Exercise 16.1: Surface Areas and Volumes

Question 1: How many balls, each of radiuses 1 cm can be made from a solid sphere of lead of radius 8 cm?

Solution:

Given that a solid sphere of radius =8 cm

With this sphere, we have to make spherical balls = 1 cm

Since we don’t know number of balls let us assume that number of balls be n

We know that,

Volume of the sphere = $43\prod r^3 \frac{4}{3}\prod r^3$

The volume of the solid sphere equal to sum of the n spherical balls.

= $n(43\prod 1^3)n(\frac{4}{3}\prod 1^3)$= $43\prod r^3 \frac{4}{3}\prod r^3$

= n= 8^3 =n= 512

Hence 512 numbers of balls can be made of radius 1 cm of a solid sphere of radius 8 cm.

Question 2: How many spherical bullets each of 5 cm in diameter can be cast from a rectangular block of metal 11dm *1m*5dm?

Solution:

Given that a metallic block which is rectangular of dimension 11dm*1m*5dm

Given that the diameter of each bullet is 5 cm

Volume of the sphere = $43\prod r^3 \frac{4}{3}\prod r^3$

Dimensions of the rectangular block = 11dm*1m*5dm

Since we know that 1 dm = 10^{-1}m

$11*10^{-1}*1*5*10^{-1} = 55*10^{-2}$ ……………………….(i)

Diameter of the bullet = 5cm

Radius of the bullet = 5/2 = 2.5 cm

So the volume of the rectangular block equals to the sum of the volumes of the n spherical bullets

Let the number of bullets be n

$55*10^{-2}$= $n(43\prod 1^3)n(\frac{4}{3}\prod 1^3)$

= n= 8400

Numbers of bullets formed were 8400.

Question.3: A spherical ball of radius 3cm is melted and recast into three spherical balls. The radii of the two balls are 2cm and 1.5cm respectively. Determine the diameter of the third ball?

Solution:

According to the question

Radius of the spherical ball=3cm

We know that the volume of the sphere= $43\Pi r^3 \frac{4}{3}\Pi r^3$

So it's volume (v)= $43\Pi r^3 \frac{4}{3}\Pi r^3$

Given,

That the ball is melted and recast into 3 spherical balls.

Volume (V_1) of first ball = $43\prod 1.5^3 \frac{4}{3}\prod 1.5^3$

Volume (V_2) of second ball = $43\prod 2^3 \frac{4}{3}\prod 2^3$

Radii of the third ball be =r cm

Volume of third ball (V_3) = $43\prod r^3 \frac{4}{3}\prod r^3$

Volume of the spherical ball is equal to the volume of the 3 small spherical balls.

$V=V_1+V_2+V_3$

$43\Pi r^3 \frac{4}{3}\Pi r^3 = 43\prod 1.5^3 \frac{4}{3}\prod 1.5^3 + 43\prod +2^3 \frac{4}{3}\prod +2^3 + 43\prod r^3 \frac{4}{3}\prod r^3$

Now,

Cancelling out the common part from both sides of the equation we get,

$(3)^3 = (2)^3 + (1.5)^3 + r^3$

$r^3 = 3^3 - 2^3 - 1.5^3\ cm^3$

$r^3 = 15.6 cm^3$

$r = (15.6)^{1/3}$ cm

r=2.5cm

we know diameter = 2* radius

=2*2.5 cm

=5.0 cm

The diameter of the third ball is 5.0 cm

Question.4: 2.2 cubic dm of brass is to drawn into a cylindrical wire of 0.25cm diameters. Find the length of the wire?

Solution:

Given,

2.2 dm^3 of brass is to be drawn into a cylindrical wire of 0.25cm diameter

Radius of the wire (r) = d/2

=0.25/2 = $0.125*10^{-2}$cm

Now, 1cm =0.01m

So, 0.1cm=0.001m

Let the length of the wire be (h)

Volume of the cylinder= $\prod r^2h \prod r^2 h$

Volume of brass of 2.2 dm^3 is equal to volume of cylindrical wire.

227 $(0.125\times10^{-2})^2\times h=2.2\times10^{-3}$ $\frac{22}{7}(0.125 \times 10^{-2})^2 \times h = 2.2 \times 10^{-3}$

H=448 m

The length of the cylindrical wire is 448m

Question 5: What length of a solid cylinder 2 cm in diameter be taken to recast into a hollow cylinder of length 16 cm, external daimeter20cm and thickness 2.5mm?

Solution:

According to the question

Diameter of the solid cylinder=2 cm

The solid cylinder is recast into a hollow cylinder of length 16 cm, external diameter of 20cm and thickness of2.5 cm

Volume of the cylinder = $\prod r^2h \prod r^2 h$

Radius of the cylinder =1 cm

So, volume of the solid cylinder=$\prod 1^2h \prod 1^2 h$

Let the length of the solid cylinder be l

Volume of the hollow cylinder = $\prod h(R^2-r^2) \prod h(R^2-r^2)$

Thickness of the cylinder = (R-r)

0.25=10-r

Internal radius of the cylinder is 9.75cm

Volume of the hollow cylinder = $\prod 16(100-95.0625) \prod 16(100-95.0625)$

Hence, the volume of the solid cylinder is equal to the volume of the hollow cylinder

Equation I = equation ii

$\prod 1^2h \prod 1^2 h$= π*16(100-95.06)

=h=79.04cm

Length of the solid cylinder is 79.04 cm.

Question 6: A cylindrical vessel having the diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel.

Sol:

Given,

The diameter is equal to the height of a cylinder = $\prod r^2 h \prod r^2 h$

So, volume = $\prod r^2 2r \prod r^2 2r$ (h=2r)……………(i)

$2\prod r^3 2\prod r^3$

Volume of each vessel = $\prod r^2 h \prod r^2 h$

Diameter=42 cm

Height=21 cm

Diameter= 2r

2r=42 cm

.r =21 cm

Volume of vessel = $\prod 21^2 \times 21 \prod 21^2 \times 21$......................(ii)

Since the volumes of equation i and ii are equal

So equating both the equations

$r^3 = (21)^3$

r =21 cm

d= 42cm

The diameter of the cylindrical vessel is 42cm

Question 7: 50 circular plates each of diameter 14cm and thickness 0.5cm are placed one above the other to form right circular cylinder. Find its total surface area

Solution:

Given that the 50 circular plates each with diameter 14 cm

Radius of circular plates =7 cm

Thickness of plates =0.5 cm

Since these plates are one above the other so total thickness of plates =0.5 *100 =25cm

Total surface area of a cylinder = $2\prod r \times h + 2\prod r^2 2\prod r \times h + 2\prod r^2$

$2\prod r(h+r) 2\prod r(h+r)$ 2× 227 ×7(25+7) $2 \times \frac{22}{7} \times 7(25+7)$

Total surface area =1408 cm^2

The total surface area of the cylinder is 1408 cm^2

Question 8: 25 circular plates each of radius 10.5cm and thickness 1.6 cm are placed one above the other to form a solid circular cylinder. Find the curved surface area and volume of the cylinder so formed.

Solution:

Given that 250 circular plates each with radius 10.5 cm

Thickness is 1.6 cm.

Since plates are placed one above the other so its height becomes =1.6*25 =40 cm

Volume of the cylinder = $\prod r^2 h \prod r^2 h$

= $\prod 10.5^2 \times 40 \prod 10.5^2 \times 40$ =13860 cm^3

Curved surface area of a cylinder = $2\prod r \times h 2 \prod r \times h$

= $2\prod 10.5 \times 40 2 \prod 10.5 \times 40$ = 2640cm^2

Volume of the cylinder is 13860 cm^3

Curved surface area of the cylinder is 2640 cm^2

Question 9: The diameter of a metallic sphere is equaled to 9 cm. It is melted and drawn into a long wire of diameter 2 mm having the uniform cross section. Find the length of the wire?

Solution:

Given,

Diameter of the metallic sphere = 9 cm

Radius of the metallic sphere = 9/2 cm =4.5 cm

Volume of the sphere = $43 \prod \times r^3 \frac{4}{3} \prod \times r^3$

= $43 \prod \times 4.5^3 \frac{4}{3} \prod \times 4.5^3$

= 381.703 cm^3 …………………………………….. (i)

Diameter of the cylindrical wire = 2 mm = 2/10 cm = 0.2 cm

The radius of the cylindrical wire be = 0.2/2 cm = 0.1 cm

Let the height of the cylindrical wire be = h cm

Volume of the cylindrical wire = ∏×r^2×h$\prod \times r^2 \times h$

= ∏×0.1^2×h$\prod \times 0.1^2 \times h$...............(ii)

Since the metallic sphere is melted and recast into a long cylindrical wire.

So comparing and equating both the above-marked equations

We get,

381.703 = ∏×0.1^2×h$\prod \times 0.1^2 \times h$...............(ii)

H= 12150 =12150 cm

The required length of the wire after recasting of the metallic sphere is 12150 cm.

Question 10: Find the number of smaller sphere required if the radius of the larger ball is four times the radius of the smaller ball.

Solution:

According to the question,

Let the radius of the smaller ball be = r cm

Now, the radius of the larger ball be = 4r cm

Volume of the smaller sphere = 43∏×$r^3$$\frac{4}{3} \prod \times r^3$ ....(i)

Volume of the larger sphere = 43∏×$(4r)^3$$\frac{4}{3} \prod \times (4r)^3$ ....(ii)

Now, dividing equation (ii) by (i) .we get,

Number of smaller balls (n)

n =43∏×$(4r)^3$$\frac{4}{3} \prod \times (4r)^3$ / 43∏×$(r)^3$$\frac{4}{3} \prod \times (r)^3$ = 64

The volume of the larger sphere is 8 times the volume of the smaller ball.

Curved surface area of the smaller sphere = 4∏×(r)$4 \prod \times (r)$ ....(iii)

Curved surface area of the larger sphere = 4∏×$(2r)^{4r}$$4 \prod \times (2r)^{4r}$ ....(iv)

Now, dividing equation (iv) by (iii). We get,

Number of smaller spheres (N)

N = $4\prod\times(2r)^2 4\prod\times(2r)^2$ / $4\prod\times(r)^2 4\prod\times(r)^2$

= 4

The curved surface area of large ball is four times the curved surface area of the smaller ball.

Question 11: A copper sphere of radius 3 cm is melted and re-casted into a right circular cone of height 3 cm. Find the radius of the base of the cone?

Solution:

Given,

Radius of the sphere = 5 cm

Volume of the sphere = $43\prod\times(r)^3\frac{4}{3}\prod\times(r)^3$

= $43\prod\times(3)^3\frac{4}{3}\prod\times(3)^3$...............(i)

The given sphere is melted and recast into a right circular cone

Height of the cone = 3cm

Volume of the right circular cone = $13\prod\times(r)^2\times h\frac{1}{3}\prod\times(r)^2\times h$

= $13\prod\times(r)^2\times 3\frac{1}{3}\prod\times(r)^2\times 3$...............(ii)

Now comparing equation (i) and (ii) we get

$r^2 = 36$

r = 6 cm

The radius of the cone is 6 cm

Question 12: A copper wire of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire?

Solution:

Given,

Diameter of the copper wire = 1 cm

Radius of the copper wire = 1 / 2 cm = 0.5 cm

Length of the copper rod = 8 cm

Volume of the cylinder = $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times 0.5^2 \times 10 \prod \times 0.5^2 \times 10$ …………………………………….. (i)

Length of the wire = 18 m = 1800cm

Volume of the wire = $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times r^2 \times 1800 \prod \times r^2 \times 1800$ …………………………………………. (ii)

Now equating both the equations. We get,

r^2 = 200.00044

r= 0.033 cm

The radius or thickness of the wire is 0.033 cm

Question 13: The diameters of the internal and external surface area of hollow spherical shell are 10cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of 2 23 $2\frac{2}{3}$, find the diameter of the cylinder?

Solution:

Given,

Internal diameter of the hollow sphere = 6 cm

Internal radius of the hollow sphere = 6/2 cm = 3 cm

External diameter of the hollow sphere = 10 cm

External radius of the hollow sphere = 10/2 cm = 5 cm

Volume of the hollow spherical shell = 43∏×R^3−r^3 $\frac{4}{3}\prod \times R^3 - r^3$

= 43∏×5^3−3^3 $\frac{4}{3}\prod \times 5^3 - 3^3$ …………………… (i)

Given the length of the solid cylinder = 83 $\frac{8}{3}$ cm

Let the radius of the solid cylinder be h cm

Volume of the cylinder = ∏×r^2×h $\prod \times r^2 \times h$

= ∏×r^2× 83 $\prod \times r^2 \times \frac{8}{3}$ …………….. (ii)

Now equating both the above marked equations in order to obtain the radius of the cylinder

r^2 = 49

r =7

d= 7 * 2 =14 cm

Question 14: How many coins 1.75cm in diameter and 2mm thick must be melted to form a cuboid 11cm*10cm*7cm?

Solution:

So its volume = 11*10*10 cm^3

Given diameter =1.75 cm

Radius = 0.875 cm

Volume of the cylinder = ∏r^2h $\prod r^2 h$

= ∏$0.875^2$0.2 $\prod 0.875^2 0.2$

V_1=v_2 *n

By calculating the above problem we get,

N=1600

Number of coins are 1600

Question 15: The surface area of the solid metallic sphere is 616 cm^3. It is melted and recast into a cone of height 28 cm. find the diameter of the base of the cone so formed?

Solution:

The height of the cone = 28 cm

Surface area of the sphere= 616 cm^3

We know that the surface area of the sphere = $4\prod r^2 4\prod r^2$

$=4\prod r^2 4\prod r^2$ = 616

= r^2= 49

= r =7

Radius of the sphere = 7 cm

Let R be the radius of the cone

Volume of the cone = $13\prod r^2 h \frac{1}{3}\prod r^2 h$

= $13\prod r^2 28 \frac{1}{3}\prod r^2 28$…………………(i)

Volume of the sphere= $43\prod r^3 \frac{4}{3}\prod r^3$

= $43\prod 7^3 \frac{4}{3}\prod 7^3$ …………………..(ii)

Comparing equation (i) and (ii)

R^2= 49

R=7

Diameter of the cone = 7*2 = 14 cm

The diameter of the base of the cone is 14 cm

Question 16: A spherical shell of internal and external diameter 6cm and 10cm respectively is melted and recast into a cylinder of diameter 14 cm. find the height of

the cylinder ?

Solution:

Internal diameter of a hollow spherical shell =6 cm

Internal radius of a hollow spherical shell = 3 cm

External diameter of a hollow spherical shell = 10 cm

External radius of a hollow spherical shell = 5 cm

Volume of the spherical shell = $43\prod\times(5^3-3^3)\frac{4}{3}\prod\times(5^3-3^3)$...............(i)

Diameter of the cylinder = 14 cm

Radius of the cylinder = 7cm

Let the height of the cylinder be (x) cm

Volume of the cylinder = $\prod\times7^2\times x\prod\times7^2\times x$..........(ii)

According to the question,

Volume of the cylinder is equals to the volume of the spherical shell.

$43\prod\times(5^3-3^3)\frac{4}{3}\prod\times(5^3-3^3)$ = 49 *x

X= 2.666 cm

Question 17: A hollow sphere of internal and external diameters 4cm and 8cm is melted into a cone of base diameter 8 cm.Calculate the height of the cone?

Solution:

Given,

Internal diameter of hollow sphere = 4 cm

Internal radius of hollow sphere = 2 cm

External diameter of hollow sphere = 8 cm

External radius of hollow sphere = 4 cm

Volume of the hollow sphere = $43\prod\times(4^3-2^3)\frac{4}{3}\prod\times(4^3-2^3)$……………………(i)

Given, diameter of the cone = 8 cm

Radius of the cone = 4 cm

Let the height of the cone be x cm

Volume of the cone = $13\prod\times4^2\times h\frac{1}{3}\prod\times4^2\times h$……………………………..(ii)

Since the volume of the hollow sphere and cone are equal.

So, equating equations i and ii, we get,

$$43\prod\times(4^2-2^2)\frac{4}{3}\prod\times(4^2-2^2)= 13\prod\times4^2\times h\frac{1}{3}\prod\times4^2\times h$$

H= 12 cm

The height of the cone so formed is having a height of 12 cm

Question 18: A path 2m wide surrounds a circular pond of diameter of 40 m. How many cubic meters of sand are required to grave the path to a depth of 20 cm?

Solution:

Given,

Diameter of the circular pond=40 m

Radius of the pond= 20m/2 =10m (r=d/2)

Thickness=2m

We know 1cm =0.01m

10cm=10/100 m =0.10m

Since the whole view of the pond looks like a hollow cylinder. So,

Thickness (t) =R-r

2=R-20

R=22m

Volume of the hollow cylinder=$\prod(R^2-r^2)\times h\prod(R^2-r^2)\times h$

$\prod(22^2-20^2)\times 0.10\prod(22^2-20^2)\times 0.10$

=52.77m³

Volume of the cylinder is 52.77m³.

Since it is a hollow cylinder the volume of the cylinder indicates the required amount of sand needed to spread across to a depth of 20 m.

Question 19: A 16 m deep well with diameter 3.5 m is dug up and the earth from it is spread evenly to form a platform 27.5 m by 7m. Find the height of the platform?

Solution:

Let us assume the well is a solid right circular cylinder

Radius(r) of the cylinder =3.5/2 m =1.75 m

Height or depth of the well (h) = 16 m

Volume of the cylinder = $\prod\times r^2\times h\prod\times r^2\times h$

$\prod\times 1.75^2\times 16\prod\times 1.75^2\times 16$ ……………………………………………………….(i)

Given,

The length of the platform (l) = 27.5 m

Breadth of the platform (b) =7 m

Let the height of the platform be x m

Volume of the rectangle = l*b*h

V = 25*5*x ……………………………………………………………………………………….(ii)

Since the well is spread evenly to form the platform

So equating equations (i) and (ii), we get .

$V_1 = V_2$

$\prod \times 1.75^2 \times 16 \prod \times 1.75^2 \times 16$ = 27.5*7*x

x = 0.8 m

Therefore x = 80 cm

The height of the platform is 80 cm.

Question 20: A well of diameter 2 m is dug 14 m deep. The earth taken out of it is evenly spread all around it to form an embankment of height 40 cm. find the width of the embankment?

Solution:

Radius of the circular cylinder (r) = 2/2 m = 1 m

Height of the well (h) = 14 m

Volume of the solid circular cylinder = $\prod \times r^2 \times h \prod \times r^2 \times h$ ……………………………………(i)

= $\prod \times 1^2 \times 14 \prod \times 1^2 \times 14$

Given,

The height of the embankment (h) = 40 cm

=0.4m

Let the width of the embankment be (x) m.

Volume of the embankment = $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times ((1+x)^2 - (1)^2 \times 0.4 \prod \times ((1+x)^2 - (1)^2 \times 0.4$……………(ii)

Since the well is spread evenly to form embankment so their volumes will be same

So equating equations i and ii, we get

$\prod \times r^2 \times h \prod \times r^2 \times h$ = $\prod \times ((2.5+x)^2 - (2.5)^2 \times 0.20 \prod \times ((2.5+x)^2 - (2.5)^2 \times 0.20$

X= 5 m

The width of the embankment is 5 m.

Question 21: A well with inner radius 4 m is dug up and 14 m deep. Earth taken out of it has been evenly all around a width of 3 m it to form an embankment. Find the height of the embankment?

Solution:

Given,

Inner radius of the well =4 m

Depth of the well = 14 m

Volume of the cylinder = $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times 4^2 \times 14 \prod \times 4^2 \times 14$ .... (i)

According to the question,

The earth taken out from the well is evenly spread all around it to form an embankment

Width of the embankment = 3 m

Outer radii of the well = 3 + 4 m = 7 m

Volume of the hollow well = $\prod \times (R^2 - r^2) \times h \prod \times (R^2 - r^2) \times h$

= $\prod \times (7^2 - 4^2) \times h \prod \times (7^2 - 4^2) \times h$ .... (ii)

Comparing both equations we get

H=6.78 m

The height of the well so formed is 6.78 m.

Question 22: A well of diameter 3m is dug up to 14m deep. The earth taken out of it has been spread evenly all around it to a width of 3m to form an embankment. Find the height of the embankment.

Sol:

Given

Diameter of the well = 3 m

Radius of the well = 3/2 m = 1.5m

Depth of the well = 14 m

Width of the embankment = 4 m

Radius of the outer surface of the embankment = 4+ 3/2 m = 5.5 m

Let the height of the embankment be = h m

Volume of the embankment = $\prod \times (R^2 - r^2) \times h \prod \times (R^2 - r^2) \times h$

= $\prod \times (5.5^2 - 1.5^2) \times h \prod \times (5.5^2 - 1.5^2) \times h$ ………… (i)

Volume of earth dug out = $\prod \times (r^2) \times h \prod \times (r^2) \times h$

= $\prod \times (2^2) \times 14 \prod \times (2^2) \times 14$ …….. (ii)

Comparing both the equations we get,

H = $98 \frac{9}{8}$ m

Height of the embankment is $98 \frac{9}{8}$ m

Question 23: Find the volume largest right circular cone that can be cut out of a cone of a cube whose edge is 9 cm.

Solution:

Given,

The side of the cube = 9 cm

The largest cone is curved from cube diameter of base of cone = side of the cube

2r = 9

r = 9/2 cm = 4.5 cm

Height of cone = side of cube

Height of cone (h) = 9 cm

Volume of the largest cone to fit in = $13 \times \prod \times r^2 \times h \frac{1}{3} \times \prod \times r^2 \times h$

= $13 \times \prod \times 4.5^2 \times 9 \frac{1}{3} \times \prod \times 4.5^2 \times 9$

=190.92 cm³

The volume of the largest cone to fit in is having a volume of 190.92 cm³

Question 24: Rain water which falls on a flat rectangular surface of length 6m and breadth 4m is transferred into a cylindrical vessel of internal radius 20 cm .What will be the height of water in the cylindrical vessel if a rainfall of 1cm has fallen?

Solution:

Given,

Length of the rectangular surface = 6 m = 600 cm

Breadth of the rectangular surface =4m = 400cm

Height of the perceived rain = 1 cm

Volume of the rectangular surface = length * breadth * height

=600*400*1 cm³

=240000 cm³ …………………………… (i)

Given,

Radius of the cylindrical vessel =20 cm

Let the height of the cylindrical vessel = h cm

Volume of the cylindrical vessel = $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times 20^2 \times h \prod \times 20^2 \times h$…………………………..(ii)

Since rains are transferred to cylindrical vessel

So, equating both the above marked equations:

240000 = $\prod \times 20^2 \times h \prod \times 20^2 \times h$

H =190.9 cm

The height of the cylindrical vessel is 190.9 cm.

Question 25: A conical flask of water. The flask has base radius(r) and height (h). The water is poured into a cylindrical flask of base radius $M_{r.}$ Find the height of water in the cylindrical flask?

Solution

Given:

Base radius of the conical flask = r m

Height of the conical flask = h m

Volume of the cone = 13∏×r²×h $\frac{1}{3}\prod \times r^2 \times h$ ....(i)

Given,

Base radius of the cylindrical flask is $M_{r.}$

Let the height of the flask be $h_{1.}$

Volume of the cylinder = ∏×r²×h $\prod \times r^2 \times h$

It's volume = 13∏×r²×h $\frac{1}{3}\prod \times r^2 \times h$.............................(ii)

Since water in conical flask is poured into cylindrical flask their volumes are same.

So, equations i and ii are same .

13∏×r²×h $\frac{1}{3}\prod \times r^2 \times h$ = 13∏×r²×h $\frac{1}{3}\prod \times r^2 \times h$

h_1= h/3m^2

The required height of the cylindrical vessel is h/3m^2.

Question 26: A rectangular tank is 15 m long and 11m in breadth is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21 m and length 5 m .Find the least height of the tank that will serve the purpose?

Solution:

Given,

Length of the rectangular tank = 15 m

Breadth of the rectangular tank = 11 m

Let the height of the rectangular tank be =h m

Volume of the rectangular tank = length *breadth * height

=15*11*h m^3 ………………………………………… (i)

Given,

Radius of the cylindrical tank = 21/2 m = 10.5 m

Height of the tank = 5 m

Volume of the cylindrical tank -= $\prod \times r^2 \times h \prod \times r^2 \times h$

= $\prod \times 10.5^2 \times 5 \prod \times 10.5^2 \times 5$ ……………………………(ii)

Since equation i and ii are having same volumes so, equating both equations

We get,

165 * h=$\prod \times 10.5^2 \times 5 \prod \times 10.5^2 \times 5$

h= 10.5 m

The height of the tank is 10.5 m.

Question 27: A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to empty the bowl?

Solution:

Given,

The internal radius of the hemispherical bowl = 9 cm

Volume of the hemisphere = 43$\prod \times r^3 \frac{4}{3} \prod \times r^3$

= 43$\prod \times 9^3 \frac{4}{3} \prod \times 9^3$…………………………..(i)

Given,

Diameter of the cylindrical bottle = 3 cm

Radius = 3/2 cm = 1.5 cm

Volume of the cylinder = ∏×r^2×h $\prod \times r^2 \times h$

=∏×1.5^2×4 $\prod \times 1.5^2 \times 4$ ....(ii)

Volume of the hemispherical bowl is equals to (n) multiplied by volume of the cylindrical bottles.

Now,

Comparing equations i and ii,

We get,

43∏×9^3 $\frac{4}{3}\prod \times 9^3$ =∏×1.5^2×4 $\prod \times 1.5^2 \times 4$

N= 54

54 cylindrical bottles are required to empty a hemispherical bowl.

Question 28: A cylindrical tube of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tube and the level of water is raised by 6.75 cm. Find the radius of the ball?

Solution:

Given,

The radius of a cylindrical tube (r) =12 cm

Level of water raised in the tube (h) = 6.75 cm

Volume of the cylinder = ∏×r^2×h $\prod \times r^2 \times h$

= ∏×12^2×6.75 $\prod \times 12^2 \times 6.75$

Let r be the radius of the spherical ball

Volume of the sphere = 43∏×r^3 $\frac{4}{3}\prod \times r^3$ ....(ii)

To find the radius of the spherical ball of unknown radius r we need to equate equations i and ii

We get,

$\prod \times 12^2 \times 6.75 \prod \times 12^2 \times 6.75$= 12.571 r^3

$r^3 = 729$

r = 9 cm

The radius of the spherical ball is 9 cm.

Question 29: 500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04m^3?

Solution:

Given,

The length of a rectangular tank (l) = 50 m

Breadth of the rectangular tank (b) = 10 m

Total displacement of water in a rectangular tank by 400 persons = 400* 0.02 m^3

=8 m^3 ……………………….(i)

Let the depth of tank be h m

Volume of the rectangular tank = length * breadth * height

= l* b* h

=50*10* h m^3 = 500hm^3 ………………………………….(ii)

Now equating equations i and ii, we get ,

8 = 500* h

H = 0.016 m

The rise in water level of the rectangular tank is 0.016m.

Question 30: A cylindrical jar of radius 6cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by 2 cm?

Solution:

Given,

The radius of the cylindrical jar (r) = 6 cm

Height of the cylindrical jar (h) = 2 cm

Let the number of balls be (n)

Volume of the cylinder = ∏×r^2×h$\prod \times r^2 \times h$

= ∏×6^2×2$\prod \times 6^2 \times 2$ ………………….(i)

Let the Radius of the sphere be 1.5 cm

Volume of the sphere = 43∏×$r^3$$\frac{4}{3} \prod \times r^3$ ………………………….(ii)

Volume of the cylindrical jar is equal to the sum of volume of (n) number of spheres.

Now, equating equations I and ii we get,

∏×6^2×2[/latex=n∗[latex]43∏×$r^3$$\prod \times 6^2 \times 2[/latex = n * [latex]\frac{4}{3} \prod \times r^3$

∏×6^2×2$\prod \times 6^2 \times 2$ = n * 4.1904

N= 16

= 16

16 spherical balls are required to raise the water level by 2 cm.

Question 31: The metallic spheres each of radius 2 cm area packed into a rectangular box of internal dimension 16 cm* 8cm* 8cm where 16 spheres are packed the box is filled with preservation liquid . Find the volume of this liquid ?

Solution:

Given,

Radius of the metallic spheres = 2 cm

Volume of the sphere = $43\prod\times(r)^3\frac{4}{3}\prod\times(r)^3$

= $43\prod\times(2)^3\frac{4}{3}\prod\times(2)^3$ ……………………………….(i)

Total volume of the 16 spheres = 16 * $43\prod\times(2)^3\frac{4}{3}\prod\times(2)^3$

Volume of the rectangular box= 16*8*8 cm^{33}……………………………………..(ii)

Subtracting equation (ii) from (i) we get the volume of the liquid

Volume of the liquid = (1024 – 536.16) cm^3 =488 cm^3

The volume of the liquid is 488 cm^3

Question 32: A vessel in the shape of a cuboids' contains some water. If 3 identical spheres are immersed in the water, the level of water is increased by 2 cm. If the area of the base of the cuboids is 160 cm^2 and its height is 12 cm. determine the radius of any of the spheres?

Solution:

Given,

The area of the cuboids'= 160 cm^2

Level of water in the vessel increased = 2 cm

Volume of the vessel = 160 *2 cm^2

=320 cm^2 ……………………………….. (i)

Volume of each sphere = $43\prod\times(r)^3\frac{4}{3}\prod\times(r)^3$

Total volume of 3 spheres = $3\times43\prod\times(r)^3 3\times\frac{4}{3}\prod\times(r)^3$ …………….(ii)

Now equating equation (i) and (ii) we get

320 = $3\times43\prod\times(r)^3 3\times\frac{4}{3}\prod\times(r)^3$

R= 2.94 cm.

The radius of the sphere so obtained is 2.94 cm.

Question 33: Water in a canal 1.5 m wide and 6 m deep is flowing with a sped of 10 km / hr .How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Solution:

Given

Water is flowing at a speed of = 10 km / hr

In 20 minutes length of the flowing standing water = 10*30/60 = 5 km = 5000 m

Width of the canal = 1.5m

Depth of the canal = 6m

Volume of water in 20 minutes = 5000 * width * depth = 45000 m^3

Irrigated area in 20 minutes if 5 cm of standing water is desired = 45000 / 0.08 m^2 = 562500 m^2

Irrigated area in 30 minutes is 562500 m^2

Question 34: A tent of height 77 dm is in the form of aright angled circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of canvas at the rate of Rs.3.5 per m^2.

Solution:

Given

The height of the tent = 77 dm

Height of the cone = 44 dm

Height of the tent without the cone = (77 – 44) dm = 33 dm

= 3.3 m

Diameter of the cylinder = 36 m

Radius of the cylinder = 36 / 2 m = 18 m

Let us assume the slant height of the cone be (l)

$L^2 = r^2 + h^2$

$L^2 = (18)^2 + (3.3)^2$

$l = 18.3$ m

The slant height of the cone is 18.3 m.

Curved surface area of the cylinder = 2πrh

= 2 *π * 18*4.4 m^2 ……………….. (i)

Curved surface area of the cone = πrh

= π * 18* 18.3 m^2…………………….. (ii)

Now adding equation (i) and (ii) we will get the value of the total surface area

Total surface area = 1532.46 m^2

Cost of canvas = 1532.46 m^2 * Rs 3.5 = Rs 5363.61

Question 35: The largest sphere is to be curved out of a right circular cylinder of radius 7 cm and height 14 cm. find the volume of the sphere.

Solution:

Given radius of the cylinder = 7 cm

Height of the cylinder = 14 cm

Largest sphere is curved out from the cylinder.

Thus the diameter of sphere = diameter of cylinder

Diameter of the sphere = 2*7 = 14 cm

Volume of the sphere = 43∏r^3 $\frac{4}{3} \prod r^3$

= 43∏7^3 $\frac{4}{3} \prod 7^3$

= 1436.75 cm^3

Volume of the sphere is 1436.75 cm^3

Question 36: A right angled triangle whose sides are 3cm, 4cm and 5 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two cones so formed. Also find the curved surfaces?

Solution:

Given:

I>

Radius of the cone = 4 cm

Height of the cone = 3 cm

Slant height of the cone = 5 cm

Volume of the cone = 13×∏×r^2×h$\frac{1}{3} \times \prod \times r^2 \times h$

= 13×∏×4^2×3$\frac{1}{3} \times \prod \times 4^2 \times 3$

= 16π cm^3

II>

Radius of the second cone = 3 cm

Height of the cone = 4 cm

Slant height of the cone = 5 cm

Volume of the cone = 13×∏×r^2×h$\frac{1}{3} \times \prod \times r^2 \times h$

= 13×∏×3^2×4$\frac{1}{3} \times \prod \times 3^2 \times 4$

=12π cm^3

Difference of the volumes of two cone = (16-12) πcm^3

=4π cm^3

Curved surface area of the first cone = π r_1 l_1

$= \pi * 4 * 5 \text{ cm}^2$

$= 20\pi \text{ cm}^2$

Curved surface area of the second cone = $\pi r_2 l_2$

$= \pi * 3 * 5 \text{ cm}^2 = 15\pi \text{ cm}^2$

Question 37: The volume of the hemisphere is 2425 12 $2425\frac{1}{2}$. find it's curved surface area.

Solution:

Given,

Volume of the hemisphere = 23∏×r3 $\frac{2}{3}\prod \times r^3$

23∏×r3 $\frac{2}{3}\prod \times r^3$ = 48512 $\frac{4851}{2}$

r^3 = 4851(3)2(2)(∏) $\frac{4851(3)}{2(2)(\prod)}$

r = (1193.18) ⅓

r = 10.5 cm

Curved surface area of the hemisphere = $2\pi r^2$

$2 * \pi * (10.5)^2 = 693 \text{ cm}^2$

The curved surface area of the hemisphere is 693 $\text{cm}^{2.}$

Question 38: The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14 cm long is 88 cm^2. If the volume of metal used in making cylinder is 176 cm^3. Find the outer and inner diameters of the cylinder?

Solution:

Given,

Height of the cylinder = 14 cm

Let the inner and outer radii of the hollow sphere be (r) and (R) respectively.

The difference between inner and outer curved surface area is 88 cm ^{2}

Curved surface area of the hollow sphere = 2π(R-r) h

88 = 2π(R-r) h

88 = 2π(R-r) 14

R-r = 1 ………………………………….. (i)

Volume of the hollow cylinder = $\prod \times R^2 - r^2 \times h \prod \times R^2 - r^2 \times h$

176 = $\prod \times R^2 - r^2 \times h \prod \times R^2 - r^2 \times h$

176 = $\prod \times R^2 - r^2 \times 15 \prod \times R^2 - r^2 \times 15$

From equation (i) substituting the values of equation. We get,

R+ r = 4……………………………………. (ii)

Now solving both the equations we get,

2R= 5

R= 2.5cm & r = 1.5 cm

Inner radius of the hollow cylinder = 1.5 cm

Inner diameter of the hollow cylinder = 2 * 1.5 cm = 3 cm

Outer radius of the hollow cylinder = 2.5 cm

Outer diameter of the hollow cylinder = 5 cm

Question 39: The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25.2 cm .The cost of painting 1 cm^2 of the surface is 10 paisa. Find the total cost to paint the vessel all over.

Solution

Given,

Internal diameter of the hollow hemisphere = 21 cm

Internal radius of the hollow hemisphere = 21/2 = 10.5 cm

External diameter of the hollow hemisphere = 25.2 cm

External radius of the hollow hemisphere = 25.2/2 =12.6 cm

Total area of the hollow hemisphere = $2\prod R^2+2\prod r^2+\prod(R^2-r^2)$
$2\prod R^2 + 2\prod r^2 + \prod(R^2 - r^2)$

= $2\prod 12.6^2+2\prod 10.5^2+\prod(12.6^2-10.5^2)$ $2\prod 12.6^2 + 2\prod 10.5^2 + \prod(12.6^2 - 10.5^2)$

= 997.51+692.72 +152.39 cm^2

=1843.38 cm^2

According to the question,

The cost of painting of 1 cm^2 of the surface is =10 p =Rs0.01

Then the total cost of painting = 1848.38 * 0.10

=Rs184.338

The total cost of painting all over the vessel is Rs.184.338.

Question 40: The difference between outer and inner curved surface area of a hollow right circular cylinder is 14cm long is 88 cm^2. If the volume of metal used in making cylinder is 176 cm^3. Find the outer and inner diameters of the cylinder?

Solution

Given,

Height of the hollow cylinder = 14 cm

Let the internal and external diameters of the hollow cylinder be r and R respectively.

Given that the difference between inner and outer curved surface =88 cm^3

Curved surface area of a hollow cylinder = $2\prod(R-r)\times h$ $2\prod(R - r) \times h$

88=2π(R-r)* h

88=2*3.1428*(R-r)*14

R-r =1 ……………………………(i)

Volume of the hollow cylinder = $\prod(R^2 - r^2) \times h \prod(R^2 - r^2) \times h$

Volume of the hollow cylinder= 176cm³

$\pi (R^2 - r^2) h = 176$

$R^2 - r^2 = 4$

R+ r = 4 ……………………………(ii)

Now, solving eq I and ii we get,

R-r = 1

R+ r = 4

…………………………………

2R = 5

R=2.5 cm

r = 1.5 cm

The internal and external radii of the hollow cylinder are 1.5 cm and 2.5 cm respectively.

Question 41: Prove that the surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

Solution:

Let the radius of the sphere be (r)

Curved surface area of the sphere $= 4\pi r^2$ ………………………… (i)

$S_1 = 4\pi r^2$

Let the radius of the cylinder be (r)

Let the height of the cylinder be (2r)

Curved surface area of the cylinder = 2πrh

$S_2 = = 2\pi r (2r)$

$=4\pi r^2$ …………………………………….. (ii)

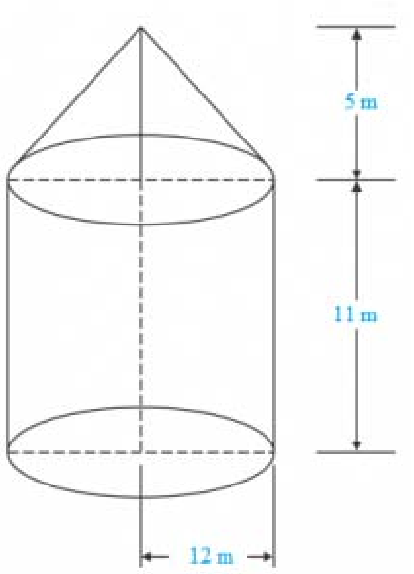
From the above equations, it is proven that surface area of the sphere is equal to the curved surface area of the circumscribed cylinder.

Exercise 16.2: Surface Areas and Volumes

Q.1 Consider a tent cylindrical in shape and surmounted by a conical top having height 16 m and radius as common for all the surfaces constituting the whole portion of the tent which is equal to 24 m. Height of the cylindrical portion of the tent is 11 m. Find the area of Canvas required for the tent.

Solution:

The diameter of the cylinder given in the question is 24m



Radius (R) = 122 $\frac{12}{2}$= 12m

The height of the Cylindrical part (H_1) given in the question is 11m

So, Height of the cone part (H_2) = 5m

Now,

Vertex of the cone above the ground= 11 + 5= 16m

Curved Surface area of the Cone (say, S_1) = πRL = 227 ×6×L $\frac{22}{7} \times 6 \times L$

Where,

L=√R2+H2 $L = \sqrt{R^2 + H_2}$ L=√12²+5² $L = \sqrt{12^2 + 5^2}$

L= 13m

So,

Curved Surface Area of Cone (S_1) = 227 ×12×13 $\frac{22}{7} \times 12 \times 13$ ……. E.1

Curved Surface Area of Cylinder (S_2) = 2πRH1

S_2= 2π(12)(11)m^2

………E.2

To find the area of Canvas required for tent

S = S_1 + S_2 = E.1 + E.2

S = 227×12×13+2×227×12×11 $\frac{22}{7} \times 12 \times 13 + 2 \times \frac{22}{7} \times 12 \times 11$

S = 490 + 829.38

S = 1319.8 m^2

S = 1320 m^2

Hence, the total Canvas required for tent (s) = 1320 m^2

Q.2 Consider a Rocket. Suppose the rocket is in the form of a Circular Cylinder Closed at the lower end with a Cone of the same radius attached to its top. The Cylindrical portion of the rocket has radius say, 2.5m and the height of that cylindrical portion of the rocket is 21m. The Conical portion of the rocket has a slant height of 8m, then calculate the total surface area of the rocket and also find the volume of the rocket.

Solution:

Given radius of the cylindrical portion of the rocket (say, R) = 2.5m

Given height of the cylindrical portion of the rocket (say, H) = 21m

Given Slant Height of the Conical surface of the rocket (say, L) = 8m

Curved Surface Area of the Cone (say S_1) = RL

S_1 = m^2 ……. E.1

Curved Surface Area of the Cone (say, S_2) = 2RH + R^2

S_2 = (2 π 2.5 21) + (π $(2.5)^2$)

S_2 = (π 105) + (π 6.25) …….. E.2

So, The total curved surface area = E.1 + E.2

S = S_1 + S_2

S = (π 20) + (π 105) + (π 6.25)

S = 62.83 + 329.86 + 19.63

S = 412.3 m^2

Hence, the total Curved Surface Area of the Conical Surface = 412.3 m^2

Volume of the conical surface of the rocket = 13 × 227 × R^2 × h $\frac{1}{3} \times \frac{22}{7} \times R^2 \times h$

V_1 = 13 × 227 × $(2.5)^2$ × h $\frac{1}{3} \times \frac{22}{7} \times (2.5)^2 \times h$ ……. E.3

Let, h be the height of the conical portion in the rocket.

Now,

$L^2 = R^2 + h^2$

$h^2 = L^2 – R^2$

h = √L^2−R^2 $\sqrt{L^2 - R^2}$

h = √8^2−2.5^2 $\sqrt{8^2 - 2.5^2}$

h = 23.685 m

Putting the value of h in E.3, we will get

Volume of the conical portion (V_1) = 13 × 227 × 2.5^2 × 23.685 $\frac{1}{3} \times \frac{22}{7} \times 2.5^2 \times 23.685$ m^2

…….. E.4

Volume of the Cylindrical Portion (V_2) = $\pi R^2 h$

V_2 = 227 × 2.5^2 × 21 $\frac{22}{7} \times 2.5^2 \times 21$

So, the total volume of the rocket = $V_1 + V_2$

V = 461.84 m^2

Hence, the total volume of the Rocket (V) is 461.84 m^2

Q.3 Take a tent structure in vision being cylindrical in shape with height 77 dm and is being surmounted by a cone at the top having height 44 dm. The diameter of the cylinder is 36 m. Find the curved surface area of the tent.

Solution:

As per the question,

Height of the tent = 77 dm
Height of a surmounted cone = 44 dm

Height of the Cylindrical Portion = Height of the tent – Height of the surmounted Cone

= 77 – 44

= 33 dm = 3.3 m

Given diameter of the cylinder (d) = 36 m

So, Radius (r) of the cylinder = 362 $\frac{36}{2}$

R = 18 m

Consider L as the Slant height of the Cone.

$L^2 = r^2 + h^2$

$L^2 = 18^2 + 3.3^2$

$L^2 = 324 + 10.89$

$L^2 = 334.89$

L = 18.3 m

Hence, Slant height of the cone (L) = 18.3 m

The Curved Surface area of the Cylinder (S_1) = 2πRh

$S_1 = 2\ \pi\ 18\ \ 4.4\ m^2$ …………… E.1

The Curved Surface area of the cone (S^2) = πRh

$S_2 = \pi\ 18\ \ 18.3\ m^2$ ……………. E.2

So, the total curved surface of the tent = $S_1 + S_2$

$S = S_1 + S_2$

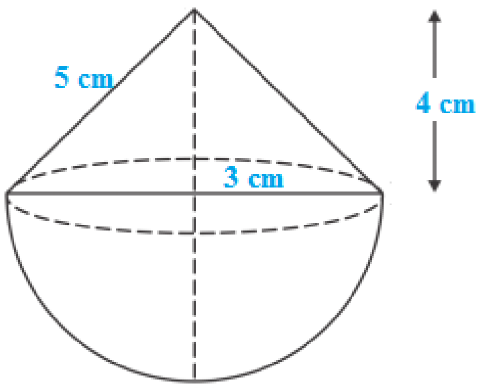
S = (2π 18 4.4) + (π 18 18.3)

S = 1532. 46 m^2

Hence, the total Curved Surface Area (S) = 1532.46 m^2

Q.4: A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy.

Solution:



Given that,

The height of the cone (h) = 4cm

Diameter of the cone (d) = 6 cm

So, radius (r) = 3 [as we know that the radius is half of the diameter]

Let, 'l' be the slant height of cone. Then,

$l = \sqrt{r^2 + h^2}$

$= \sqrt{3^2 + 4^2}$

l = 5cm

So, slant height of the cone (l) = 5 cm

Curved surface area of the cone (S_1) = $\prod rl$

$S_1 = \prod(3)(5)$

S_1 = 47.1 cm^2

Curved surface area of the hemisphere (S_2) = $2\prod r^2$

$S_2 = 2\prod(3)^2$

S_2 = 56.23 cm^2

So, the total surface area (S) = $S_1 + S_2$

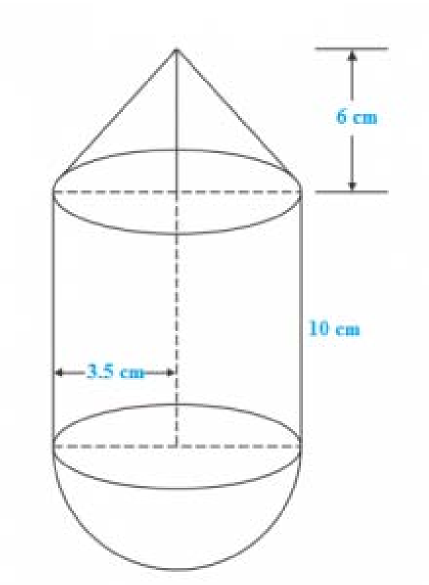
$S = 47.1 + 56.23$

$S = 103.62 \text{ cm}^2$

Therefore, the curved surface area of the toy = S = 103.62 cm^2

Q.5: A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the height of the cylindrical and conical portions are 10 cm and 6 cm, respectively. Find the total surface area of the solid. (Use ∏=227 $\prod = \frac{22}{7}$).

Solution:



Given that,

Radius of the common base (r) = 3.5 cm

Height of the cylindrical part (h) = 10 cm

Height of the conical part (H) = 6 cm

Let, 'l' be the slant height of the cone, then

l=√r2+H2 $l = \sqrt{r^2 + H^2}$

= √3.52+62 $\sqrt{3.5^2 + 6^2}$

l = 48.25 cm

Curved surface area of the cone (S_1) = ∏rl $\prod rl$

$S_1 = \prod(3.5)(48.25)\prod(3.5)(48.25)$

$S_1 = 76.408\ cm^2$

Curved surface area of the hemisphere (S_2) = $2\prod rh 2\prod rh$

$S_2 = 2\prod(3.5)(10)2\prod(3.5)(10)$

$S_2 = 220\ cm^2$

So, the total surface area (S) = $S_1 + S_2$

S = 76.408 + 220

$S = 373.408\ cm^2$

Q.6: A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical parts are 5cm and 13 cm, respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the tor if the total height of the toy is 30 cm.

Solution:

As per the parameters given in the question, we have

Height of the Cylindrical portion = 13 cm

Radius of the Cylindrical portion = 5 cm

Height of the whole solid = 30 cm

Then,

The curved surface area of the Cylinder (say S_1) = $2\pi rh$

$S_1 = 2\pi(5)(13)$

$S_1 = 408.2\ cm^2$

The curved surface area of the cone (say S_2) = πrL

$S_2 = \pi\ (6)\ L$

For conical part, we have

h = 30 – 13 – 5 = 12 cm

We know that,

$L = \sqrt{r^2+h_{22}}\sqrt{r^2 + h_2^2}$

$L = \sqrt{5^2+12^2}\sqrt{5^2 + 12^2}$

$L = \sqrt{25+144}\sqrt{25 + 144}$

$L = \sqrt{169}\sqrt{169}$

L = 13 cm

So, The curved surface area of the cone (say S_2) = πrL

S_2 = π (5) (13) cm^2

S_2 = 204.1 cm^2

The curved surface area of the hemisphere (say S_3) = 2 πr^2

S_3 = 2 π $(5)^2$

S_3 = 157 cm^2

The total curved surface area (say S) = $S_1 + S_2 + S_3$

S = (408.2 + 204.1 + 157)

S = 769.3 cm^2

Therefore, the surface area of the toy (S) = 769.3 cm^2

Q- 7. Consider a cylindrical tub having radius as 5 cm and its length 9.8 cm. It is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in tub. If the radius of the hemisphere is 3.5 cm and the height of the cone outside the hemisphere is 5 cm. Find the volume of water left in the tub.

Solution:

As per the parameters given in the question, we have

The radius of the Cylindrical tub (r) = 5 cm

Height of the Cylindrical tub (say H) = 9.8 cm

Height of the cone outside the hemisphere (say h) = 5 cm

Radius of the hemisphere = 5 cm

Now,

The volume of the Cylindrical tub (say V_1) = $\pi r^2 H$

$V_1 = \pi(5)^2 9.8$

$V_1 = 770\ cm^3$

The volume of the Hemisphere (say V_2) = $23 \times \prod \times r^3 \frac{2}{3} \times \prod \times r^3$

$V_2 = 23 \times 227 \times 3.5^3 \frac{2}{3} \times \frac{22}{7} \times 3.5^3$

$V_2 = 89.79\ cm^3$

The volume of the Hemisphere (say V_3) = $23 \times \prod \times r^2 h \frac{2}{3} \times \prod \times r^2 h$

$V_3 = 23 \times 227 \times 3.5^2 \times 5 \frac{2}{3} \times \frac{22}{7} \times 3.5^2 \times 5$

$V_3 = 64.14\ cm^3$

Therefore, The total volume = Volume of the cone + Volume of the hemisphere = $V_2 + V_3$

$V = 89.79 + 64.14\ cm^3 = 154\ cm^3$

Hence, the total volume of the solid = $154\ cm^3$

To find the volume of the water left in the tube, we have to subtract the volume of the hemisphere and the cone from the volume of the cylinder.

Hence, the volume of water left in the tube = $V = V_1 – V_2$

V = 770 – 154

V = 616 cm3

Therefore, the volume of water left in the tube is $616\ cm^3$.

Q-8. A circus tent has a cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 cm. The height of the cylindrical and conical portions is 4.2 cm and 2.1 cm. Find the volume of that circus tent.

Solution:

As per the parameters given in the question, we have

Radius of the cylindrical portion (say R) = 20 m

Height of the cylindrical portion (say h_1) = 4.2 m

Height of the conical portion (say h_2) = 2.1 m

Now,

Volume of the Cylindrical portion (say V_1) = $\pi r^2 h_1$

$V_1 = \pi$

$(20)^2$ 4.2

V_1 = 5280 m^3

Volume of the conical part (say V_2) = 13 × 227 × r^2 × h_2 $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h_2$

V_2 = 13 × 227 × 20^2 × 2.1 $\frac{1}{3} \times \frac{22}{7} \times 20^2 \times 2.1$

V_2 = 880 m^3

Therefore, the total volume of the tent (say V) = volume of the conical portion + volume of the Cylindrical portion

$V = V_1 + V_2$

V = 6160 m^3

Volume of the tent = V = 6160 m^3

Q-9. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with the conical ends, each of axis 9 cm. Determine the capacity of the tank.

Solution:

As per the parameters given in the question, we have

Base diameter of the Cylinder = 21 cm

Radius (say r) = diameter2 $\frac{diameter}{2}$ = 252 $\frac{25}{2}$ = 11.5 cm

Height of the Cylindrical portion of the tank (say h_1) = 18 cm

Height of the Conical portion of the tank (say h_2) = 9 cm

Now,

The volume of the Cylindrical portion (say V_1) = $\pi r^2 h_1$

$V_1 = \pi (11.5)^2 18$

$V_1 = 7474.77\ cm^3$

The volume of the Conical portion (say V_2) = 13 × 227 × r^2×h_2 $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h_2$

V_2 = 13 × 227 × 11.5^2×9 $\frac{1}{3} \times \frac{22}{7} \times 11.5^2 \times 9$

$V_2 = 1245.795\ cm^3$

Therefore, the total volume of the tank (say V) = volume of the conical portion + volume of the Cylindrical portion

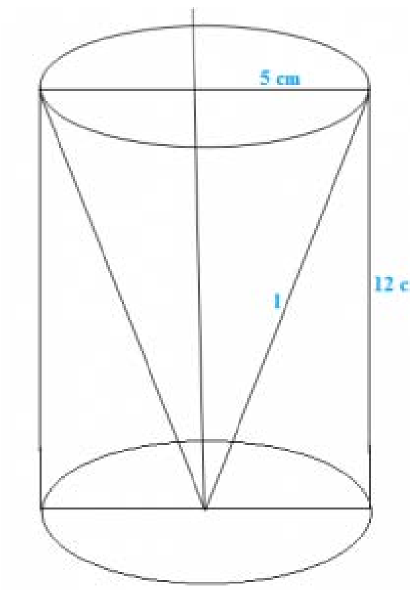
$V = V_1 + V_2$

$V = 8316\ cm^3$

So, the capacity of the tank = V = 8316 cm^3

Q-10: A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and base radius of the cone are also the same. Find the whole surface and volume of the remaining Cylinder.

Solution:



As per the parameters given in the question, we have

Height of the circular Cylinder (say h_1) = 12 cm

Base radius of the circular Cylinder (say r) = 5 cm

Height of the conical hole = Height of the circular cylinder, i.e., $h_1 = h_2 = 12$ cm

Base radius of the conical hole = Base radius of the circular Cylinder = 5 cm

Let us consider, L as the slant height of the conical hole.

$L = \sqrt{r^2 + h^2}$

$L = \sqrt{5^2 + 12^2}$

$L = \sqrt{25 + 144}$

L = 13 cm

Now,

The total surface area of the reamining portion in the circular cylinder (say V_1) = $\pi r^2 + 2\pi rh + \pi rL$

$V_1 = \pi(5)^2 + 2\pi(5)(12) + \pi(5)(13)$

$V_1 = 210\pi\ cm^2$

Volume of the remaining portion of the circular cylinder = volume pof the cylinder – volume of the conical hole

$V = \pi r^2 h - \frac{1}{3} \times \frac{22}{7} \times r^2 \times h$

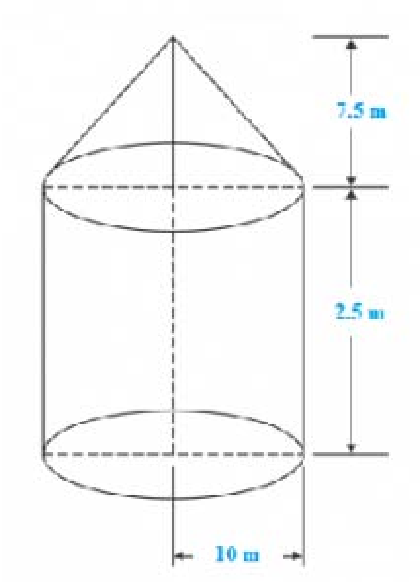
$V = \pi(5)^2(12) - \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 12$

$V = 200\pi\ cm^2$

Therefore, the volume of the remaining portion of the cylindrical part = $V = 200\pi\ cm^2$

Q-11. A tent is in the form of a cylinder of diameter 20m and height 2.5m surmounted by a cone of equal base and height 7.5m. Find the capacity of tent and the cost of canvas as well at a price of Rs.100 per square meter.

Solution:



As per the parameters given in the question, we have

Diameter of the cylinder = 20 m

Radius of the cylinder = 10 m

Height of the cylinder (say h_1) = 2.5 m

Radius of the cone = Radius of the cylinder (say r) = 15 m

Height of the Cone (say h_2) =7.5 m

Let us consider L as the slant height of the Cone, then

$L = \sqrt{r^2 + h_2^2}$

$L = \sqrt{15^2 + 7.5^2}$

L = 12.5 m

Volume of the cylinder = $\pi r^2 h_1 = V_1$

$$V_1 = \pi(10)^2\, 2.5$$

$V_1 = 250\pi\ m^3$

Volume of the Cone = $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h_2 = V_2$

$V_2 = \frac{1}{3} \times \frac{22}{7} \times 10^2 \times 7.5$

$V_2 = 250\ \pi\ m^3$

Therefore, The total capacity of the tent = volume of the cylinder + volume of the cone $= V_1 + V_2$

$V = 250\ \pi + 250\ \pi$

$V = 500\ \pi\ m^3$

Hence, the total capacity of the tent = $V = 4478.5714\ m^3$

The total area of the canvas required for the tent is $S = 2\ \pi r h_1 + \pi r L$

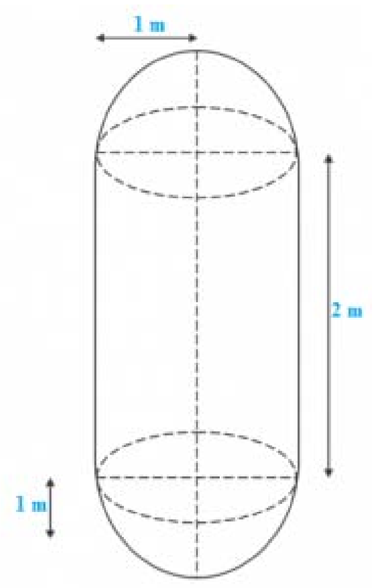
$S = 2(\pi)(10)(2.5) + \pi(10)(12.5)$

$S = 550\ m^2$

Therefore, the total cost of the canvas is (100) (550) = Rs. 55000

Q- 12. Consider a boiler which is in the form of a cylinder having length 2 m and there's a hemispherical ends each of having a diameter of 2m. Find the volume of the boiler.

Solution:



As per the parameters given in the question, we have

Diameter of the hemisphere = 2 m

Radius of the hemisphere (say r) = 1 m

Height of the cylinder (say h_1) = 2 m

The volume of the Cylinder $= \pi r^2\ h_1 = V_1$

$V_1 = \pi\ (1)^2\ 2$

V_1 = 227×2=447 $\frac{22}{7} \times 2 = \frac{44}{7} m^3$

Since, at each of the ends of the cylinder, hemispheres are attached.

So,

The volume of two hemispheres = 2×23×227×r³ $2 \times \frac{2}{3} \times \frac{22}{7} \times r^3$ = V_2

V_2 = 2×23×227×1³ $2 \times \frac{2}{3} \times \frac{22}{7} \times 1^3$

V_2 = 227×43=8821 $\frac{22}{7} \times \frac{4}{3} = \frac{88}{21} m^3$

Therefore, the volume of the boiler = volume of the cylindrical portion + volume of the two hemispheres = V

$V = V_1 + V_2$

V = 447 $\frac{44}{7}$ + 8821 $\frac{88}{21}$

V = 22021 $\frac{220}{21}$ m^3

So, The volume of the boiler = V = 22021 $\frac{220}{21}$ m^3

Q-13. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is 143 $\frac{14}{3}$ and the diameter of the hemisphere is 3.5m. Calculate the volume and the internal surface area of the solid.

Solution:

As per the parameters given in the question, we have

Diameter of the hemisphere = 3.5 m

Radius of the hemisphere (say r) = 1.75 m

Height of the cylinder (say h) = 143 $\frac{14}{3}$ m

The volume of the Cylinder = $\pi r^2 h_1 = V_1$

$V_1 = \pi (1.75)^2$ 143 $\frac{14}{3}$ m^3

The volume of two hemispheres = 2×23×227×r³ $2 \times \frac{2}{3} \times \frac{22}{7} \times r^3$ = V_2

V_2 = 2× 23 × 227 ×$1.75^3$$2 \times \frac{2}{3} \times \frac{22}{7} \times 1.75^3$ m^3

Therefore, The total volume of the vessel = volume of the cylinder + volume of the two hemispheres = V

$V = V_1 + V_2$

V = 56 m^3

Therefore, Volume of the vessel = V = 56 m^3

Internal surface area of solid (S) = 2 πr h_1 + 2 πr^2

S = Surface area of the cylinder + Surface area of the hemisphere

S = 2 π (1.75) (143 $\frac{14}{3}$) + 2 π(1.75 $)^2$

S = 70.51 m^3

Hence, the internal surface area of the solid = S = 70.51 m^3

Q-14. Consider a solid which is composed of a cylinder with hemispherical ends. If the complete length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm. Find the cost of polishing its surface at the rate of Rs. 10 per dm^2.

Solution:

As per the parameters given in the question, we have

Radius of the hemispherical end (say r) = 7 cm

Height of the solid = (h + 2r) = 104 cm

The curved surface area of the cylinder (say S) = 2 πr h

S = 2 π (7) h ……. E.1

⇒h+2r=104$\Rightarrow h + 2r = 104$ ⇒h=104−(2×7)$\Rightarrow h = 104 - (2 \times 7)$

h = 90 cm

Put the value of h in E.1, we will get

S = 2 π (7) (90)

S = 3948.40 cm^2

So, the curved surface area of the cylinder = S = 3948.40 cm^2

Curved surface area of the two hemisphere (say SA) = 2 (2 πr^2)

SA = 22π $(7)^2$

SA = 615.75 cm^2

Therefore, the total curved surface area of the solid = Curved surface area of the cylinder + Curved surface area of the two hemisphere = TSA

TSA = S + SA

TSA = 3948.40 + 615.75

TSA = 4571.8cm^2 = 45.718 dm^2

The cost of polishing the 1 dm^2 surface of the solid is Rs. 15

So, the cost of polishing the 45.718 dm^2 surface of the solid = 10 45.718 = Rs. 457.18

Hence,

The cost of polishing the whole surface of the solid is Rs. 457.18.

Q-15. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16cm and height of 42 cm. The total space between the two vessels is filled with Cork dust for heat insulation purposes. Find how many cubic cms of the Cork dust will be required?

Solution:

As per the parameters given in the question, we have

Depth of the cylindrical vessel = Height of the cylindrical vessel = h = 42 cm

Inner diameter of the cylindrical vessel = 14 cm

Inner radius of the cylindrical vessel = r_1 = 142 $\frac{14}{2}$ = 7 cm (as we know that the radius is half of the diameter)

Outer diameter of the cylindrical vessel = 16 cm

Outer radius of the cylindrical vessel = r_2 = 162 $\frac{16}{2}$ = 8 cm (as we know that the radius is half of the diameter)

Now,

The volume of the cylindrical vessel = ∏×(r_{22}−r_{21})×h$\prod \times \left(r_2^2 - r_1^2\right) \times h$ = V

$V = \prod \times (8^2 – 7^2) \times 42 \prod \times \left(8^2 – 7^2\right) \times 42$

V = 1980 cm^3

Therefore, Volume of the vessel = V = 1980 cm^3 = Amount of cork dust required.

Q-16. A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54cm and the thickness of the iron sheet used in making roller is 9 cm. Find the mass of the road roller if 1 cm^3 of the iron has 7.8 gm mass.

Solution:

As per the parameters given in the question, we have]

Height of the cylindrical road roller = h = 1 m = 100 cm

Internal Diameter of the cylindrical road roller = 54 cm

Internal radius of the cylindrical road roller = 27 cm = r (as we know that the radius is half of the diameter)

Given the thickness of the road roller (T) = 9 cm

Let us assume that the outer radii of the cylindrical road roller be R.

T = R – r

9 = R – 27

R = 27 + 9

R = 36 cm

Now,

The volume of the iron sheet = $\prod \times (R^2 − r^2) \times h \prod \times \left(R^2 - r^2\right) \times h$ = V

$V = \prod \times (36^2 − 27^2) \times 100 \prod \times \left(36^2 - 27^2\right) \times 100$

V = 1780.38 cm^3

So, the volume of the iron sheet = V = 1780.38 cm^3

Mass of 1 cm^3 of the iron sheet = 7.8 gm

So, the mass of 1780.38 cm^3 of the iron sheet = 1388696.4gm = 1388.7 kg

Hence, the mass of the road roller (m) = 1388.7 kg

Q-17. A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13cm. Find the inner surface area of the vessel.

Solution:

As per the parameters given in the question itself, we have

Diameter of the hemisphere = 14 cm

Radius of the hemisphere = 7 cm (as we know that the radius is half of the diameter)

Total height of the vessel = 13 cm = h + r

Now,

Inner surface area of the vessel = 2r (h + r) = SA

SA = 2 (13) (7)

SA = 182 cm^2 = 572 cm^2

Therefore, the inner surface area of the vessel = SA = 572 cm^2

Q-18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:

As per the parameters given in the question, we have

Radius of the conical portion of the toy = 3.5 cm = r

Total height of the toy = 15.5 cm = h

Length of the cone = L = 15.5 – 3.5 = 12 cm

Now,

The curved surface area of the cone = πrL = SA

SA = π (3.5) (12)

SA = 131.94 cm^2

The curved surface area of the hemisphere = $2\pi r^2$ = S

S = $2\pi (3.5)^2$

S = 76.96 cm^2

Therefore, The total surface area of the toy = Curved surface area of the cone + curved surface area of the hemisphere = TSA

TSA = 131.94 + 76.96

TSA = 208.90 cm^2

Hence, the total surface area of the children's toy = TSA = 209 cm^2

Q-19. The difference between outside and inside surface areas of the cylindrical metallic pipe 14 cm long is 44 dm^2. If pipe is made of 99 cm^2 of metal. Find outer and inner radii of the pipe.

Solution:

Let, inner radius of the pipe be r_1.

Radius of outer cylinder be r_2.

Length of the cylinder (h) = 14 cm

Difference between the outer and the inner surface area is 44 dm^2.

So,

Surface area = 2∏h(r_{22}–r_{21})$2\prod h\left(r_2^2 - r_1^2\right)$ = 44

2∏14(r_{22}–r_{21})$2\prod 14\left(r_2^2 - r_1^2\right)$ = 44

(r_2–r_1)$(r_2 - r_1)$ = 12 $\frac{1}{2}$ ………… E.1

So,

Volume of the metal used is 99 cm^2,

∏h(r_{22}–r_{21})$\prod h\left(r_2^2 - r_1^2\right)$ = 99

∏h(r_2–r_1)(r_2+r_1)$\prod h\left(r_2 - r_1\right)\left(r_2 + r_1\right)$ = 99

227 14(12)(r_2+r_1)=99$\frac{22}{7} 14\left(\frac{1}{2}\right)\left(r_2 + r_1\right) = 99$

Therefore,

(r_2+r_1)= 92 $(r_2 + r_1) = \frac{9}{2}$ …………..E.2

Solve E.1 and E.2 to get,

$r_2 = \frac{5}{2}$ cm

$r_1 = 2$ cm

Q-20. A right circular cylinder having diameter 12 cm and height 15 cm is full ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Solution:

Given,

Radius of cylinder (r_1) = 6 cm

Radius of hemisphere (r_2) = 3 cm

Height of cylinder (h) = 15 cm

Height of the cones (l) = 12 cm

Volume of cylinder = $\prod r_1^2 h$

= $\prod 6^2 15$ ……… E.1

Volume of each cone = Volume of cone + Volume of hemisphere

= $\frac{1}{3} \prod r_2^2 l + \frac{2}{3} \prod r_2^3$

= $\frac{1}{3} \prod 6^2 12 + \frac{2}{3} \prod 3^3$ ……. E.2

Let, number of cones be ‘n’

n(Volume of each cone) = Volume of cylinder

$n\left(\frac{1}{3} \prod 3^2 12 + \frac{2}{3} \prod 3^3\right) = \prod (6)^2 15$

n = $\frac{50}{5} = 10$

So, the number of cones being filled with the cylinder = n = 10

Q-21. Consider a solid iron pole having cylindrical portion 110 cm high and the base diameter of 12 cm is surmounted by a cone of 9 cm height. Find the mass of the pole.

Assume that the mass of 1 cm^3 of iron pole is 8 gm.

Solution:

As per the data given in the question, we have

Base diameter of the cylinder = 12 cm

Radius of the cylinder = 6 cm = r (as we know that the radius os half of the diameter)

Height of the cylinder = 110 cm = h

Length of the cone = 9 cm = L

Now,

The volume of the cylinder = ∏×r²×h$\prod \times r^2 \times h$ = V_1

V_1 = ∏×6²×110$\prod \times 6^2 \times 110$ cm^3 ……. E-1

The volume of the cone = V_2 = 13×∏r²L$\frac{1}{3} \times \prod r^2 L$

V_2 = 13×∏6²12$\frac{1}{3} \times \prod 6^2 12$

V_2 = 108π cm^3

Volume of the pole = volume of the cylinder + volume of the cone = V_1 + V_2 = V

V = 108π + π $(6)^2$ 110

V = 12785.14 cm^3

Given mass of 1 cm^3 of the iron pole = 8 gm

Then, mass of 12785.14 cm^3 of the iron pole = 8 12785.14 = 102281.12 gm = 102.2 kg

Therefore, the mass of the iron pole = 102.2 kg

Q-22. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the toy, find how much more space it will cover?

Solution:

Given that,

Radius of the cone, cylinder and hemisphere (r) = 2 cm

Height of the cone (l) = 2 cm

Height of the cylinder (h) = 4 cm

Volume of the cylinder = ∏×r^2×h$\prod \times r^2 \times h$ = V_1

V_1 = ∏×2^2×4$\prod \times 2^2 \times 4$ cm^3 ……… E-1

Volume of the cone = V_2 = 13×∏r^2L$\frac{1}{3} \times \prod r^2 L$

V_2 = 13×∏$2^2$2$\frac{1}{3} \times \prod 2^2 2$

V_2 = 13×∏4×2$\frac{1}{3} \times \prod 4 \times 2$ cm^3 ………. E-2

Volume of the hemisphere V_3 = 23∏$r_{32}$$\frac{2}{3} \prod r_2^3$

V_3 = 23∏$2^3$$\frac{2}{3} \prod 2^3$ cm^3

V_3 = 23∏×8$\frac{2}{3} \prod \times 8$ cm^3 ………. E-3

So, remaining volume of the cylinder when the toy is inserted to it = $V_1 – (V_2 + V_3)$

V = 16π – 8 π = 8 π cm^3

Hence, remaining volume of the cylinder when toy is inserted into it = V = 8 π cm^3

Q-23. Consider a solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm, is placed upright in the right circular cylinder full of water such that it touches bottoms. Find the volume of the water left in the cylinder, if radius of the cylinder is 60 cm and its height is 180 cm.

Solution:

As per the data given in the question, we have

Radius of the circular cone = r = 60 cm

Height of the circular cone = L = 120 cm

Radius of the hemisphere = r = 60 cm

Radius of the cylinder = R = 60 cm

Height of the cylinder = H = 180 cm

Now,

Volume of the circular cone = V_1 = 13×∏r^2L$\frac{1}{3} \times \prod r^2 L$

V_1 = 13×∏60^2×120$\frac{1}{3} \times \prod 60^2 \times 120$

V_1 = 452571.429 cm^3

Volume of the hemisphere = V_2 = 23×∏$r^3$$\frac{2}{3} \times \prod r^3$

V_2 = 23×∏$60^3$$\frac{2}{3} \times \prod 60^3$

V_2 = 452571.429 cm^3

Volume of the cylinder = ∏×R^2×H$\prod \times R^2 \times H$ = V_3

V_3 = ∏×60^2×180$\prod \times 60^2 \times 180$

V_3 = 2036571.43 cm^3

Volume of water left in the cylinder = Volume of the cylinder – (volume of the circular cone + volume of the hemisphere) = V

V = V_3 – (V_1 + V_2)

V = 2036571.43 – (452571.429 + 452571.429)

V = 2036571.43 – 905142.858

V = 1131428.57 cm^3

V = 1.1314 m^3

Therefore, the volume of the water left in the cylinder = V = 1.1314 m^3

Q-24. Consider a cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 8 cm and height 6 cm is completely immersed in water. Find the value of water when:

(i) Displaced out of the cylinder (ii) Left in the cylinder

Solution:

As per the parameters given in the question, we have

Internal diameter of the cylindrical vessel = 10 cm

Radius of the cylindrical vessel = r = 5 cm (as we know that the radius is half of the diameter)

Height of the cylindrical vessel = h = 10.5 cm

Base diameter of the solid cone = 7 cm

Radius of the solid cone = R = 3.5 cm (as we know that the radius is half of the diameter)

Height of the cone = L = 6 cm

(i) Volume of water displaced out from the cylinder = Volume of the cone = V_1

V_1 = 13×∏R^2L$\frac{1}{3} \times \prod R^2 L$

V_1 = 13×∏3.5^2×6$\frac{1}{3} \times \prod 3.5^2 \times 6$

V_1 = 77 cm^3

Therefore, the volume of the water displaced after immersion of the solid cone in the cylinder = V_1 =77 cm^3

(ii) Volume of the cylindrical vessel = ∏×r^2×h$\prod \times r^2 \times h$ = V_2

V_2 = ∏×5^2×10.5$\prod \times 5^2 \times 10.5$

V_2 = 824.6 cm^3 = 825 cm^3

Volume of the water left in the cylinder = Volume of the cylindrical vessel – Volume of the solid cone

V = V_2 – V_1

V = 825- 77

V = 748 cm^3

Therefore, the volume of the water left in cylinder = V = 748 cm^3

Q-25. A hemispherical depression is cut from one face of a cubical wooden block of the edge 21 cm such that the diameter of the hemispherical surface is equal to the edge of

the cubical surface. Determine the volume and the total surface area of the remaining block.

Solution:

As per the data given in the question itself, we have

Edge of the cubical wooden block = e = 21 cm

Diameter of the hemisphere = Edge of the cubical wooden block = 21 cm

Radius of the hemisphere = 10.5 cm = r (as we know that the radius is half of the diameter)

Now,

Volume of the remaining block = Volume of the cubical block – Volume of the hemisphere

V = $e^3 – (23 \prod r^3) e^3 - \left(\frac{2}{3} \prod r^3\right)$

V = $21^3 – (23 \prod 10.5^3) 21^3 - \left(\frac{2}{3} \prod 10.5^3\right)$

V = 6835.5 cm^3

Surface area of the block = $6(e^2) 6\left(e^2\right)$ = SA

SA = $6(21^2) 6\left(21^2\right)$ ………………… E-1

Curved surface area of the hemisphere = CSA = $2 \prod r^2 2 \prod r^2$

CSA = $2 \prod 10.5^2 2 \prod 10.5^2$ …………………… E-2

Base area of the hemisphere = BA = $\prod r^2 \prod r^2$

BA = $\prod 10.5^2 \prod 10.5^2$ …………………… E-3

So, Remaining surface area of the box = SA – (CSA + BA)

= $6(21^2) 6\left(21^2\right) – (2 \prod 10.5^2 2 \prod 10.5^2 + \prod 10.5^2 \prod 10.5^2)$

= 2992.5 cm^2

Therefore, the remaining surface area of the block = 2992.5 cm^2

Volume of the remaining block = V = 6835.5 cm^3

Q-26. A boy is playing with a toy which is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere. Calculate the height of the cone and surface area of the toy.

Solution:

As per the parameter given in the question itself, we have

Radius of the cone = 21 cm = R

Radius of the hemisphere = Radius of the cone = 21 cm

Volume of the cone = 2/3 of the hemisphere

We know that,

The volume of the cone = V_1 = $13\times\prod R^2L\frac{1}{3}\times\prod R^2L$

V_1 = $13\times\prod 21^2L\frac{1}{3}\times\prod 21^2L$

Also, we know that,

The volume of the hemisphere = V_2 = $23\times\prod R^3\frac{2}{3}\times\prod R^3$

V_2 = $23\times\prod 21^3\frac{2}{3}\times\prod 21^3$ cm^3

Now, as per the condition

V_1 = $23V_2\frac{2}{3}V_2$

V_1 = $23\times169714.286\frac{2}{3}\times169714.286$

$13\times\prod 21^2L\frac{1}{3}\times\prod 21^2L$ = $23\times\prod 21^3\frac{2}{3}\times\prod 21^3$

L = 28 cm

Curved surface area of the Cone = CSA_1 = $\prod RL\prod RL$

CSA_1 = $\prod\times21\times28\prod\times21\times28$ cm^2

Curved Surface area of the hemisphere = CSA_2 = $2\prod R^2 2\prod R^2$

CSA_2 = $2\prod(21^2)2\prod\left(21^2\right)$ cm^2

Now, the total surface area = S = CSA_1 + CSA_2

S = $\prod \times 21 \times 28 \prod \times 21 \times 28$ + $2\prod(21^2) 2\prod(21^2)$

S = 5082 cm^2

Therefore, the curved surface area of the toy = S = 5082 cm^2

Q-27. Consider a solid which is in the form of a cone surmounted on hemisphere. The radius of each of them is being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Solution:

As per the data given in the question, we have

Radius of the hemisphere = 3.5 cm = R

Radius of the cone = Radius of the hemisphere = 3.5 cm = R

Total height of the solid = 9.5 cm = H

Then,

Length of the cone = Total height – Radius of the cone

L = 9.5 – 3.5 = 6 cm

Now,

The volume of the cone = V_1 = $13 \times \prod R^2 L \frac{1}{3} \times \prod R^2 L$

V_1 = $13 \times \prod 3.5^2 6 \frac{1}{3} \times \prod 3.5^2 6$ cm^3 …….. E-1

The volume of the hemisphere = V_2 = $23 \times \prod R^3 \frac{2}{3} \times \prod R^3$

V_2 = $23 \times \prod 5^3 \frac{2}{3} \times \prod 5^3$ cm^3 ……… E-2

Total volume of the solid = Volume of the cone + Volume of the hemisphere = V

V = $V_1 + V_2$

V = $13 \times \prod 3.5^2 6 \frac{1}{3} \times \prod 3.5^2 6$ + $23 \times \prod 5^3 \frac{2}{3} \times \prod 5^3$

V = 166.75 cm^3

So, the volume of the solid = V = 166.75 cm^3

Q-28. A wooden toy is made by scooping out a hemisphere of same radius from each end of the solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of the wood in the toy.

Solution:

Given that,

Radius of the cylinder = Radius of the hemisphere = 3.5 cm = r

Height of the hemisphere = 10 cm = h

Volume of the cylinder = $\prod \times r^2 \times h \prod \times r^2 \times h$ = V_1

V_1 = $\prod \times 3.5^2 \times 10 \prod \times 3.5^2 \times 10$ ………… E-1

Volume of the hemisphere = V_2 = $23 \times \prod r^3 \frac{2}{3} \times \prod r^3$

V_2 = $23 \times \prod 3.5^3 \frac{2}{3} \times \prod 3.5^3$ cm^3 ……… E-2

So,

Volume of the wood in the toy = Volume of the cylinder – 2(Volume of the hemisphere)

$V = V_1 - V_2$

V = 205.33 cm^3

Q-29. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left.

Solution:

Given that,

Diameter of the wooden solid = 7 cm

Radius of the wooden solid = 3.5 cm

Volume of the cube = $e^3 e^3$

V_1 = $3.5^3 3.5^3$ …………. E-1.

Volume of sphere = $43 \times \prod \times r^3 \frac{4}{3} \times \prod \times r^3$ = V_2

V_2 = 43×∏×3.5^3 $\frac{4}{3} \times \prod \times 3.5^3$ E-2

Volume of the wood left = Volume of the cube – Volume of sphere

V = 3.5^3−43×∏×3.5^3 $3.5^3 - \frac{4}{3} \times \prod \times 3.5^3$

V = 163.33 cm^3

Q-30. From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid.

Solution:

Given that,

Height of the cylinder = 2.8 cm = Height of the cone

Diameter of the cylinder = 4.2 cm

Radius of the cylinder = 2.1 cm = Radius of the cone

CSA of the cylindrical part = CSA_1 = 2πRH

CSA_1 = 2π(2.8)(2.1) cm^2

Curved surface area of the Cone = CSA_2 = ∏RL $\prod RL$

CSA_2 = ∏×2.1×2.8 $\prod \times 2.1 \times 2.8$ cm^2

Area of the cylindrical base = πr^2 = $\pi(2.1)^2$

Total surface area of the remaining solid = CSA of the cylindrical part + Curved surface area of the Cone + Area of the cylindrical base

TSA = 2π(2.8)(2.1) + ∏×2.1×2.8 $\prod \times 2.1 \times 2.8$ + $\pi(2.1)^2$

TSA = 36.96 + 23.1 + 13.86

TSA = 73.92 cm^2

Q-31. The largest cone is carved out from one face of the solid cube of side 21 cm. Find the volume of the remaining solid.

Solution:

Given that,

The radius of the largest possible cone is carved out of a solid cube is equal to the half of the side of the cube.

Diameter of the cone = 21 cm

Radius of the cone = 10.5 cm

The height of the cone is equal to the side of the cone.

Volume of the cube = $e^3 e^3$

V_1 = $10.5^3 10.5^3$ .... E-1.

Volume of the cone = V_2 = 13×∏r^{2}L $\frac{1}{3} \times \prod r^2 L$

V_2 = 13×∏10.5^{2}21 $\frac{1}{3} \times \prod 10.5^2 21$ cm^3 .... E-2

Volume of the remaining solid = Volume of cube – Volume of cone

V = $10.5^3 10.5^3$ – 13×∏10.5^{2}21 $\frac{1}{3} \times \prod 10.5^2 21$

V = 6835.5 cm^3

Q-32. A solid wooden toy is in the form of a hemisphere surmounted by a cone of the same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the making of toy is 166 56 cm^3 $166\frac{5}{6} cm^3$. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs. 10 per square cm.

Solution:

Given that,

Radius of the hemisphere = 3.5 cm

Volume of the solid wooden toy = 166 56 cm^3 $166\frac{5}{6} cm^3$

As, Volume of the solid wooden toy = Volume of the cone + Volume of the hemisphere = 166 56 cm^3 $166\frac{5}{6} cm^3$

13×∏r^{2}L $\frac{1}{3} \times \prod r^2 L$ + 23×∏r^3 $\frac{2}{3} \times \prod r^3$ = 10016

13×∏3.5^{2}L $\frac{1}{3} \times \prod 3.5^2 L$ + 23×∏3.5^3 $\frac{2}{3} \times \prod 3.5^3$ = 10016

L + 7 = 13

L = 6 cm

Height of the solid wooden toy = Height of the cone + Radius of the hemisphere

= 6 + 3.5

= 9.5 cm

Now, curved surface area of the hemisphere = 2∏R^2 $2\prod R^2$

CSA_2 = 2∏(3.5^2) $2\prod\left(3.5^2\right)$ = 77 cm^2

Cost of painting the hemispherical part of the toy = 10 77 = Rs. 770

Q-33. How many spherical bullets can be made out of a solid cube of lead whose edge measures 55 cm and each of the bullet being 4 cm in diameter?

Solution:

Let, the total number of bullets be a.

Diameter of the bullet = 4 cm

Radius of the spherical bullet = 2 cm (as we know that the radius is half of the diameter)

Now,

Volume of a spherical bullet = 43 ×∏×r^3 $\frac{4}{3}\times\prod\times r^3$ = V

V = 43 ×∏×2^3 $\frac{4}{3}\times\prod\times 2^3$

V = 43 × 227 ×2^3 $\frac{4}{3}\times\frac{22}{7}\times 2^3$

V = 33.5238 cm^3

Volume of ‘a’ number of the spherical bullets = V a

V_1 = (33.5238 a) cm^{23}

Volume of the solid cube = $(55)^3$ = 166375 cm^3

Volume of ‘a’ number of the spherical bullets = Volume of the solid cube

33.5238 a = 166375

A = 4962.892

Hence, total number of the spherical bullets = 4963

Q-34. Consider a children's toy which is in the form of a cone at the top having a radius of 5 cm mounted on a hemisphere which is the base of the toy having the same radius. The total height of the toy is 20 cm. Find the total surface area of the toy.

Solution:

As per the parameters given in the question, we have

Radius of the conical portion of the toy = 5 cm = r

Total height of the toy = 20 cm = h

Length of the cone = L = 20 – 5 = 15 cm

Now,

The curved surface area of the cone = πrL = SA

SA = π (5) (15)

SA = 235.7142 cm^2

The curved surface area of the hemisphere = $2\pi r^2$ = S

S = $2\pi (5)^2$

S = 157.1428 cm^2

Therefore, The total surface area of the toy = Curved surface area of the cone + curved surface area of the hemisphere = TSA

TSA = 235.7142 + 157.1428

TSA = 392.857 cm^2

Hence, the total surface area of the children's toy = TSA = 392.857 cm^2

Q-35. A boy is playing with a toy conical in shape and is surmounted with hemispherical surface. Consider a cylinder in with the toy is inserted. The diameter of cone is the same as that of the radius of cylinder and hemispherical portion of the toy which is 8 cm. The height of the cylinder is 6 cm and the height of the conical portion of the toy is 3 cm. Assume a condition in which the boy's toy is inserted in the cylinder, then find the volume of the cylinder left vacant after insertion of the toy.

Solution:

As per the parameter given in the question, we have

Diameter of the cone = Diameter of the Cylinder = Diameter of the Hemisphere = 8 cm

Radius of the cone = Radius of the cylinder = Radius of the Hemisphere = 4 cm = r (as we know that the radius is half of the diameter)

Height of the conical portion = 3 cm = L

Height of the cylinder = 6 cm = H

Now,

Volume of the cylinder = $\prod \times r^2 \times H$ = V_1

V_1 = $\prod \times 4^2 \times 6$

V_1 = 301.7142 cm^3

Volume of the Conical portion of the toy = V_2 = $\frac{1}{3} \times \prod r^2 L$

V_2 = $\frac{1}{3} \times \prod 4^2 \times 3$

V_2 = 50.2857 cm^3

Volume of the hemispherical portion of the toy = V_3 = $\frac{2}{3} \times \prod r^3$

V_3 = $\frac{2}{3} \times \prod 4^3$

V_3 = 134.0952 cm^3

So, the remaining volume of the cylinder when the toy (conical portion + hemispherical portion) is inserted in it = Volume of cylinder – (volume of the conical portion + volume of the hemispherical portion)

$V = V_1 - (V_2 + V_3)$

V = 301.7142 – (50.2857 + 134.0952)

V = 301.7142 – 184.3809

V = 117.3333 cm^3

Therefore, the remaining portion of the cylinder after insertion of the toy in it = V = 117.3333 cm^3

Q-36. Consider a solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm, is placed upright in the right circular cylinder full of water such that it touches bottoms. Find the volume of the water left in the cylinder, if radius of the cylinder is 60 cm and its height is 180 cm.

Solution:

As per the data given in the question, we have

Radius of the circular cone = r = 60 cm

Height of the circular cone = L = 120 cm

Radius of the hemisphere = r = 60 cm

Radius of the cylinder = R = 60 cm

Height of the cylinder = H = 180 cm

Now,

Volume of the circular cone = V_1 = 13×∏r^2L $\frac{1}{3} \times \prod r^2 L$

V_1 = 13×∏60^2×120 $\frac{1}{3} \times \prod 60^2 \times 120$

V_1 = 452571.429 cm^3

Volume of the hemisphere = V_2 = 23×∏r^3 $\frac{2}{3} \times \prod r^3$

V_2 = 23×∏60^3 $\frac{2}{3} \times \prod 60^3$

V_2 = 452571.429 cm^3

Volume of the cylinder = ∏×R^2×H $\prod \times R^2 \times H$ = V_3

V_3 = ∏×60^2×180 $\prod \times 60^2 \times 180$

V_3 = 2036571.43 cm^3

Volume of water left in the cylinder = Volume of the cylinder – (volume of the circular cone + volume of the hemisphere) = V

V = V_3 – (V_1 + V_2)

V = 2036571.43 – (452571.429 + 452571.429)

V = 2036571.43 – 905142.858

V = 1131428.57 cm^3

V = 1.1314 m^3

Therefore, the volume of the water left in the cylinder = V = 1.1314 m^3

Q-37. Consider a cylindrical vessel with internal diameter 20 cm and height 12 cm is full of water. A solid cone of base diameter 8 cm and height 7 cm is completely immersed in water. Find the value of water when

(i) Displaced out of the cylinder

(ii) Left in the cylinder

Solution:

As per the parameters given in the question, we have

Internal diameter of the cylindrical vessel = 20 cm

Radius of the cylindrical vessel = r = 10 cm (as we know that the radius is half of the diameter)

Height of the cylindrical vessel = h = 12 cm

Base diameter of the solid cone = 8 cm

Radius of the solid cone = R = 4 cm (as we know that the radius is half of the diameter)

Height of the cone = L = 7 cm

(i) Volume of water displaced out from the cylinder = Volume of the cone = V_1

V_1 = 13×∏R^2L$\frac{1}{3} \times \prod R^2 L$

V_1 = 13×∏4^2×7$\frac{1}{3} \times \prod 4^2 \times 7$

V_1 = 117.3333 cm^3

Therefore, the volume of the water displaced after immersion of the solid cone in the cylinder = V_1 =117.3333 cm^3

(ii) Volume of the cylindrical vessel = ∏×r^2×h$\prod \times r^2 \times h$ = V_2

V_2 = ∏×10^2×12$\prod \times 10^2 \times 12$

$V_2 = 3771.4286\ cm^3$

Volume of the water left in the cylinder = Volume of the cylindrical vessel – Volume of the solid cone

$V = V_2 – V_1$

V = 3771.4286 – 117.3333

$V = 3654.0953\ cm^3$

Therefore, the volume of the water left in cylinder = V = $3654.0953\ cm^3$

Exercise 16.3: Surface Areas and Volumes

Q1. A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also find the cost of tin sheet for making the bucket at the rate of Rs 1.20 per dm^2

Soln:

Given

Diameter to top of bucket = 40 cm

Radius (r_1) = $\frac{40}{2} = 20cm$

Diameter of bottom part of the bucket = 20 cm

Radius (r_2) = $\frac{30}{2} = 10cm$

Depth of the bucket (h) = 12 cm

Volume of the bucket = $\frac{\Pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) h$

= $\frac{\Pi}{3}\left(20^2 + 10^2 + 20 \times 10\right) 12$

= $8800\ cm^3$

Let ‘L’ be slant height of the bucket

=>L = $\sqrt{(r_1 - r_2)^2 + h^2}$

=>L = $\sqrt{(20 - 10)^2 + 12^2}$

=>L = 15.620cm

Total surface area of bucket = $\Pi(r_1 + r_2) \times L + \Pi \times r_2^2$

= $\Pi(20 + 10) \times 15.620 + \Pi \times 10^2$

= $\frac{1320\sqrt{61} + 2200}{7}$

= $17.87\ dm^2$

Given that cost of tin sheet used for making bucket per dm^2 = Rs 1.20

So total cost for $17.87\ dm^2$ = 1.20×17.87

= Rs 21.40.

$\therefore$

Cost for $17.87\ dm^2$ = Rs 21.40

Q2. A frustum of a right circular cone has a diameter of base 20cm, of top 12 cm and height 3 cm. find the area of its whole surface and volume.

Sol:

Given base diameter of cone (d_1)= 20cm

Radius (r_1) = $\frac{20}{2}cm = 10cm$

Top diameter of Cone (d_2) = 12 cm

Radius (r_2) = $\frac{12}{2}cm = 6cm$

Height of the cone (h)= 3cm

Volume of the frustum right circular cone = $\frac{\Pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) h$

= $\frac{\Pi}{3}\left(10^2 + 6^2 + 10 \times 6\right) 3$

=616 cm^3

Let 'L' be the slant height of cone

=>L = $\sqrt{(r_1 - r_2)^2 + h^2}$

=>L = $\sqrt{(10 - 6)^2 + 3^2}$

=>L = $\sqrt{25}$

=>L = 5 cm

∴∵ Slant height of cone (L)= 5 cm

Total surface area of the cone = $\Pi\left(r_1 + r_2\right) \times L + \Pi \times r_1^2 + Pi \times r_2^2$

= $\Pi\left(10 + 6\right) \times 5 + \Pi \times 10^2 + Pi \times 6^2$

= $\Pi\left(80 + 100 + 36\right)$

= $\Pi\left(216\right)$

= $678.85 cm^2$

∴∵

Total surface area of the cone = 678.85 cm^2

Q 3. The slant height of the frustum of a cone is 4 cm and perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.

Soln:

Given slant height of frustum of cone (l) = 4 cm
Let ratio of the top and bottom circles be r_1 and r_2

Given perimeters of its ends as 18 cm and 6 cm

=> $2\Pi r_1$ = 18 cm ; $2\Pi r_2$ = 6 cm

=> Πr_1 = 9 cm — (a) ; Πr_2 = 3 cm —(b)

Curved surface area of frustum cone = $\Pi(r_1+r_2)l$

= $\Pi(r_1+r_2)l$

= $(\Pi r_1+\Pi r_2)l$

= (9 + 3) $\times 4$

= (12) $\times 4$

= $48 cm^2$

∴ Curved surface area of the frustum cone = $48 cm^2$

Q4. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Soln:

Given:

Perimeters of ends of frustum right circular cone are 44 cm and 33 cm

Height of the frustum cone = 16 cm

Perimeter = $2\Pi r$

$2\Pi r_1$ = 44 ; $2\Pi r_2$ = 33

r_1 = 7 cm ; r_2 = 5025 cm

Let the slant height of frustum right circular cone be L

L = $\sqrt{(r_1-r_2)^2+h^2}$

L = $\sqrt{(7-5.25)^2+16^2}$

L = 16.1 cm

$\therefore$ Slant height of the frustum cone = 20.37 cm

Curved surface area of the frustum cone = $\Pi\left(r_1+r_2\right)L$

= $\Pi\left(7+5.25\right)16.1$

Curved surface area of the frustum cone = 619.65 cm^3

Volume of the frustum cone = $\frac{1}{3}\Pi\left(r_1^2+r_2^2+r_1r_2\right)h$

= $\frac{1}{3}\Pi\left(7^2+5.25^2+7\times 5.25\right)\times 16$

= 1898.56 cm^3

$\therefore$ volume of the cone = 1898.56 cm^3

Total surface area of the frustum cone = $\Pi\left(r_1+r_2\right)L+\Pi r_1^2+\Pi r_2^2$

= $\Pi\left(7+5.25\right)\times 16.1+\Pi 7^2+\Pi 5.25^2$

= $\Pi\left(7+5.25\right)\times 16.1+\Pi\left(7^2+5.25^2\right)$

= 860.27 cm^2

$\therefore$ total surface area of the frustum cone = 860.27 cm^2

Q.5: If the radius of circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm. Find the capacity of the bucket.

Soln:

Given

Height of the conical bucket = 45 cm

Radii of the 2 circular ends of the conical bucket is 28 cm and 7 cm

r_1 = 28 cm

r_2 = 7 cm

Volume of the conical bucket = $\frac{1}{3}\Pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$

= $\frac{1}{3}\Pi(28^2 + 7^2 + 28 \times 7)45$

= 15435Π

Volume = 48510 cm^3

Q7. If the radii of circular end of a bucket 24 cm high are 5 and 15 cm. Find surface area of the bucket.

Soln:

Given height of the bucket (h) = 24 cm

Radius of the circular ends of the bucket 5 cm and 15 cm

$r_1 = 5cm$; $r_2 = 15cm$

Let 'L' be the slant height of the bucket

L = $\sqrt{(r_1 - r_2)^2 + h^2}$

=>L = $\sqrt{(5 - 15)^2 + 24^2}$

=>L = $\sqrt{(100 + 576)}$

=>L = 26 cm

Curved surface area of the bucket = $\Pi\left(r_1 + r_2\right)L + \Pi r_2^2$

= $\Pi\left(5 + 15\right)26 + \Pi 15^2$

= $\Pi\left(520 + 225\right)$

= $745\Pi\ cm^2$

Curved surface area of the bucket = $745\Pi cm^2$

Q8. The radii of circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 15 cm. Find the total surface area and volume of frustum.

Soln:

Let slant height of the frustum cone be 'L'

Given height of frustum cone = 12 cm

Radii of a frustum cone are 12 cm and 3 cm

$r_1 = 12\ cm$; $r_2 = 3\ cm$

$L = \sqrt{(r_1 - r_2)^2 + h^2}$

L=Extra close brace or missing open brace Extra close brace or missing open brace

$L = \sqrt{81 + 144}$ = 15 cm

L = 15 cm

Total surface area of cone = $\Pi(r_1 + r_2)L + \Pi r_1^2 + \Pi r_2^2$

= $\Pi(12 + 3)15 + \Pi 12^2 + \Pi 3^2$

Total surface area = 378 cm^2

Volume of frustum cone = $\frac{1}{3}\Pi\left(r_1^2 + r_2^2 + r_1 r_2\right) \times h$

= $\frac{1}{3}\Pi\left(12^2 + 3^2 + 12 \times 3\right) \times 12$ = $756\Pi cm^3$

Volume of the frustum cone = $756\Pi cm^3$

Q9. A tent consists of a frustum of a cone capped by a cone. If radii of ends of frustum be 13 m and 7 m the height of frustum be 8 m and the slant height of the conical cap be 12 m. Find canvas required for the tent.

Soln:

Given height of frustum (h) = 8 m

Radii of the frustum cone are 13 cm and 7 cm

$r_1 = 13m$; $r_2 = 7$

Let ‘L’ be slant height of the frustum cone

=>L = $\sqrt{(r_1 - r_2)^2 + h^2}$

=>L = $\sqrt{(13-7)^2 + 8^2}$

=>L = $\sqrt{36+64}$

=>L = 10 m

Curved surface area of frustum $(s_1) = \Pi(r_1 + r_2) \times L$

= $\Pi(13+7) \times 10$

= $200\Pi m^2$

Curved surface area of frustum (s_1) = $200\Pi m^2$

Given slant height of conical cap = 12 m

Base radius of upper cap cone = 7 m

Curved surface area of upper cap cone $(s_2) = \Pi rL$

= $\Pi \times 7 \times 12$

= $264\ m^2$

Total canvas required for tent $(S) = s_1 + s_2$

S = $200\Pi + 264 = 892.57m^2$

∴∵ total canvas = $892.57m^2$

Q10. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. Radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making.

Soln:

Radii of top circular ends (r_1) = 20 cm

Radii of bottom circular end of bucket (r_2) = 12 cm

Let height of bucket be ‘h’

Volume of frustum cone = $\frac{1}{3}\Pi\left(r_1^2 + r_2^2 + r_1 r_2\right) h$

= $\frac{1}{3}\Pi\left(20^2 + 12^2 + 20 \times 12\right) h$

= $\frac{784}{3}\Pi h\ cm^3$ —-(a)

Given capacity/ volume of the bucket = 12308.8 cm^3 —–(b)

Equating (a) and (b)

=> $\frac{784}{3}\Pi h\ cm^3$ = 12308.8

=>h = $\frac{12308.8 \times 3}{784 \times \Pi}$

=> h =15cm

∴∵. height of the bucket (h) = 15 cm

Let ‘L’ be slant height of bucket

=> $L^2 = (r_1 - r_2)^2 + h^2$

=> L = $\sqrt{(r_1 - r_2)^2 + h^2}$

=> L = 17 cm

∴ length of the bucket/ slant height of the bucket (L) =17 cm

Curved surface area of bucket = $\Pi\left(r_1 + r_2\right) L + \Pi r_2^2$

= $\Pi\left(20 + 12\right) 17 + \Pi \times 12^2$

= $\Pi\left(9248 + 144\right)$

= 2160.32 cm²

∴ curved surface area = 2160.32 cm²

Q11. A bucket made of aluminum sheet is of height 20 cm and its upper and lower ends are of radius 25 cm and 10 cm. Find cost of making if the aluminum sheet costs

Rs 70 per 100 cm^2.

Soln:

Given height of bucket (h) = 20 cm

Upper radius of bucket (r_1) = 25 cm

Lower radius of the bucket (r_2) = 10 cm

Let 'L' be slant height of the bucket

L = $\sqrt{(r_1 - r_2)^2 + h^2}$

= $\sqrt{(25 - 10)^2 + 20^2}$

= 25 cm

∴ slant height of bucket (L) = 25 cm

Curved surface area of bucket = $\Pi(r_1 + r_2) \times L + \Pi r_2^2$

= $\Pi(25 + 10) \times 25 + \Pi \times 10^2$

= $3061.5 cm^2$

∴ curved surface area = 3061.5 cm^2

Cost of making bucket per 100 cm^2 = Rs 70

Cost of making bucket per 3061.5 cm^2 = $\frac{3061.5}{100} \times 70$

= Rs 2143.05

Total cost for 3061.5 cm^2 = Rs 2143.05

Q12. Radii of circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Finds its total surface area.

Soln:

Given slant height of frustum cone = 10 cm

Radii of circular ends of frustum cone are 33 cm and 27 cm

$r_1 = 33cm$; $r_2 = 27cm$

Total surface area of a solid frustum of cone

$= \Pi(r_1 + r_2) \times L + \Pi r_1^2 + \Pi r_2^2$

$= \Pi(33 + 27) \times 10 + \Pi 33^2 + \Pi 27^2$

$= \Pi(60) \times 10 + \Pi 33^2 + \Pi 27^2$

$= \Pi(600 + 1089 + 729)$

= 2418 Πcm^2

∴ Total surface area = 7599.42 cm^2

Q13. A bucket made up of a metal sheet is in from of a frustum of cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm. Find the volume of the bucket. Also find the cost of the bucket if the cost of metal sheet used id Rs 20 per 100 cm^2.

Soln:

Given height of frustum cone = 16 cm

Diameter of the lower end of the bucket $d_1 = 16cm$

Lower end radius $r_1 = \frac{16}{2} = 8\ cm$

Upper end radius $r_2 = \frac{340}{2} = 20\ cm$

Let 'L' be slant height of frustum of cone

$L = \sqrt{(r_1 - r_2)^2 + h^2}$

$L = \sqrt{(20 - 8)^2 + 16^2}$

$L = \sqrt{(114 + 256)}$

L = 20 cm

∴∵ slant height of the frustum cone L = 20 cm

Volume of the frustum cone = $\frac{1}{3}\Pi\left(r_1^2 + r_2^2 + r_1 \times r_2\right) \times h$

= $\frac{1}{3}\Pi\left(8^2 + 20^2 + 8 \times 20\right) \times 16$

Volume of the frustum cone = 10449.92 cm^3

Curved surface area of the frustum cone = $\Pi\left(r_1 + r_2\right) \times L + \Pi r_1^2$

= $\Pi\left(20 + 8\right) \times 20 + \Pi 8^2$

= $\Pi(560 + 64)$

= 624 Πcm^2

Cost of the metal sheet per 100 cm^2 = Rs 20

Cost of the metal sheet per 624Πcm^2 = $\frac{624\Pi}{100} \times 20$

∴∵ Cost of the metal sheet = 391.9

Q14. A solid is in the shape of a frustum of a cone. The diameters of two circular ends are 60 cm and 36 cm and height is 9 cm. find area of its whole surface and volume.

Soln:

given height of the frustum cone = 9 cm

Lower end radius $r_1 = \frac{60}{2} = 30\ cm$

Upper end radius $r_2 = \frac{36}{2} = 18\ cm$

Let slant height of the frustum cone be L

$L = \sqrt{\left(r_1 - r_2\right)^2 + h^2}$

$L = \sqrt{(18-30)^2 + 9^2}$

$L = \sqrt{144 + 81}$

L = 15 cm

Volume of the frustum cone = $\frac{1}{3}\Pi\left(r_1^2 + r_2^2 + r_1 \times r_2\right) \times h$

= $\frac{1}{3}\Pi\left(30^2 + 18^2 + 30 \times 18\right) \times 9$

= $5292\Pi cm^3$

Volume = $5292\Pi cm^3$

Total surface area of frustum cone = $\Pi\left(r_1 + r_2\right) \times L + \Pi r_1^2 + \Pi r_2^2$

= $\Pi\left(30 + 18\right) \times 15 + \Pi 30^2 + \Pi 18^2$

= $\Pi\left(720 + 900 + 324\right)$

= $1944 Pi cm^2$

∴∵ total surface area = $1944\Pi cm^2$

Q15. A milk container is made of metal sheet in the shape of frustum cone whose volume is 10459 cm^3. The radii of its lower and upper circular ends are 8 cm and 20 cm. Find the cost of metal sheet used in making container at a rate of Rs 1.40 per cm^2.

Soln:

Given,

Lower end radius r_1 = 8 cm

upper end radius r_2 = 20 cm

Let the height of the container be ‘h’

$V_1 = \frac{1}{3}\Pi\left(8^2 + 20^2 + 8(20)\right) h cm^3$ —- (1)

Volume of the milk container = $10459\frac{3}{4}cm^3$

V_2 = $\frac{73216}{7}cm^3$

$V_1 - V_2$

$\frac{1}{3}\Pi\left(8^2+20^2+8(20)\right)hcm^3$

=> h = $\frac{10459.42}{653.45}$

=> h= 16 cm

∴∵. Height of frustum cone (h) = 16 cm

Let slant height of frustum cone be ‘L’

L = $\sqrt{(r_1-r_2)^2+h^2}$

L = $\sqrt{(20-8)^2+16^2}$

L = 20 cm

∴∵. Slant height of frustum cone (L) = 20 cm

Total surface area of the frustum cone

$=\Pi(r_1+r_2)\times L+\Pi r_1^2+\Pi r_2^2$

$= \Pi(20+8\times 20+\Pi 20^2+\Pi 8^2$

$= \Pi(560+400+64)$

$= 1024\Pi$

$= 3216.99cm^2$

Total surface area of the frustum = $3216.99cm^2$

Q.16: A reservoir in form of frustum of a right circular cone contains 44×10^7 liters of water which fills it completely. The radii of bottom and top of the reservoir are 50 m and 100 m. Find the depth of water and lateral surface area of the reservoir.

Soln:

Let depth of frustum cone be h

Volume of first cone (V) = 13Π(r21+r22+r1×r2)×h $\frac{1}{3}\Pi\left(r_1^2+r_2^2+r_1\times r_2\right)\times h$

r1=50m $r_1 = 50m$; r2=100 $r_2 = 100$

V= 13 227(50²+100²+50×100)h $\frac{1}{3}\frac{22}{7}\left(50^2+100^2+50\times 100\right)h$

V= 13 227 $\frac{1}{3}\frac{22}{7}$ (2500+1000+5000)h —- (a)

Volume of the reservoir = 44×10⁷ 44×10^7 liters —- (b)

Equating (a)and(b)

13Π $\frac{1}{3}\Pi$ (8500)h =44×10⁷ 44×10^7

h = 24

Q.17: A metallic right circular cone 20 cm high and whose vertical angle is 90° 90° is cut into two parts at the middle point of its axis by a plane parallel to the base. If frustum so obtained be drawn into a wire of diameter 116 $\frac{1}{16}$ cm. Find the length of the wire.

Soln:

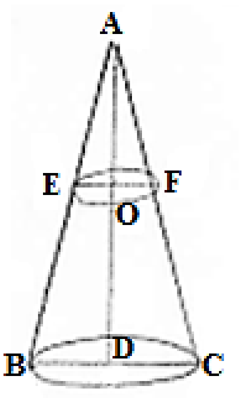
let ABC be the cone. Height of metallic cone

AO = 20 cm

-> Cone is cut into two parts at the middle point of its axis

Hence height of frustum cone AD= 10 cm

Since angle A is right angles. So each angles B and C = 45° 45°



Angles E and F = 45° 45°

Let radii of top and bottom circle of frustum

Cone be r1 r_1 and r2 r_2

From ΔADE ΔADE => DEAD =Cot45° $\frac{DE}{AD} = Cot 45^\circ$

=> r110 =1 $\frac{r_1}{10} = 1$

=> r1=10cm $r_1 = 10cm$

From ΔAOB ΔAOB

=> OEOA =Cot45° $\frac{OE}{OA} = Cot 45^\circ$

=> r220 =1 $\frac{r_2}{20} = 1$

=> r2=20cm $r_2 = 20cm$