Real Numbers Formulas

Euclid’s Division Algorithm (lemma): According to Euclid’s Division Lemma if we have two positive integers a and b, then there exist unique integers q and r such that $a = bq + r$, where $0 \leq r \leq b$. (Here, a = dividend, b = divisor, q = quotient and r = remainder.)

Here you can check the important formulas related to Class 10 Maths Real Numbers.

<table>
<thead>
<tr>
<th>S. No</th>
<th>Type of Numbers</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Natural Numbers</td>
<td>$N = {1,2,3,4,5}$ &gt; It is the counting numbers</td>
</tr>
<tr>
<td>2</td>
<td>Whole numbers</td>
<td>$W = {0,1,2,3,4,5}$ &gt; It is the counting numbers + zero</td>
</tr>
<tr>
<td>3</td>
<td>Integers</td>
<td>All whole numbers including Negative number + Positive number $\ldots ... -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots$ so on. Like whole numbers, integers don’t include fractions or decimals.</td>
</tr>
<tr>
<td>4</td>
<td>Positive integers</td>
<td>$Z = 1, 2, 3, 4, 5, \ldots$</td>
</tr>
<tr>
<td>5</td>
<td>Negative integers</td>
<td>$Z = -1, -2, -3, -4, -5, \ldots$</td>
</tr>
<tr>
<td>6</td>
<td>Rational Number</td>
<td>A number is called rational if it can be expressed in the form $p/q$ where $p$ and $q$ are integers ($q &gt; 0$). Ex: $p/q, 4/5$</td>
</tr>
<tr>
<td>7</td>
<td>Irrational Number</td>
<td>A number is called rational if it cannot be expressed in the form $p/q$ where $p$ and $q$ are integers ($q &gt; 0$). Ex: $\sqrt{2}, \pi, \ldots$ etc</td>
</tr>
<tr>
<td>8</td>
<td>Real Numbers</td>
<td>A real number is a number that can be found on the number line. Real Numbers are the numbers that we normally use and apply in real-world applications. Real Numbers include Natural Numbers, Whole Numbers, Integers, Fractions, Rational Numbers and Irrational Numbers</td>
</tr>
</tbody>
</table>

HCF (Highest common factor)

HCF of two positive integers can be find using the Euclid’s Division Lemma algorithm. We know that for any two integers a, b, we can write following expression

- $a = bq + r, \ 0 \leq r < b$
- If $r = 0$, then $\text{HCF}(a, b) = b$
- If $r \neq 0$, then $\text{HCF}(a, b) = \text{HCF}(b, r)$

Again expressing the integer $b \cdot r$ in Euclid’s Division Lemma, we get
\[ b = pr + rl \]
\[ \text{HCF} (b,r) = \text{HCF} (r,rl) \]

Similarly, successive Euclid's division can be written until we get the remainder zero, the divisor at that point is called the HCF of the a and b.

- HCF (a,b) = 1 – Then a and b are co-primes.
- Product of primes theorem of arithmetic – Composite Number = Product of Primes.
- HCF and LCM by prime factorization method:
  - HCF = Product of the smallest power of each common factor in the numbers
  - LCM = Product of the greatest power of each prime factor involved in the numbers.
- Important formula: \[ \text{HCF} (a,b) \times \text{LCM} (a,b) = a \times b \]
- Important concept for rational number – Terminating decimal expression can be written in the form \( p/2^n \).

### Polynomials Formulas

- \((a + b)^2 = a^2 + 2ab + b^2\)
- \((a - b)^2 = a^2 - 2ab + b^2\)
- \(a^2 - b^2 = (a + b)(a - b)\)
- \((a + b)^3 = a^3 + b^3 + 3ab(a + b)\)
- \((a - b)^3 = a^3 - b^3 - 3ab(a - b)\)
- \(a^3 + b^3 = (a + b)(a^2 - ab + b^2)\)
- \(a^3 - b^3 = (a - b)(a^2 + ab + b^2)\)
- \(a^2 - b^2 = (a^2 - b^2) = (a^2 + b^2)(a - b)\)
- \((a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\)
- \((a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca\)
- \((a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca\)
- \(a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\)

### Linear Equations in Two Variables Formulas

For the pair of linear equations \(a_1 x + b_1 y + c_1 = 0\) and \(a_2 x + b_2 y + c_2 = 0\), the nature of roots (zeroes) or solutions is determined as follows:

- If \(a_1/a_2 \neq b_1/b_2\) then we get a unique solution and the pair of linear equations in two variables are consistent. Here, the graph consists of two intersecting lines.
- If \(a_1/a_2 \neq b_1/b_2 \neq c_1/c_2\), then there exists no solution and the pair of linear equations in two variables are said to be inconsistent. Here, the graph consists of parallel lines.
- If \(a_1/a_2 = b_1/b_2 = c_1/c_2\), then there exists infinitely many solutions and the pair of lines are coincident and therefore, dependent and consistent. Here, the graph consists of coincident lines.

### Quadratic Equation Formulas

For a quadratic equation, \(ax^2 + bx + c = 0\)
\[ ax + bx + c = 0 \text{ where } a \neq 0 \text{ and } x = [-b \pm \sqrt{(b^2 - 4ac)}/2a] \]

- Sum of roots = \(-b/a\)
- Product of roots = \(c/a\)
- If roots of a quadratic equation are given, then the quadratic equation can be represented as:
  \[ x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0 \]
- If Discriminant > 0, then the roots of the quadratic equation are real and unequal/unique.
- If Discriminant = 0, then the roots of the quadratic equation are real and equal.
- If Discriminant < 0, then the roots of the quadratic equation are imaginary (not real).
Arithmetic Progression Formulas

- **nth Term of an Arithmetic Progression:** For a given AP, where \(a\) is the first term, \(d\) is a common difference, \(n\) is the number of terms, its \(n\)th term \((a_n)\) is given as
  \[a_n = a + (n-1)d\]

- **Sum of First \(n\) Terms of an Arithmetic Progression, \(S_n\)** is given as:
  \[S_n = \frac{n}{2} [a + (n-1)d]\]

The similarity of Triangles Formulas

- If two triangles are similar then the ratio of their sides is equal.
  \[\triangle ABC \sim \triangle PQR\] then \[\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}\]

- Theorem on the area of similar triangles: If two triangles are similar, then the ratio of the area of both triangles is proportional to the square of the ratio of their corresponding sides.
  \[
  \frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2
  \]

Coordinate Geometry Formulas

- **Distance Formulae:** Consider a line having two points \(A(x_1, y_1)\) and \(B(x_2, y_2)\), then the distance of these points is given as:
  \[AB = \sqrt{(x_2- x_1)^2 + (y_2- y_1)^2}\]

- **Section Formula:** If a point \(P\) divides a line \(AB\) with coordinates \(A(x_1, y_1)\) and \(B(x_2, y_2)\), in ratio \(m:n\), then the coordinates of the point \(P\) are given as:
  \[P=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)\]

- **Midpoint Formula:** The coordinates of the mid-point of a line \(AB\) with coordinates \(A(x_1, y_1)\) and \(B(x_2, y_2)\), are given as:
  \[P=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\]

- **Area of a Triangle:** Consider the triangle formed by the points \(A(x_1, y_1)\) and \(B(x_2, y_2)\) and \(C(x_3, y_3)\) then the area of a triangle is given as:
  \[\triangle ABC=\frac{1}{2} |x_1(y_2- y_3)+x_2(y_3- y_1)+x_3(y_1- y_2)|\]

Trigonometry Formulas

In a right-angled triangle, the Pythagoras theorem states

\[(\text{perpendicular})^2 + (\text{base})^2 = (\text{hypotenuse})^2\]

**Important trigonometric properties:** (with \(P = \text{perpendicular}, B = \text{base}\) and \(H = \text{hypotenuse}\))

- \(\sin A = P/H\)
- \(\cos A = B/H\)
- \(\tan A = P/B\)
- \(\cot A = B/P\)
- \(\cosec A = H/P\)
- \(\sec A = H/B\)

**Trigonometric Identities:**

- \(\sin^2 A + \cos^2 A = 1\)
\[ \tan^2 A + 1 = \sec^2 A \]
\[ \cot^2 A + 1 = \csc^2 A \]

**Relations between trigonometric identities are given below:**
- \( \sec \theta = 1 / \cos \theta \)
- \( \cot \theta = 1 / \tan \theta \)
- \( \csc \theta = 1 / \sin \theta \)
- \( \tan \theta = \sin \theta / \cos \theta \)

**Trigonometric Ratios of Complementary Angles are given as follows:**
- \( \sin (90^\circ - A) = \cos A \)
- \( \cos (90^\circ - A) = \sin A \)
- \( \tan (90^\circ - A) = \cot A \)
- \( \cot (90^\circ - A) = \tan A \)
- \( \sec (90^\circ - A) = \csc A \)
- \( \cosec (90^\circ - A) = \sec A \)

**Values of Trigonometric Ratios of 0° and 90° are tabulated below:**

<table>
<thead>
<tr>
<th>Angle</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinθ</td>
<td>0</td>
<td>1/2</td>
<td>1/√2</td>
<td>√3/2</td>
<td>1</td>
</tr>
<tr>
<td>Cosθ</td>
<td>1</td>
<td>√3/2</td>
<td>1/2</td>
<td>½</td>
<td>0</td>
</tr>
<tr>
<td>Tanθ</td>
<td>0</td>
<td>1/√3</td>
<td>1</td>
<td>√3</td>
<td>Undefined</td>
</tr>
<tr>
<td>Cotθ</td>
<td>Undefined</td>
<td>√3</td>
<td>1</td>
<td>1/√3</td>
<td>0</td>
</tr>
<tr>
<td>Secθ</td>
<td>1</td>
<td>2/√3</td>
<td>√2</td>
<td>2</td>
<td>Undefined</td>
</tr>
<tr>
<td>Cosecθ</td>
<td>Undefined</td>
<td>2</td>
<td>√2</td>
<td>2/√3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Circles Formulas**
- **Circumference of the circle** = \( 2\pi r \)
- **Area of the circle** = \( \pi r^2 \)
- **Area of the sector of angle** \( \theta \) = \( (\theta/360) \times \pi r^2 \)
- **Length of an arc of a sector of angle** \( \theta \) = \( (\theta/360) \times 2\pi r \) (\( r = \) radius of the circle)

**Areas Related to Circles Formulas**
- The equal chord of a circle is equidistant from the center.
- The perpendicular drawn from the center of a circle bisects the chord of the circle.
- The angle subtended at the center by an arc = Double the angle at any part of the circumference of the circle.
- Angles subtended by the same arc in the same segment are equal.
- To a circle, if a tangent is drawn and a chord is drawn from the point of contact, then the angle made between the chord and the tangent is equal to the angle made in the alternate segment.
The sum of the opposite angles of a cyclic quadrilateral is always 180°.

- Area of a Segment of a Circle: If AB is a chord that divides the circle into two parts, then the bigger part is known as the major segment and the smaller one is called the minor segment.

Here, Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

**Surface Areas and Volumes Formulas**

The common formulas from the surface area and volumes chapter in Class 10 Maths include the following:

| Sphere Formulas |
|-----------------|-----------------|
| Diameter of sphere | 2r              |
| Circumference of Sphere | 2πr            |
| The surface area of a sphere | 4πr²           |
| Volume of Cylinder | 4/3πr³         |

| Cylinder Formulas |
|-------------------|-----------------|
| Circumference of Cylinder | 2πrh           |
| The curved surface area of Cylinder | 2πr²           |
The total surface area of Cylinder: Circumference of Cylinder + Curved surface area of Cylinder = \(2 \pi rh + 2 \pi r^2\)

Volume of Cylinder: \(\pi r^2h\)

<table>
<thead>
<tr>
<th>• Cone Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>The slant height of a cone: (l = \sqrt{r^2 + h^2})</td>
</tr>
<tr>
<td>The curved surface area of a cone: (\pi rl)</td>
</tr>
<tr>
<td>The total surface area of a cone: (\pi r(l + r))</td>
</tr>
<tr>
<td>Volume of cone: (\frac{1}{3} \pi r^2h)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>• Cuboid Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter of cuboid: (4(l + b + h))</td>
</tr>
<tr>
<td>Length of the longest diagonal of a cuboid: (\sqrt{l^2 + b^2 + h^2})</td>
</tr>
<tr>
<td>The total surface area of the cuboid: (2(lb + bh + lh))</td>
</tr>
<tr>
<td>Volume of Cuboid: (l \times b \times h)</td>
</tr>
</tbody>
</table>

Here, \(l =\) length, \(b =\) breadth and \(h =\) height In case of Cube, put \(l = b = h = a\), as cube all its sides of equal length, to find the surface area and volumes.

**Statistics Formulas**

**Mean:** The mean value of a variable is defined as the sum of all the values of the variable divided by the number of values.

\[
a_m = \frac{a_1 + a_2 + a_3 + a_4 + \ldots + a_n}{n}
\]

**Median:** The median of a set of data values is the middle value of the data set when it has been arranged in ascending order. That is, from the smallest value to the highest value.

Median is calculated as

\[
\frac{1}{2}(n + 1)
\]

Where \(n\) is the number of values in the data. If the number of values in the data set is even, then the median is the average of the two middle values.

**Mode:** Mode of a statistical data is the value of that variable which has the maximum frequency

For Grouped Data:

**Mean:** If \(x_1, x_2, x_3, \ldots, x_n\) are observations with respective frequencies \(f_1, f_2, f_3, \ldots, f_n\) then mean is given as:

\[
\bar{x} = \frac{\sum f_i x_i}{\sum f_i}
\]
Median: For the given data, we need to have a class interval, frequency distribution, and cumulative frequency distribution. Then, the median is calculated as

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

or

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

where

\[\sum_{i=1}^{n} f_i = f_1 + f_2 + f_3 + \ldots + f_n\]

**Median**

\[\text{Median} = l + \left( \frac{n}{2} - cf \right) \frac{h}{f} \]

Where

- \(l\) = lower limit of the median class,
- \(n\) = number of observations,
- \(cf\) = cumulative frequency of the class preceding the median class,
- \(f\) = frequency of the median class,
- \(h\) = class size (assuming class size to be equal).

Mode: Modal class: The class interval having the highest frequency is called the modal class and Mode is obtained using the modal class.

**Mode**

\[M_o = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h\]

Where

- \(l\) = lower limit of the modal class,
- \(h\) = size of the class interval (assuming all class sizes to be equal),
- \(f_1\) = frequency of the modal class,
- \(f_0\) = frequency of the class preceding the modal class,
- \(f_2\) = frequency of the class succeeding in the modal class.

**Probability Formulas**

Probability of an event, \(P(E) = \frac{\text{Number of outcomes favorable to } E}{\text{Total number of outcomes}}\)