac{180}{3}$

$= 60^\circ$

Therefore, $\angle A = \angle B = \angle C = 60^\circ$

Hence, each angle of an equilateral triangle is 60° .

8. Question

Angles A, B, C of a triangle ABC are equal to each other. Prove that $\triangle ABC$ is equilateral.

Answer

Given that A, B, C of a triangle ABC are equal to each other.

We have to prove that, ΔABC is equilateral.

We have,

$$\angle A = \angle B = \angle C$$

Now,

$$\angle A = \angle B$$

$BC = AC$ (Opposite sides to equal angles are equal)

$$\angle B = \angle C$$

$AC = AB$ (Opposite sides to equal angles are equal)

From the above, we get

$$AB = BC = AC$$

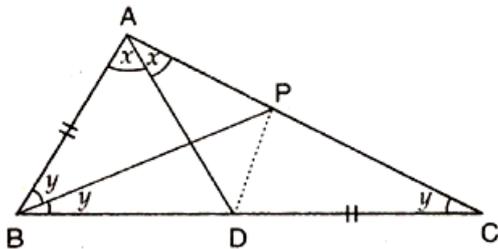
Therefore, ΔABC is an equilateral triangle.

Hence, proved

9. Question

ABC is a triangle in which $\angle B = 2\angle C$. D is a point on BC such that AD bisects $\angle BAC$ and $AB = CD$. Prove that $\angle BAC = 72^\circ$.

Answer



Given that, in ΔABC ,

$$\angle B = 2\angle C \text{ and,}$$

D is a mid-point on BC such that AD bisects $\angle BAC$ and $AB = CD$.

We have to prove that, $\angle BAC = 72^\circ$

Now draw the angular bisector of $\angle ABC$, which meets AC in P

Join PD

$$\text{Let, } \angle ACB = y$$

$$\angle B = \angle ABC = 2\angle C = 2y$$

$$\text{Also, } \angle BAD = \angle DAC = x$$

$$\angle BAC = 2x \text{ (Therefore, } AD \text{ is the bisector of } \angle BAC)$$

Now, in ΔBPC

$$\angle CBP = y \text{ (Therefore, } BP \text{ is the bisector of } \angle ABC)$$

$$\angle PCB = y$$

$$\angle CBP = \angle PCB = y$$

Therefore, $PC = BP$

Consider $\triangle ABP$ and $\triangle DCP$, we have

$$\angle ABP = \angle DCP = y$$

$$AB = DC \text{ (Given)}$$

$$PC = BP \text{ (From above)}$$

So, by SAS theorem, we have

$$\triangle ABP \cong \triangle DCP$$

Now,

$$\angle BAP = \angle CDP$$

$$\text{And, } AP = DP \text{ (By c.p.c.t)}$$

$$\angle BAP = \angle CDP = 2x$$

Now, in $\triangle ABD$

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

$$\angle ADB + \angle ADC = 180^\circ \text{ (Straight angle)}$$

$$2x + 2y + y = 180^\circ \text{ (Therefore, } \angle A = 2x, \angle B = 2y, \angle C = y)$$

$$2y + 3y = 180^\circ \text{ (Therefore, } x = y)$$

$$5y = 180^\circ$$

$$y = \frac{180}{5}$$

$$y = 36^\circ$$

$$\text{Therefore, } x = y = 36^\circ$$

Now,

$$\angle A = \angle BAC = 2x = 2 * 36^\circ = 72^\circ$$

$$\text{Therefore, } \angle BAC = 72^\circ$$

Hence, proved

10. Question

ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Answer

Given that ABC is a right angled triangle

Such that,

$$\angle A = 90^\circ$$

And,

$$AB = AC$$

Since,

$$AB = AC$$

$\triangle ABC$ is also isosceles triangle

Therefore, we can say that $\triangle ABC$ is a right angled isosceles triangle.

$$\angle C = \angle B$$

And,

$$\angle A = 90^\circ$$

Now, we have

Sum of angles in a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$90^\circ + \angle B + \angle B = 180^\circ \text{ (From i)}$$

$$90^\circ + 2\angle B = 180^\circ$$

$$2\angle B = 90^\circ$$

$$\angle B = 45^\circ$$

Therefore, $\angle B = \angle C = 45^\circ$

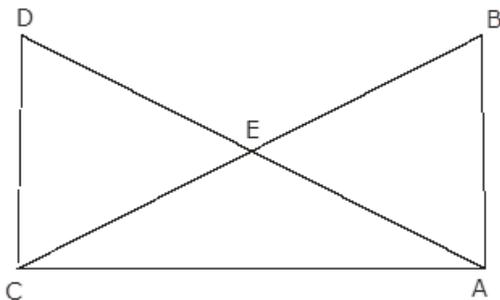
Exercise 10.4

1. Question

In Fig. 10.92, it is given that $AB = CD$ and $AD = BC$. Prove that $\triangle ADC \cong \triangle CBA$.

Answer

Given, in the figure



$$AB = CD$$

And,

$$AD = BC$$

To prove: $\triangle ADC \cong \triangle CBA$

Proof: Consider, $\triangle ADC$ and $\triangle CBA$

$$AB = CD \text{ (Given)}$$

$$BC = AD \text{ (Given)}$$

$$AC = AC \text{ (Common)}$$

By SSS theorem,

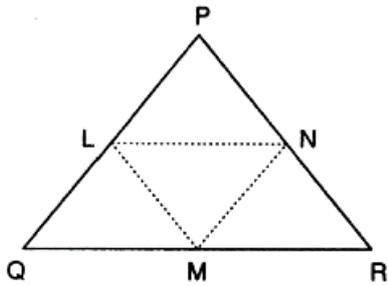
$$\triangle ADC \cong \triangle CBA$$

Hence, proved

2. Question

In $\triangle PQR$, if $PQ = QR$ and L , M and N are the mid-point of the sides PQ , QR and RP respectively. Prove that $LN = MN$.

Answer



Given that,

In ΔPQR

$PQ = QR$

And,

L, M, and N are the mid points of PQ, QR and RP respectively

To prove: $LM = MN$

Construction: Join L and M, M and N and N and L

Proof: We have,

$PL = LQ, QM = MR$ and $RN = NP$

Since, L, M and N are mid points of PQ, QR and RP respectively.

And, also $PQ = QR$

$$PL = LQ = QM = MR = \frac{PQ}{2} = \frac{QR}{2} \text{ (i)}$$

Using mid-point theorem, we have

$MN \parallel PQ$

And,

$$MN = \frac{1}{2}PQ = MN = PL = LQ \text{ (ii)}$$

Similarly, we have

$LN \parallel QR$

And,

$$LN = \frac{1}{2}QR = LN = QM = MR \text{ (iii)}$$

From equations (i), (ii) and (iii), we have

$$PL = LQ = QM = MR = MN = LN$$

Therefore, $LN = MN$

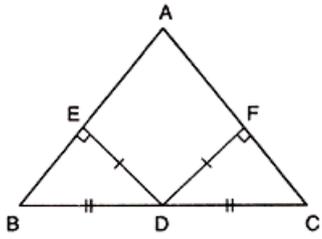
Hence, proved

Exercise 10.5

1. Question

ABC is a triangle and D is the mid-point of BC . The perpendicular from D to AB and AC are equal. Prove that the triangle is isosceles.

Answer



Given,

ABC is a triangle and D is the mid-point of BC

Perpendicular from D to AB and AC are equal.

To prove: Triangle is isosceles

Proof: Let DE and DF be perpendiculars from A on AB and AC respectively.

In order to prove that $AB = AC$, we will prove that $\triangle BDE \cong \triangle CDF$.

In these two triangles, we have

$$\angle BED = \angle CFD = 90^\circ$$

$$BD = CD \text{ (Therefore, } D \text{ is the mid-point of } BC)$$

$$DE = DF \text{ (Given)}$$

So, by RHS congruence criterion, we have

$$\triangle BDE \cong \triangle CDF$$

$$\angle B = \angle C \text{ (By c.p.c.t)}$$

$$AC = AB \text{ (By c.p.c.t)}$$

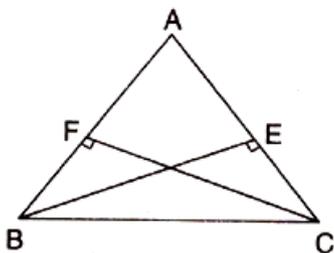
As opposite sides and opposite angles of the triangle are equal.

Therefore, $\triangle ABC$ is isosceles

2. Question

ABC is a triangle in which BE and CF are, respectively, the perpendiculars to the sides AC and AB . If $BE = CF$, prove that $\triangle ABC$ is isosceles.

Answer



Given that ABC is a triangle in which BE and CF are perpendiculars to the side AC and AB respectively.

Such that,

$$BE = CF$$

We have to prove that, $\triangle ABC$ is isosceles triangle.

Now, consider $\triangle BCF$ and $\triangle CBE$

We have,

$$\angle BFC = \angle CEB = 90^\circ \text{ (Given)}$$

$$BC = CB \text{ (Given)}$$

CF = BE (Given)

So, by RHS congruence rule, we have

$$\triangle BFC \cong \triangle CEB$$

Now,

$$\angle FBC = \angle ECB \text{ (By c.p.c.t)}$$

$$\angle ABC = \angle ACB \text{ (By c.p.c.t)}$$

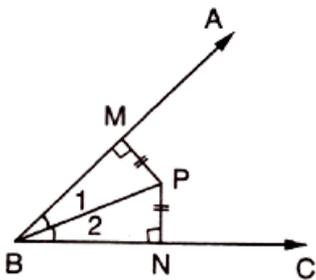
AC = AB (Opposite sides of equal angles are equal in a triangle)

Therefore, $\triangle ABC$ is isosceles.

3. Question

If perpendiculars from any point within an angle on its arms are congruent, prove that it lies on the bisector of that angle.

Answer



Given that perpendiculars from any point within an angle on its arms are congruent.

We have to prove that it lies on the bisector of that angle.

Now, let us consider an $\angle ABC$ and let BP be one of the arm within the angle.

Draw perpendicular PN and PM On the arms BC and BA

Such that,

They meet BC and BA in N and M respectively.

Now, in $\triangle BPM$ and $\triangle BPN$

We have,

$$\angle BMP = \angle BNP = 90^\circ \text{ (Given)}$$

$$BP = BP \text{ (Common)}$$

$$MP = NP \text{ (Given)}$$

So, by RHS congruence rule, we have

$$\triangle BPM \cong \triangle BPN$$

$$\angle MBP = \angle NBP \text{ (By c.p.c.t)}$$

BP is the angular bisector of $\angle ABC$

Hence, proved

4. Question

In Fig. 10.99, $AD \perp CD$ and $CB \perp CD$. If $AQ = BP$ and $DP = CQ$, prove that $\angle DAQ = \angle CBP$.

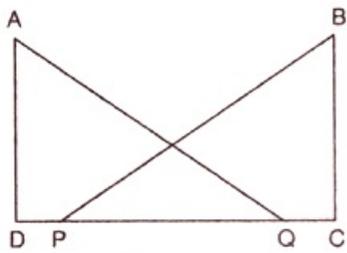


Fig. 10.99

Answer

Given that in figure,

$AD \perp CD$ and $CB \perp CD$

And,

$AQ = BP$, $DP = CQ$

WE have to prove that,

$\angle DAQ = \angle CBP$

Given that, $DP = QC$

Adding PQ on both sides, we get

$DP + PQ = PQ + QC$

$DQ = PC$ (i)

Now consider $\triangle DAQ$ and $\triangle CBP$, we have

$\angle ADQ = \angle BCP = 90^\circ$ (Given)

$AQ = BP$ (Given)

And,

$DQ = PC$ (From i)

So, by RHS congruence rule, we have

$\triangle DAQ \cong \triangle CBP$

Now,

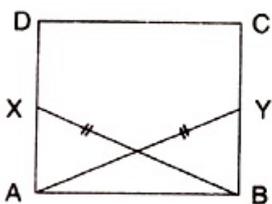
$\angle DAQ = \angle CBP$ (By c.p.c.t)

Hence, proved

5. Question

$ABCD$ is a square, X and Y are points on sides AD and BC respectively such that $AY = BX$. Prove that $BY = AX$ and $\angle BAY = \angle ABX$.

Answer



Given that $ABCD$ is a square, X and Y are points on the sides AD and BC respectively.

Such that,

$$AY = BX$$

We have to prove: $BY = AX$ and $\angle BAY = \angle ABX$

Join B and X, A and Y

Since, ABCD is a square

$$\angle DAB = \angle CBA = 90^\circ$$

$$\angle XAB = \angle YBA = 90^\circ \text{ (i)}$$

Now, consider $\triangle XAB$ and $\triangle YBA$

We have,

$$\angle XAB = \angle YBA = 90^\circ \text{ [From (i)]}$$

$$BX = AY \text{ (Given)}$$

$$AB = BA \text{ (Common side)}$$

So, by RHS congruence rule, we have

$$\triangle XAB \cong \triangle YBA$$

$$BY = AX \text{ and } \angle BAY = \angle ABX \text{ (By c.p.c.t)}$$

Hence, proved

6. Question

Which of the following statements are true (T) and which are false (F):

- (i) Sides opposite to equal angles of a triangle may be unequal.
- (ii) Angles opposite to equal sides of a triangle are equal.
- (iii) The measure of each angle of an equilateral triangle is 60° .
- (iv) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle may be isosceles.
- (v) The bisectors of two equal angles of a triangle are equal.
- (vi) If the bisector of the vertical angle of a triangle bisects the base, then the triangle may be isosceles.
- (vii) The two altitudes corresponding to two equal sides of a triangle need not be equal.
- (viii) If any two sides of a right triangle are respectively equal to two sides of other right triangle, then the two triangles are congruent.
- (ix) Two right triangles are congruent if hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle.

Answer

- (i) False: Sides opposite to equal angles of a triangle are equal.
- (ii) True: Since, the sides are equal, the corresponding opposite angles must be equal.
- (iii) True: Since, all the three angles of an equilateral triangle are equal and sum of the three angles is 180° , so each angle will be equal to $\frac{180}{3} = 60^\circ$
- (iv) False: Here, the altitude from the vertex is also the perpendicular bisector of the opposite side. Here the triangle must be isosceles and may be an equilateral triangle.
- (v) True: Since, it is an isosceles triangle, the length of bisector of the two angles are equal.
- (vi) False: The angular bisector of the vertex angle is also a median. The triangle must be an isosceles and an equilateral triangle.
- (vii) False: Since, two sides are equal the triangle must be an isosceles triangle. The two altitudes

corresponding to two equal sides must be equal.

(viii) False: The two right triangles may or may not be congruent.

(ix) True: According to RHS congruence the given statement is true.

7. Question

Fill in the blanks in the following so that each of the following statements is true.

(i) Sides opposite to equal angles of a triangle are

(i) Sides opposite to equal angles of a triangle are

(iii) In an equilateral triangle all angles are

(iv) In a ΔABC , if $\angle A = \angle C$, then $AB =$

(v) If altitudes CE and BF of a triangle ABC are equal, then, $AB =$

(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is CE .

(vii) In right triangles ABC and DEF , if hypotenuse $AB = EF$ and side $AC = DE$, then $\Delta ABC \cong \Delta$

Answer

(i) Sides opposite to equal angles of a triangle are equal

(ii) Sides opposite to equal angles of a triangle are equal

(iii) In an equilateral triangle all angles are equal

(iv) In a ΔABC , if $\angle A = \angle C$, then $AB = BC$

(v) If altitudes CE and BF of a triangle ABC are equal, then, $AB = AC$

(vi) In an isosceles triangle ABC with $AB = AC$, if BD and CE are its altitudes, then BD is equal to CE

(vii) In right triangles ABC and DEF , if hypotenuse $AB = EF$ and side $AC = DE$, then $\Delta ABC \cong \Delta EFD$

Exercise 10.6

1. Question

In ΔABC , if $\angle A = 40^\circ$ and $\angle B = 60^\circ$. Determine the longest and shortest sides of the triangle.

Answer

Given that in ΔABC

$$\angle A = 40^\circ \text{ and } \angle B = 60^\circ$$

We have to find shortest and longest side.

We know that,

$$\text{Sum of angles of triangle} = 180^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$40^\circ + 60^\circ + \angle C = 180^\circ$$

$$100^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 100^\circ$$

$$= 80^\circ$$

Now,

$$40^\circ < 60^\circ < 80^\circ$$

$$\angle A < \angle B < \angle C$$

$\angle C$ is greater angle and $\angle A$ is smaller angle.

$$\text{As, } \angle A < \angle B < \angle C$$

$BC < AC < AB$ (Therefore, side opposite to greater angle is larger and side opposite to smaller angle is smaller)

Therefore, AB is longest and BC is smallest or shortest side.

2. Question

In a $\triangle ABC$, if $\angle B = \angle C = 45^\circ$, which is the longest side?

Answer

Given that in $\triangle ABC$,

$$\angle B = \angle C = 45^\circ$$

We have to find longest side.

We know that,

$$\text{Sum of angles in a triangle} = 180^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 45^\circ + 45^\circ = 180^\circ$$

$$\angle A + 90^\circ = 180^\circ$$

$$\angle A = 90^\circ$$

Therefore, BC is the longest side because side opposite to greater angle is larger.

3. Question

In $\triangle ABC$, side AB is produced to D so that $BD = BC$. If $\angle B = 60^\circ$ and $\angle A = 70^\circ$, prove that:

(i) $AD > CD$ (ii) $AD > AC$

Answer

Given that in $\triangle ABC$, side AB is produced to D so that $BD = BC$ and $\angle B = 60^\circ$, $\angle A = 70^\circ$

We have to prove that,

(i) $AD > CD$

And, (ii) $AD > AC$

First join C and D

Now,

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ \text{ (Sum of all angles of triangle)}$$

$$\angle C = 180^\circ - 70^\circ - 60^\circ$$

$$= 50^\circ$$

$$\angle C = 50^\circ$$

$$\angle ACB = 50^\circ \text{ (i)}$$

And also in $\triangle BDC$,

$$\angle DBC = 180^\circ - \angle ABC \text{ (Therefore, } \angle ABD \text{ is straight angle)}$$

$$= 180^\circ - 60^\circ$$

$$= 120^\circ$$

$$BD = BC \text{ (Given)}$$

$$\angle BCD = \angle BDC \text{ (Therefore, angle opposite to equal sides are equal)}$$

Now,

$$\angle DBC + \angle BCD + \angle BDC = 180^\circ \text{ (Sum of all sides of triangle)}$$

$$120^\circ + \angle BCD + \angle BDC = 180^\circ$$

$$2\angle BCD = 180^\circ - 120^\circ$$

$$2\angle BCD = 60^\circ$$

$$\angle BCD = 30^\circ$$

$$\text{Therefore, } \angle BCD = \angle BDC = 30^\circ \text{ (ii)}$$

Now, consider $\triangle ADC$,

$$\angle BAC = \angle DAC = 70^\circ \text{ (Given)}$$

$$\angle BDC = \angle ADC = 30^\circ \text{ [From (ii)]}$$

$$\angle ACD = \angle ACB + \angle BCD$$

$$= 50^\circ + 30^\circ \text{ [From (i) and (ii)]}$$

$$= 80^\circ$$

Now,

$$\angle ADC < \angle DAC < \angle ACD$$

$AC < DC < AD$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

$$AD > CD$$

And,

$$AD > AC$$

Hence, proved

We have,

$$\angle ACD > \angle DAC$$

And,

$$\angle ACD > \angle ADC$$

$$AD > DC$$

And,

$AD > AC$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

4. Question

Is it possible to draw a triangle with sides of length 2 cm, 3 cm, and 7 cm?

Answer

Given, Length of sides are 2cm, 3cm and 7cm.

We have to check whether it is possible to draw a triangle with the given length of sides.

We know that,

A triangle can be drawn only when the sum of any two sides is greater than the third side.

Here,

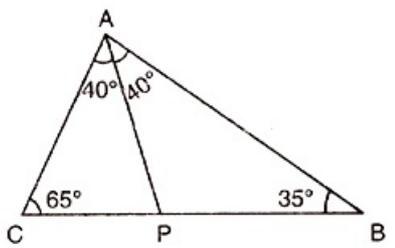
$$2 + 3 > 7$$

So, the triangle does not exist.

5. Question

In $\triangle ABC$, $\angle B = 35^\circ$, $\angle C = 65^\circ$ and the bisector of $\angle ABC$ meets BC in P . Arrange AP , BP and CP in descending order.

Answer



Given: $\angle B = 35^\circ$

$\angle C = 65^\circ$

The bisector of $\angle ABC$ meets BC in P

We have to arrange AP , BP and CP in descending order

In $\triangle ACP$, we have

$$\angle ACP > \angle CAP$$

$$AP > CP \text{ (i)}$$

In $\triangle ABP$, we have

$$\angle BAP > \angle ABP$$

$$BP > AP \text{ (ii)}$$

From (i) and (ii), we have

$$BP > AP > CP$$

6. Question

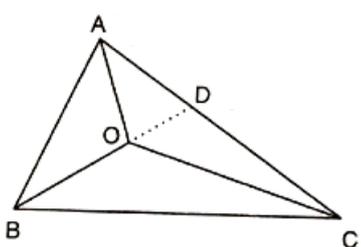
O is any point in the interior of $\triangle ABC$. Prove that

(i) $AB + AC > OB + OC$

(ii) $AB + BC + CA > OA + OB + OC$

(iii) $OA + OB + OC > \frac{1}{2} (AB + BC + CA)$

Answer



Given that, O is any point in the interior of $\triangle ABC$

We have to prove:

$$(i) AB + AC > OB + OC$$

$$(ii) AB + BC + CA > OA + OB + OC$$

$$(iii) OA + OB + OC > \frac{1}{2} (AB + BC + CA)$$

We know that,

In a triangle sum of any two sides is greater than the third side.

So, we have

In $\triangle ABC$

$$AB + BC > AC$$

$$BC + AC > AB$$

$$AC + AB > BC$$

In $\triangle OBC$,

$$OB + OC > BC \text{ (i)}$$

In $\triangle OAC$,

$$OA + OC > AC \text{ (ii)}$$

In $\triangle OAB$,

$$OA + OB > AB \text{ (iii)}$$

Now, extend BO to meet AC in D.

In $\triangle ABD$, we have

$$AB + AD > BD$$

$$AB + AD > BO + OD \text{ (iv) [Therefore, } BD = BO + OD\text{]}$$

Similarly,

In $\triangle ODC$, we have

$$OD + DC > OC \text{ (v)}$$

(i) Adding (iv) and (v), we get

$$AB + AD + OD + DC > BO + OD + OC$$

$$AB + (AD + DC) > OB + OC$$

$$AB + AC > OB + OC \text{ (vi)}$$

Similarly, we have

$$BC + BC > OA + OC \text{ (vii)}$$

And,

$$CA + CB > OA + OB \text{ (viii)}$$

(ii) Adding (vi), (vii) and (viii), we get

$$AB + AC + BC + BA + CA + CB > OB + OC + OA + OC + OA + OB$$

$$2AB + 2BC + 2CA > 2OA + 2OB + 2OC$$

$$2(AB + BC + CA) > 2(OA + OB + OC)$$

$$AB + BC + CA > OA + OB + OC$$

(iii) Adding (i), (ii) and (iii), we get

$$OB + OC + OA + OC + OA + OB > BC + AC + AB$$

$$2OA + 2OB + 2OC > AB + BC + CA$$

$$2(OA + OB + OC) > AB + BC + CA$$

$$\text{Therefore, } (OA + OB + OC) > \frac{1}{2}(AB + BC + CA)$$

7. Question

Prove that the perimeter of a triangle is greater than the sum of its altitudes.

Answer

Given: A $\triangle ABC$ in which AD perpendicular BC and BE perpendicular AC and CF perpendicular AB.

To prove: $AD + BE + CF < AB + BC + AC$

Proof: We know that all the segments that can be drawn into a given line, from a point not lying on it, perpendicular distance i.e. the perpendicular line segment is the shortest. Therefore,

AD perpendicular BC

$$AB > AD \text{ and } AC > AD$$

$$AB + AC > 2AD \text{ (i)}$$

Similarly,

BE perpendicular AC

$$BA > BE \text{ and } BC > BE$$

$$BA + BC > 2BE \text{ (ii)}$$

And also

CF perpendicular AB

$$CA > CF \text{ and } CB > CF$$

$$CA + CB > 2CF \text{ (iii)}$$

Adding (i), (ii) and (iii), we get

$$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$$

$$2AB + 2BC + 2CA > 2(AD + BE + CF)$$

$$2(AB + BC + CA) > 2(AD + BE + CF)$$

$$AB + BC + CA > AD + BE + CF$$

The perimeter of the triangle is greater than the sum of its altitudes.

Hence, proved

8. Question

Prove that in a quadrilateral the sum of all the sides is greater than the sum of its diagonals.

Answer

Given: Let ABCD is a quadrilateral with AC and BD as its diagonals

To Prove: Sum of all the sides of a quadrilateral is greater than the sum of its diagonals

Proof: Consider a quadrilateral ABCD where AC and BD are the diagonals

$AB+BC > AC$ (i) (Sum of two sides is greater than the third side)

$AD+DC > AC$ (ii)

$AB+AD > BD$ (iii)

$DC+BC > BD$ (iv)

Adding (i), (ii), (iii), and (iv)

$AB+BC+AD+DC+AB+AD+DC+BC > AC+AC+BD+BD$

$2(AB+BC+CD+DA) > 2(AC+BD)$

$AB+BC+CD+DA > AC+BC$

Hence, proved that the Sum of all the sides of a quadrilateral is greater than the sum of its diagonals

9. Question

In Fig. 10.131, prove that:

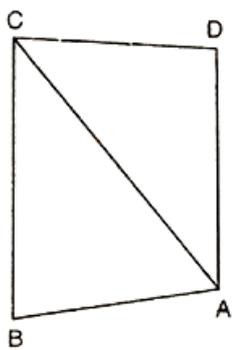


Fig. 10.131

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD+DA +AB > BC$

Answer

Given to prove,

(i) $CD + DA + AB + BC > 2AC$

(ii) $CD+DA +AB > BC$

From the given figure,

We know that,

In a triangle sum of any two sides is greater than the third side.

(i) So,

In $\triangle ABC$, we have

$AB + BC > AC$ (1)

In $\triangle ADC$, we have

$CD + DA > AC$ (2)

Adding (1) and (2), we get

$AB + BC + CD + DA > AC + AC$

$CD + DA + AB + BC > 2 AC$

(ii) Now, in $\triangle ABC$, we have

$$CD + DA > AC$$

Add AB on both sides, we get

$$CD + DA + AB > AC + AB > BC$$

$$CD + DA + AB > BC$$

Hence, proved

10. Question

Which of the following statements are true (T) and which are false (F)?

- (i) Sum of the three sides of a triangle is less than the sum of its three altitudes.
- (ii) Sum of any two sides of a triangle is greater than twice the median drawn to the third side.
- (iii) Sum of any two sides of a triangle is greater than the third side.
- (iv) Difference of any two sides of a triangle is equal to the third side.
- (v) If two angles of a triangle are unequal, then the greater angle has the larger side opposite to it.
- (vi) Of all the line segments that can be drawn from a point to a line not containing it, the perpendicular line segment is the shortest one.

Answer

- (i) False: Sum of three sides of a triangle is greater than sum of its three altitudes.
- (ii) True
- (iii) True
- (iv) False: The difference of any two sides of a triangle is less than the third side.
- (v) True: The side opposite to greater angle is longer and smaller angle is shorter in a triangle.
- (vi) True: The perpendicular distance is the shortest distance from a point to a line not containing it.

11. Question

Fill in the blanks to make the following statements true:

- (i) In a right triangle the hypotenuse is theside.
- (ii) The sum of three altitudes of a triangle is than its perimeter.
- (iii) The sum of any two sides of a triangle is than the third side.
- (iv) If two angles of a triangle are unequal, then the smaller angle has the side opposite to it.
- (v) Difference of any two sides of a triangle is than the third side.
- (vi) If two sides of a triangle are unequal, then the larger side has angle opposite to it.

Answer

- (i) In a right triangle the hypotenuse is the **largest** side.
- (ii) The sum of three altitudes of a triangle is **less** than its perimeter.
- (iii) The sum of any two sides of a triangle is **greater** than the third side.
- (iv) If two angles of a triangle are unequal, then the smaller angle has the **smaller** side opposite to it.
- (v) Difference of any two sides of a triangle is **less** than the third side.
- (vi) If two sides of a triangle are unequal, then the larger side has **greater** angle opposite to it.

CCE - Formative Assessment

1. Question

In two congruent triangles ABC and DEF , if $AB = DE$ and $BC = EF$. Name the pairs of equal angles.

Answer

By c.p.c.t. that is corresponding part of congruent triangles, the pair of equal angles are:

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$$

2. Question

In two triangles ABC and DEF , it is given that $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. Are the two triangles necessarily congruent?

Answer

No, the two triangles are not necessarily congruent in the given case as knowing only angle-angle-angle (AAA) does not work because it can produce similar but not congruent triangles.

3. Question

If ABC and DEF are two triangles such that $AC = 2.5$ cm, $BC = 5$ cm, $\angle C = 75^\circ$, $DE = 2.5$ cm, $DF = 5$ cm and $\angle D = 75^\circ$. Are two triangles congruent?

Answer

Yes, the given triangles are congruent as $AC=DE$, $BC = DF$ and $\angle D$ is equal to $\angle C$. Hence, By SAS theorem triangle ABC is congruent to triangle EDF .

4. Question

In two triangles ABC and ADC , if $AB = AD$ and $BC = CD$. Are they congruent?

Answer

Yes, the given triangles are congruent as $AB= AD$, $BC= CD$ and AC is a common side. Hence, by SSS theorem triangle ABC is congruent to triangle ADC .

5. Question

In triangles ABC and CDE , if $AC = CE$, $BC = CD$, $\angle A= 60^\circ$, $\angle C= 30^\circ$ and $\angle D = 90^\circ$. Are two triangles congruent?

Answer

Yes, the two given triangles are congruent because $AC = CE$, $BC= CD$

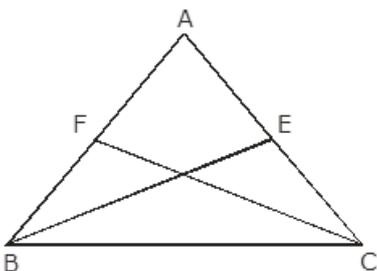
And $\angle B = \angle D= 90^\circ$.

Therefore by SSA criteria triangle ABC is congruent to triangle CDE .

6. Question

ABC is an isosceles triangle in which $AB = AC$. BE and CF are its two medians. Show that $BE=CF$.

Answer



Given,

ABC is an isosceles triangle

$AB = AC$

BE and CF are two medians

To prove: $BE = CF$

Proof: In $\triangle BEC$ and $\triangle CFB$

$CE = BF$ (Since, $AC = AB = \frac{1}{2}AC = \frac{1}{2}AB = CE = BF$)

$\angle ECB = \angle FBC$ (Angle opposite to equal sides are equal)

$BC = BC$ (Common)

Therefore, By SAS theorem

$\triangle BEC \cong \triangle CFB$

$BE = CF$ (By c.p.c.t)

7. Question

Find the measure of each angle of an equilateral triangle.

Answer

Let ABC is an equilateral triangle

We know that all the angles in an equilateral triangle are equal.

Therefore,

$\angle A = \angle B = \angle C$ (i)

Now,

$\angle A + \angle B + \angle C = 180^\circ$ (Sum of all angles of a triangle)

$\angle A + \angle A + \angle A = 180^\circ$

$3\angle A = 180^\circ$

$\angle A = \frac{180}{3}$

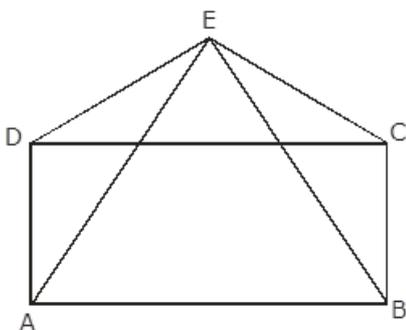
$= 60^\circ$

Hence, the measure of each angle of an equilateral triangle is 60° .

8. Question

CDE is an equilateral triangle formed on a side CD of a square ABCD. Show that $\triangle ADE \cong \triangle BCE$.

Answer



Given: An equilateral triangle CDE is on side CD of square ABCD

To prove: $\triangle ADE \cong \triangle BCE$

Proof: $\angle EDC = \angle DCE = \angle CED = 60^\circ$ (Angles of equilateral triangle)

$$\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ \text{ (Angles of square)}$$

$$\angle EDA = \angle EDC + \angle CDA$$

$$= 60^\circ + 90^\circ$$

$$= 150^\circ \text{ (i)}$$

Similarly,

$$\angle ECB = 150^\circ \text{ (ii)}$$

In $\triangle ADE$ and $\triangle BCE$

$ED = EC$ (Sides of equilateral triangle)

$AD = BC$ (Sides of square)

$\angle EDA = \angle ECB$ [From (i) and (ii)]

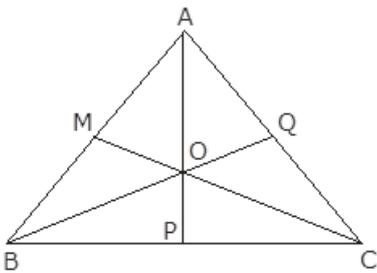
Therefore, By SAS theorem

$$\triangle ADE \cong \triangle BCE$$

9. Question

Prove that the sum of three altitudes of a triangle is less than the sum of its sides.

Answer



In $\triangle APB$,

AP is a median

Angle APB is greater than angle ABP

So angle opposite to greater side is longer

Therefore, AB is greater than AP

Hence proved that in triangle APB the side of the triangle is more than the median or altitude AP of the triangle.

Similarly, in the same manner the other triangles can also be proved.

10. Question

In Fig. 10.134, if $AB = AC$ and $\angle B = \angle C$. Prove that $BQ = CP$.

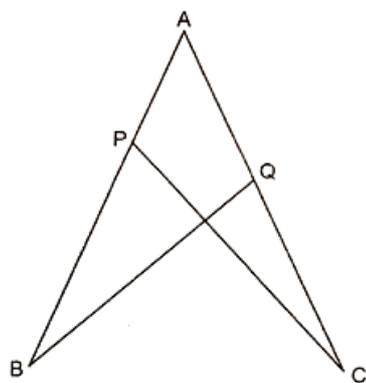


Fig. 10.134

Answer

Given,

$$\angle B = \angle C$$

$$AB = AC$$

To Prove: $BQ = CP$

Proof: In $\triangle ABQ$ and $\triangle ACP$

$$\angle B = \angle C \text{ (Given)}$$

$$AB = AC \text{ (Given)}$$

$$\angle A = \angle A \text{ (Common)}$$

Hence, by A.S.A. Theorem

$$\triangle ABQ \cong \triangle ACP$$

$$BQ = CP \text{ (By c.p.c.t)}$$

1. Question

If $\triangle ABC \cong \triangle LKM$, then side of $\triangle LKM$ equal to side AC of $\triangle ABC$ is

- A. LK
- B. KM
- C. LM
- D. None of these

Answer

Since, by corresponding part of congruent triangle AC of $\triangle ABC$ is equal to the LM of $\triangle LKM$.

2. Question

If $\triangle ABC \cong \triangle ACB$, then $\triangle ABC$ is isosceles with

- A. $AB = AC$
- B. $AB = BC$
- C. $AC = BC$
- D. None of these

Answer

AB and AC are the equal sides of equilateral triangle ABC because only then after reverting it to form $\triangle ACB$, the two triangles can be proved congruent.

3. Question

If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔPQR , then which of the following not true:

- A. $BC = PQ$
- B. $AC = PR$
- C. $AB = PQ$
- D. $QR = BC$

Answer

Since, BC and PQ are the non c.p.c.t. part of the two given triangles, hence we cannot judge them to be equal or not from the given information about them.

4. Question

In triangle ABC and PQR three equality relations between some parts are as follows:

$$AB = QP, \angle B = \angle P \text{ and } BC = PR$$

State which of the congruence conditions applies:

- A. SAS
- B. ASA
- C. SSS
- D. RHS

Answer

Since, two adjacent sides and the so formed angle with the two sides are shown equal while proving them congruent. Hence, by S.A.S. theorem the two triangles can be proved congruent.

5. Question

In triangles ABC and PQR, if $\angle A = \angle R$, $\angle B = \angle P$ and $AB = RP$, then which one of the following congruence conditions applies:

- A. SAS
- B. ASA
- C. SSS
- D. RHS

Answer

Since, the two adjacent angles and the contained side are shown equal while proving the two triangles congruent. Hence, by A.S.A. theorem the two triangles can be proved congruent.

6. Question

In $\Delta PQR \cong \Delta EFD$ then $ED =$

- A. PQ
- B. QR
- C. PR
- D. None of these

Answer

Since, by corresponding part of congruent triangle ED of ΔEFD is equal to the PR of ΔPQR .

7. Question

In $\Delta PQR \cong \Delta EFD$ then $\angle E =$

- A. $\angle P$
- B. $\angle Q$
- C. $\angle R$
- D. None of these

Answer

Since, by corresponding part of congruent triangle $\angle E$ of $\triangle EFD$ is equal to the $\angle P$ of $\triangle PQR$.

8. Question

In a $\triangle ABC$, if $AB = AC$ and BC is produced to D such that $\angle ACD = 100^\circ$, then $\angle A =$

- A. 20°
- B. 40°
- C. 60°
- D. 80°

Answer

$$\angle ACB + \angle ACD = 180^\circ$$

On solving we get,

$$\angle ACB = 80^\circ$$

$$\angle ABC = \angle ACB = 80^\circ \text{ (Angles opposite to equal sides are equal)}$$

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + 80^\circ + 80^\circ = 180^\circ$$

$$\angle A = 20^\circ$$

9. Question

In an isosceles triangle, if the vertex angle is twice the sum of the base angles, then the measure of vertex angle of the triangle is

- A. 100°
- B. 120°
- C. 110°
- D. 130°

Answer

Let the base angles be x each,

$$\text{Vertex angle} = 2(x + x) = 4x$$

Now, since the sum of all the angles of a triangle is 180°

$$x + x + 4x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

Therefore, vertex angle = $4x = 120^\circ$

10. Question

D, E, F are the mid-point of the sides BC, CA and AB respectively of $\triangle ABC$. Then $\triangle DEF$ is congruent to

triangle

A. ABC

B. AEF

C. BFD, CDE

D. AFE, BFD, CDE

Answer

Since, the so formed triangle divides the complete triangle ABC into four congruent triangles.

11. Question

Which of the following is not criterion for congruence of triangles?

A. SAS

B. SSA

C. ASA

D. SSS

Answer

Since the two triangles with two adjacent sides and an angle adjacent to any one side among them are shown equal, then the two triangles will be similar but not necessarily congruent.

12. Question

In Fig. 10.135, the measure of $\angle B'AC'$ is

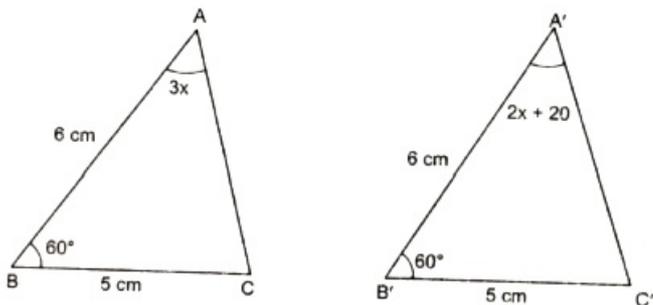


Fig. 10.135

A. 50°

B. 60°

C. 70°

D. 80°

Answer

In $\triangle ABC$ and $\triangle A'B'C'$,

$AB = A'B'$ (Given)

$\angle B = \angle B'$ (Given)

$BC = B'C'$ (Given)

Hence, by S.A.S. theorem,

$\triangle ABC \cong \triangle A'B'C'$

Therefore,

By c.p.c.t

$$\angle A = \angle A'$$

$$3x = 2x + 20$$

$$x = 20^\circ$$

Therefore angle $A' = 2x + 20$

$$= 60^\circ$$

13. Question

If ABC and DEF are two triangles such that $\triangle ABC \cong \triangle FDE$ and $AB = 5$ cm, $\angle B = 40^\circ$ and $\angle A = 80^\circ$, Then, which of the following is true?

A. $DF = 5$ cm, $\angle F = 60^\circ$

B. $DE = 5$ cm, $\angle E = 60^\circ$

C. $DF = 5$ cm, $\angle E = 60^\circ$

D. $DE = 5$ cm, $\angle D = 40^\circ$

Answer

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180$$

$$80^\circ + 40^\circ + \angle C = 180^\circ$$

$$\angle C = 60^\circ$$

Now, by c.p.c.t

$$AB = DF = 5\text{cm}$$

And

$$\angle C = \angle E = 60^\circ$$

14. Question

In Fig. 10.136, $AB \perp BE$ and $FE \perp BE$. If $BC = DE$ and $AB = FE$, then $\triangle ABD$ is congruent to

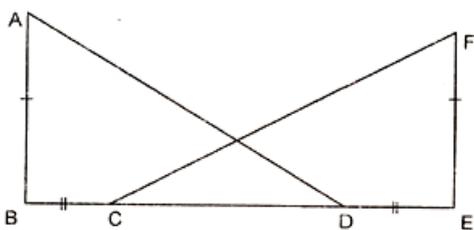


Fig. 10.136

A. $\triangle EFC$

B. $\triangle ECF$

C. $\triangle CEF$

D. $\triangle FEC$

Answer

In $\triangle ABD$ and $\triangle FEC$,

$$AB = FE \text{ (Given)}$$

$$\angle B = \angle E \text{ (Each } 90^\circ)$$

$$BC = DE \text{ (Given)}$$

Add CD both sides, we get

$$BD = EC$$

Therefore, by S.A.S. theorem,

$$\triangle ABD \cong \triangle FEC$$

18. Question

ABC , is an isosceles triangle such that $AB=AC$ and AD is the median to base BC . Then, $\angle BAD=$

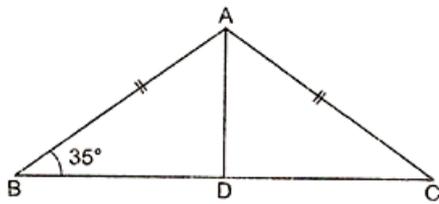


Fig. 10.137

- A. 55°
- B. 70°
- C. 35°
- D. 110°

Answer

For an isosceles triangle $\angle ABC = \angle ACB = 35^\circ$

Let the $\angle ADB$ be x

Then,

$$\angle ADC = 180^\circ - x$$

As AD is median so $BD = CD$

And for isosceles triangle $AB = AC$

So,

$$\frac{AB}{AC} = \frac{BD}{CD} = 1$$

By angle bisector theorem,

$$\angle BAD = \angle CAD = y \text{ (Let)}$$

For $\triangle BAD$

$$35 + x + y = 180 \text{ (i)}$$

For $\triangle DAC$

$$35 + 180 - x + y = 180 \text{ (ii)}$$

$$35 + y = x$$

Therefore,

$$35 + 34 + y + y = 180^\circ$$

$$2y + 70 = 180^\circ$$

$$2y = 100^\circ$$

$$y = 50^\circ$$

Therefore,

$$\angle BAD = \angle CAD = 55^\circ$$

16. Question

In Fig. 10.138, if $AE \parallel DC$ and $AB = AC$, the value of $\angle ABD$ is

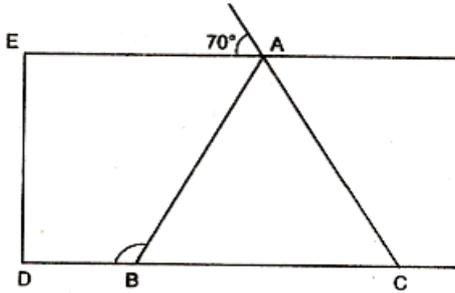


Fig. 10.138

A. 75°

B. 110°

C. 120°

D. 130°

Answer

$$\angle EAC = \angle BCA \text{ (Corresponding angles)}$$

$$\angle BCA = 70^\circ$$

$$\angle CBA = \angle BCA \text{ (Angles opposite to equal sides are equal)}$$

$$\angle CBA = 70^\circ$$

Now,

$$\angle ABD + \angle CBA = 180^\circ$$

$$\angle ABD + 70 = 180^\circ$$

$$\angle ABD = 110^\circ$$

17. Question

In Fig. 10.139, ABC is an isosceles triangle whose side AC is produced to E . Through C , CD is drawn parallel to BA . The value of x is

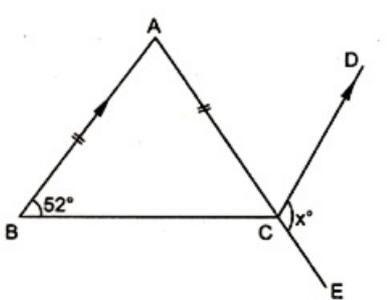


Fig. 10.139

A. 52°

B. 76°

C. 156°

D. 104°

Answer

$\angle B = \angle C$ (Angles opposite to equal sides are equal)

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A = 76^\circ$$

Now,

$$\angle BAC = \angle ACD \text{ (Alternate angles)}$$

$$\angle ACD = 76^\circ$$

Now,

$$\angle ACD + \angle ECD = 180^\circ$$

$$x = 180 - 76^\circ$$

$$x = 104^\circ$$

18. Question

In Fig. 10.140, X is a point in the interior of square $ABCD$. $AXYZ$ is also a square. If $DY = 3$ cm and $AZ = 2$ cm, then $BY =$

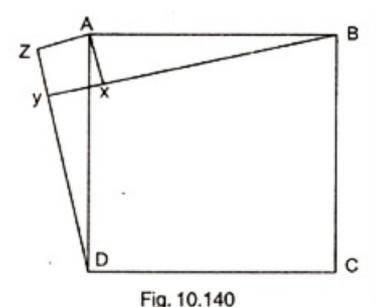


Fig. 10.140

- A. 5 cm
- B. 6 cm
- C. 7 cm
- D. 8 cm

Answer

$\angle Z = 90^\circ$ (Angle of square)

Therefore, AZD is a right angle triangle,

By Pythagoras theorem,

$$AD^2 = AZ^2 + ZD^2$$

$$AD^2 = 2^2 + (2+3)^2$$

$$AD^2 = 4 + 25$$

$$AD = \sqrt{29}$$

In $\triangle AXB$, with X as right angle,

By Pythagoras theorem,

$$AB^2 = AX^2 + XB^2$$

$$XB^2 = 29 - 4$$

$$XB = 5$$

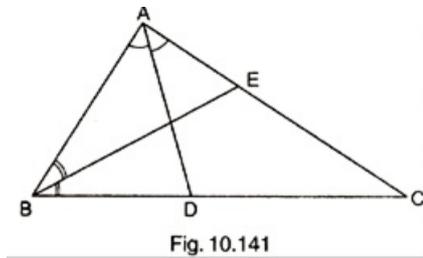
$$BY = XB + XY$$

$$= 5 + 2$$

$$= 7\text{cm}$$

19. Question

In Fig. 10.141, ABC is a triangle in which $\angle B = 2\angle C$. D is a point on side BC such that AD bisects $\angle BAC$ and $AB = CD$. BE is the bisector of $\angle B$. The measure of $\angle BAC$ is



[Hint: $\triangle ABE \cong \triangle DCE$]

A. 72°

B. 73°

C. 74°

D. 95°

Answer

Given that $\triangle ABC$

BE is bisector of $\angle B$ and AD is bisector of $\angle BAC$

$$\angle B = 2\angle C$$

By exterior angle theorem in triangle ADC

$$\angle ADB = \angle DAC + \angle C \text{ (i)}$$

In $\triangle ADB$,

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

$$2\angle C + \angle BAD + \angle DAC + \angle C = 180^\circ \text{ [From (i)]}$$

$$3\angle C + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 3\angle C \text{ (ii)}$$

Therefore,

$$AB = CD$$

$$\angle C = \angle DAC$$

$$\angle C = \frac{1}{2}\angle BAC \text{ (iii)}$$

Putting value of Angle C in (ii), we get

$$\angle BAC = 180^\circ - \frac{1}{2}\angle BAC$$

$$\angle BAC + \frac{3}{2}\angle BAC = 180^\circ$$

$$\frac{5}{2}\angle BAC = 180^\circ$$

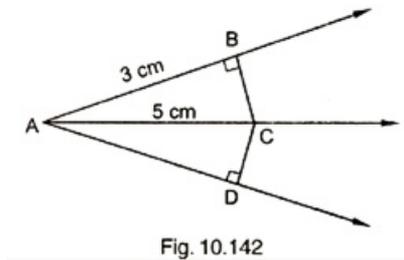
$$\angle BAC = \frac{180 \times 2}{5}$$

$$= 72^\circ$$

$$\angle BAC = 72^\circ$$

20. Question

In Fig. 10.142, if AC is bisector of $\angle BAD$ such that $AB=3$ cm and $AC=5$ cm, then $CD=$



A. 2 cm

B. 3 cm

C. 4 cm

D. 5 cm

Answer

In $\triangle ABC$ using Pythagoras theorem, we get

$$AB^2 + BC^2 = AC^2$$

$$9 + BC^2 = 25$$

$$BC = 4 \text{ cm}$$

In $\triangle ABC$ and $\triangle ADC$

$\angle BAC = \angle CAD$ (Therefore, AC is bisector of $\angle A$)

$$\angle B = \angle D = 90^\circ$$

$$\angle ABC + \angle BCA + \angle CAB = 180^\circ$$

$$\angle CAD + \angle ADC + \angle DCA = 180^\circ$$

$$\angle ABC + \angle BCA + \angle CAB = \angle CAD + \angle ADC + \angle DCA$$

$$\angle BCA = \angle DCA \text{ (i)}$$

In $\triangle ABC$ and $\triangle ADC$

$\angle CAB = \angle CAD$ (Therefore, AC is bisector of $\angle A$)

$$\angle BCA = \angle DCA \text{ [From (i)]}$$

$$AC = AC \text{ (Common)}$$

By ASA theorem, we have

$$\triangle ABC \cong \triangle ADC$$

$$BC = CD \text{ (By c.p.c.t)}$$

$$CD = 4 \text{ cm}$$