RD Sharma Solutions for Class 8 Math Chapter 17 - Understanding Shapes Iii (special Types Of Quadrilaterals)

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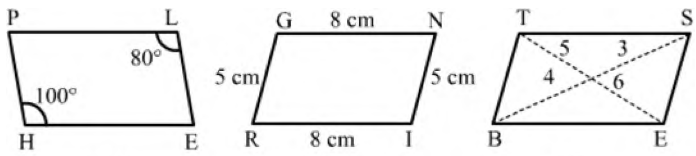
Question 3:

Can the following figures be parallelograms. Justify your answer.



(i) (ii) (iii)

ANSWER:

(i)
No. *This is because the* opposite angles are not equal.
(ii)
Yes. *This is because the opposite sides are equal.*
(ii)
No, *This is because the* diagonals do not bisect each other.

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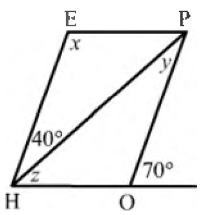
Question 4:

In the adjacent figure *HOPE* is a parallelogram. Find the angle measures *x*,*y* and *z*. State the geometrical truths you use to find them.



ANSWER:

$\angle HOP + 70^{\circ} = 180^{\circ}$ (linear pair)
$\angle HOP = 180^{\circ} - 70^{\circ} = 110^{\circ}$
$x = \angle HOP = 110^{\circ}$ (opposite angles of a parallelogram are equal)
$\angle EHP + \angle HEP = 180^{\circ}$ (sum of adjacent angles of a parallelogram is 180°)
$110^{\circ} + 40^{\circ} + z = 180^{\circ}$
$z = 180^{\circ} - 150^{\circ} = 30^{\circ}$
$y = 40^{\circ}$ (alternate angles)

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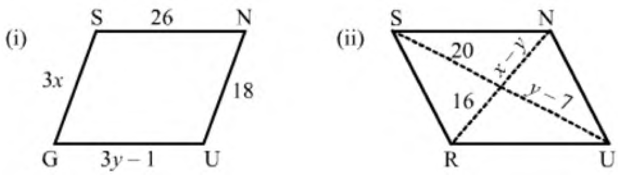
Question 5:

In the following figures *GUNS* and *RUNS* are parallelograms. Find *x* and *y*.



ANSWER:

(i)
Opposite sides are equal in a parallelogram.
$\therefore\ 3y - 1 = 26$
$3y = 27$
$y = 9$
Similarly, $3x = 18$
$x = 6$
(ii)
Diagonals bisect each other in a parallelogram.
$\therefore\ y - 7 = 20$
$y = 27$
$x - y = 16$
$x - 27 = 16$
$x = 43$

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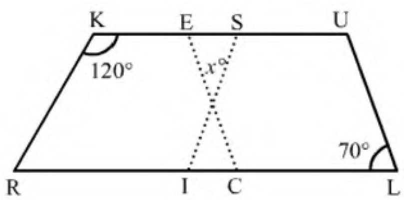
Question 6:

In the following figure *RISK* and *CLUE* are parallelograms. Find the measure of *x*.



ANSWER:

In the parallelogram RISK :
$\angle ISK + \angle RKS = 180^\circ$ (sum of adjacent angles of a parallelogram is 180°)
$\angle ISK = 180^\circ - 120^\circ = 60^\circ$
Similarly, in parallelogram CLUE :
$\angle CEU = \angle CLU = 70^\circ$ (opposite angles of a parallelogram are equal)
In the triangle :
$x + \angle ISK + \angle CEU = 180^\circ$
$x = 180^\circ - (70^\circ + 60^\circ)$
$x = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$

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Question 7:

Two opposite angles of a parallelogram are (3*x* − 2)° and (50 − *x*)°. Find the measure of each angle of the parallelogram.

ANSWER:

Oppostie angles of a parallelogram are congurent.
$\therefore\ (3x - 2)^\circ = (50 - x)^\circ$
$3x^\circ - 2^\circ = 50^\circ - x^\circ$
$3x^\circ + x^\circ = 50^\circ + 2^\circ$
$4x^\circ = 52^\circ$
$x^\circ = 13^\circ$
Putting the value of x in one angle :
$3x^\circ - 2^\circ = 39^\circ - 2^\circ$
$= 37^\circ$
Opposite angles are congurent :
$\therefore\ 50 - x^\circ$
$= 37^\circ$
Let the remaining two angles be $y\, and\, z$.
Angles y and z are congurent because they are also opposite angles.
$\therefore\ y = z$
The sum of adjacent *angles* of *a parallelogram* is equal to 180°.
$\therefore 37^\circ + y = 180^\circ$
$y = 180^\circ - 37^\circ$
$y = 143^\circ$
So, the anlges measure are :
$37^\circ,\ 37^\circ,\ 143^\circ\ and\ 143^\circ$

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Question 8:

If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

ANSWER:

Two adjacent angles of a parallelogram add up to 180°.
Let x be the angle.
$\therefore\ x + \frac{2x}{3} = 180^\circ$
$\frac{5x}{3} == 180^\circ$
$x = 72^\circ$
$\frac{2x}{3} = \frac{2\times72^\circ}{3} = 108^\circ$
Thus, two of the angles in the parallelogram are 108° and the other two are 72°.

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Question 9:

The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles?

ANSWER:

Given that one angle of the parallelogram is 70°.
Since opposite angles have same value, if one is 70°, then the one directly opposite will also be 70°.
So, let one angle be $\mathbf{x}^\circ$.
$\mathbf{x}^\circ + 70^\circ = 180^\circ$ (the sum of adjacent angles of a parallelogram is 180°)
$\mathbf{x}^\circ = 180^\circ - 70^\circ$
$\mathbf{x}^\circ = 110^\circ$
Thus, the remaining angles are 110°, 110° and 70°.

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Question 10:

Two adjacent angles of a parallelogram are as 1 : 2. Find the measures of all the angles of the parallelogram.

ANSWER:

Let the angle be A and B.
*The a*ngles are in the ratio of $1:2$.
*M*easures of $\angle A$ and $\angle B$ are $\mathbf{x}^\circ$ and $2\mathbf{x}^\circ$.
Then, $\angle C = \angle A$ and $\angle D = \angle B$ $\left(\textit{o}\text{pposite angles of } a \text{ parallelogram are congruent}\right)$
As we know that the sum of adjacent angle*s* of a parallelogram is 180°.
$\therefore \angle A + \angle B = 180^\circ$
$\Rightarrow \mathbf{x}^\circ + 2\mathbf{x}^\circ = 180^\circ$
$\Rightarrow 3\mathbf{x}^\circ = 180^\circ$
$\Rightarrow \mathbf{x}^\circ = \frac{180^\circ}{3} = 60^\circ$
Thus, measure of $\angle A = 60^\circ$, $\angle B = 120^\circ$, $\angle C = 60^\circ$ *and* $\angle D = 120^\circ$.

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Question 11:

In a parallelogram *ABCD, ∠D* = 135°, determine the measure of *∠A* and *∠B.*

ANSWER:

In *a* parallelogram, opposite angles have the same value.
$\therefore \angle D = \angle B$
$= 135^\circ$
Also, $\angle A + \angle B + \angle C + \angle D = 360^\circ$
$\angle A + \angle D = 180^\circ$ (opposite angles have the same value)
$\angle A = 180^\circ - 135^\circ = 45^\circ$
$\angle A = 45^\circ$

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Question 12:

ABCD is a parallelogram in which *∠A* = 70°. Compute *∠B, ∠C* and *∠D.*

ANSWER:

Opposite angles of a parallelogram are equal.
$\therefore \angle C = 70^\circ = \angle A.$
$\angle B = \angle D$
Also, the sum of the adjacent angles of a parallelogram is 180°.
$\therefore \angle A + \angle B = 180^\circ$
$70^\circ + \angle B = 180^\circ$
$\angle B = 110^\circ$
$\therefore \angle B = 110^\circ, \angle C = 70^\circ$ and $\angle D = 110^\circ$

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Question 13:

The sum of two opposite angles of a parallelogram is 130°. Find all the angles of the parallelogram.

ANSWER:

Let the angles be A, B, C and D.
It is given that *the* sum of two opposite angles is 130°.
$\therefore \angle A + \angle C = 130^\circ$
$\angle A + \angle A = 130^\circ$ (opp*o*site angle*s* of *a* parallelogram are same)
$\angle A = 65^\circ$
and $\angle C = 65^\circ$
*The s*um of *adjacent angles of a paralle*log*ram is* 180°.
$\angle A + \angle B = 180^\circ$
$65^\circ + \angle B = 180^\circ$
$\angle B = 180^\circ - 65^\circ$
$\angle B = 115^\circ$
$\angle D = 115^\circ$
$\therefore \angle A = 65^\circ$, v$\angle B = 115^\circ, \angle C = 65^\circ$ and $\angle D = 115^\circ$.

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Question 14:

All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

ANSWER:

Let the angle be x.
*A*ll the angles are equal.
$\therefore x + x + x + x = 360^\circ$
$4x = 360^\circ$
$x = 90^\circ$
So, each angle is 90° and quadrilateral is a parallelogram. It is a rectangle.

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Question 15:

Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

ANSWER:

We know that the opposite sides of a parallelogram are equal.
Two sides are given, i.e. 4 cm and 3 cm.
Therefore, the rest of the sies will also be 4 cm and 3 cm.
$\therefore$ *P*erimeter = *S*um of all *the* sides of *a* parallelogram
$= 4 + 3 + 4 + 3$
$= 14$ cm

Question 16:

The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

ANSWER:

Opposite sides of a parallelogram are same.
Let two sides of the parallelogram be x and y.
Given :
$x = y + 25$
Also, $x + y + x + y = 150$ $\left(Perimeter = \text{ } Sum\, of\, all\, the\, sides\, of\, a\, parallelogram\right)$
$y + 25 + y + y + 25 + y = 150$
$4y = 150 - 50$
$4y = 100$
$y = \frac{100}{4} = 25$
$\therefore\ x = y + 25 = 25 + 25 = 50$
Thus, the lengths of the sides of the parallelogram are 50 cm and 25 cm.

Question 17:

The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

ANSWER:

Given :
Shorter side $= 4.8$ cm
Longer side $= \frac{4.8}{2} + 4.8 = 7.2$ cm
Perimeter = Sum of all the sides
$= 4.8 + 4.8 + 7.2 + 7.2$
$= 24$ cm

Question 18:

Two adjacent angles of a parallelogram are $(3x - 4)°$ and $(3x + 10)°$. Find the angles of the parallelogram.

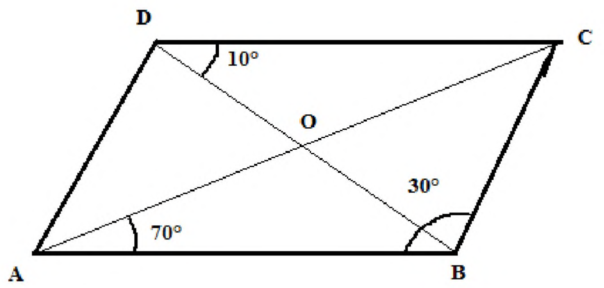
ANSWER:

We know that the adjacent angles of a parallelogram are supplementry.
Hence, $(3x + 10)^\circ$ *and* $(3x - 4)^\circ$ are supplementry.
$(3x + 10)^\circ + (3x - 4)^\circ = 180^\circ$
$6x^\circ + 6^\circ = 180^\circ$
$6x^\circ = 174^\circ$
$x = 29^\circ$
First angle $=$ $(3x + 10)^\circ = (3 \times 29^\circ + 10^\circ) = 97^\circ$
Second angle $=$ $(3x - 4)^\circ = 83^\circ$
Thus, the angles of the parallelogram are 97°, 83°, 97° and 83°.

Question 19:

In a parallelogram *ABCD*, the diagonals bisect each other at *O*. If $\angle ABC = 30°$, $\angle BDC = 10°$ and $\angle CAB = 70°$. Find:
$\angle DAB, \angle ADC, \angle BCD, \angle AOD, \angle DOC, \angle BOC, \angle AOB, \angle ACD, \angle CAB, \angle ADB, \angle ACB, \angle DBC$ and $\angle DBA$.

ANSWER:



$$\angle ABC = 30^\circ$$
$$\therefore\ \angle ADC = 30^\circ \ \text{(opposite angle of } \textit{the} \text{ parallelogram)}$$
$$\text{and } \angle BDA = \angle \mathrm{ADC} - \angle \mathrm{BDC} = 30^\circ - 10^\circ = 20^\circ$$
$$\angle BAC = \angle ACD = 70^\circ \ \ (\textit{alternate angle})$$
$$\text{In } \triangle \ \mathrm{ABC}:$$
$$\angle \mathrm{CAB} + \angle \mathrm{ABC} + \angle \mathrm{BCA} = 180^\circ$$
$$70^\circ + 30^\circ + \angle \mathrm{BCA} = 180^\circ$$
$$\therefore\ \angle \mathrm{BCA} = 80^\circ$$
$$\angle \mathrm{DAB} = \angle \mathrm{DAC} + \angle \mathrm{CAB} = 70^\circ + 80^\circ = 150^\circ$$
$$\angle \mathrm{BCD} = 150^\circ \ \text{(opposite angle of } \textit{the} \text{ parallelogram)}$$
$$\angle DCA = \angle CAB = 70^\circ$$
$$\text{In } \triangle \mathrm{DOC}:$$
$$\angle \mathrm{ODC} + \angle \mathrm{DOC} + \angle \mathrm{OCD} = 180$$
$$10^\circ + 70^\circ + \angle \mathrm{DOC} = 180^\circ$$
$$\therefore \angle \mathrm{DOC} = 100^\circ$$
$$\angle \mathrm{DOC} + \angle \mathrm{BOC} = 180^\circ$$
$$\angle \mathrm{BOC} = 180^\circ - 100^\circ$$
$$\angle \mathrm{BOC} = 80^\circ$$
$$\angle AOD = \angle BOC = 80^\circ \ \ \text{(vertically opposite angles)}$$
$$\angle \mathrm{AOB} = \angle \mathrm{DOC} = 100^\circ \ \text{(vertically opposite angles)}$$
$$\angle \mathrm{CAB} = 70^\circ \ \text{(given)}$$
$$\angle \mathrm{ADB} = 20^\circ$$
$$\angle DBA = \angle BDC = 10^\circ \ \ (\textit{alternate angle})$$
$$\angle ADB = \angle DBC = 20^\circ \ \ (\textit{alternate angle})$$

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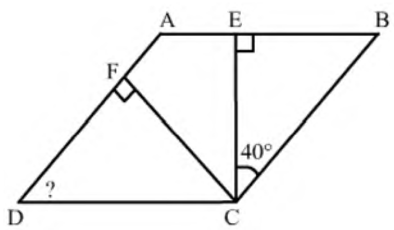
Question 20:

Find the angles marked with a question mark shown in Fig. 17.27



ANSWER:

In $\triangle$ CEB :
$\angle ECB + \angle CBE + \angle BEC = 180^\circ$ (*angle sum property of a triangle*)
$40^\circ + 90^\circ + \angle EBC = 180^\circ$
$\therefore \angle EBC = 50^\circ$
Also, $\angle EBC = \angle ADC = 50^\circ$ (opposite angle of a parallelogram)
In $\triangle$ FDC :
$\angle FDC + \angle DCF + \angle CFD = 180^\circ$
$50^\circ + 90^\circ + \angle DCF = 180^\circ$
$\therefore \angle DCF = 40^\circ$
Now, $\angle BCE + \angle ECF + \angle FCD + \angle FDC = 180^\circ$ (in a parallelogram, the sum of alternate angles is 180°)
$50^\circ + 40^\circ + \angle ECF + 40^\circ = 180^\circ$
$\angle ECF = 180^\circ - 50^\circ + 40^\circ - 40^\circ = 50^\circ$

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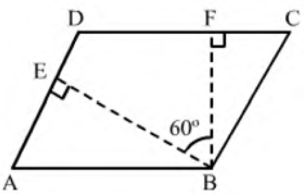
Question 21:

The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

ANSWER:



Draw a parallelogram ABCD.
Drop a perpendicular from B to the side AD, at the point E.
Drop perpendicular from B to the side CD, at the point F.
In the quadrilateral BEDF :
$\angle EBF = 60^\circ, \angle BED = 90^\circ$
$\angle BFD = 90^\circ$
$\angle EDF = 360^\circ - (60^\circ + 90^\circ + 90^\circ) = 120^\circ$
In a parallelogram, opposite angles are congruent and adjacent angles are supplementary.
In the parallelogram ABCD :
$\angle B = \angle D = 120^\circ$
$\angle A = \angle C = 180^\circ - 120^\circ = 60^\circ$

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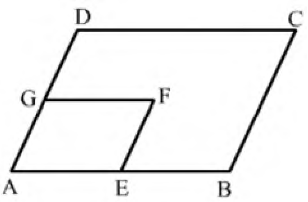
Question 22:

In Fig. 17.28, *ABCD* and *AEFG* are parallelograms. If $\angle C$ = 55°, what is the measure of $\angle F$?

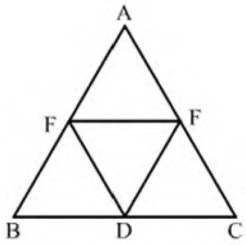


ANSWER:

Both the parallelograms ABCD and AEFG are similar.
$\therefore \angle C = \angle A = 55^\circ$ (opposite angles of a parallelogram are equal)
$\therefore \angle A = \angle F = 55^\circ$ (opposite angles of a parallelogram are equal)

Question 23:

In Fig. 17.29, *BDEF* and *DCEF* are each a parallelogram. Is it true that *BD* = *DC*? Why or why not?



ANSWER:

In parallelogram BDEF
$\therefore$ BD = EF ...(i) (opposite sides of a parallelogram are equal)
In parallelogram DCEF
CD = EF ...(ii) (opposite sides of a parallelogram are equal)
From equations (i) and (ii)
BD = CD

Question 24:

In Fig. 17.29, suppose it is known that *DE* = *DF*. Then, is Δ*ABC* isosceles? Why or why not?
Fig. 17.29

ANSWER:

In Δ FDE :
DE = DF
$\therefore \angle FED = \angle DFE$............(i) (angles opposite to equal sides)
In the II^{gm} BDEF :
$\angle FBD = \angle FED$.......(ii) (opposite angles of a parallelogram are equal)
In the II^{gm} DCEF :
$\angle DCE = \angle DFE$......(iii) (opposite angles of a parallelogram are equal)
From equations (i), (ii) and (iii) :
$\angle FBD = \angle DCE$
In $\triangle$ ABC :
If $\angle FBD = \angle DCE$, then AB = AC (sides opposite to equal angles).
Hence, $\triangle$ ABC is isosceles.

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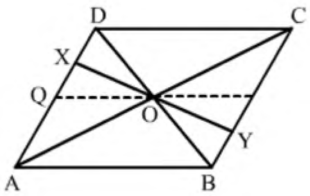
Question 25:

Diagonals of parallelogram *ABCD* intersect at *O* as shown in Fig. 17.30. *XY* contains *O*, and *X*, *Y* are points on opposite sides of the parallelogram. Give reasons for each of the following:
(i) *OB* = *OD*
(ii) ∠*OBY* = ∠*ODX*
(iii) ∠*BOY* = ∠*DOX*
(iv) Δ*BOY* ≅ Δ*DOX*
Now, state if *XY* is bisected at *O*.



ANSWER:

(i) Diagonals of a parallelogram bisect each other.
(ii) Alternate angles
(iii) Vertically opposite angles
(iv)

In Δ BOY and Δ DOX :
$OB = OD$ (diagonals of a parallelogram bisect each other)
$\angle OBY = \angle ODX$ (alternate angles)
$\angle BOY = \angle DOX$ (vertically opposite angles)

ASA congruence:
XO = YO (c.p.c.t)
So, XY is bisected at O.

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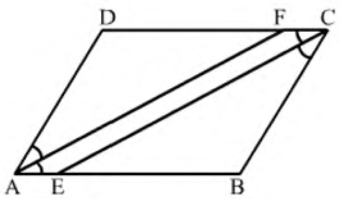
Question 26:

In Fig. 17.31, *ABCD* is a parallelogram, *CE* bisects ∠*C* and *AF* bisects ∠*A*. In each of the following, if the statement is true, give a reason for the same:



(i) $\angle A = \angle C$
(ii) $\angle FAB = \frac{1}{2}\angle A$
(iii) $\angle DCE = \frac{1}{2}\angle C$
(iv) $\angle CEB = \angle FAB$
(v) *CE* || *AF*

ANSWER:

(i) True, since opposite angles of a parallelogram are equal.
(ii) True, as AF is the bisector of ∠A.
(iii) True, as CE is the bisector of ∠C.
(iv) True

∠CEB = ∠DCE........(i) (alternate angles)
∠DCE= ∠ FAB.........(ii) (opposite angles of a parallelogram are equal)

From equations (i) and (ii):
∠CEB = ∠FAB

(v) True, as corresponding angles are equal (∠CEB = ∠FAB).

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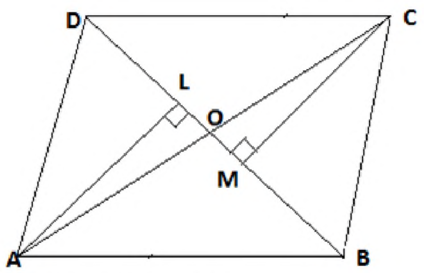
Question 27:

Diagonals of a parallelogram *ABCD* intersect at *O*. *AL* and *CM* are drawn perpendiculars to *BD* such that *L* and *M* lie on *BD*. Is *AL* = *CM*? Why or why not?

ANSWER:



In ΔAOL and ΔCMO :

$\angle AOL = \angle COM$(vertically opposite angle)....(i)

$\angle ALO = \angle CMO = 90^\circ$ (each right angle).....(ii)

Using angle sum property :

$\angle AOL + \angle ALO + \angle LAO = 180^\circ$..........(iii)

$\angle COM + \angle CMO + \angle OCM = 180^\circ$......(iv)

From equations (iii) and (iv) :

$\angle AOL + \angle ALO + \angle LAO = \angle COM + \angle CMO + \angle OCM$

$\angle LAO = \angle OCM$ (from equations (i) and (ii))

In ΔAOL and ΔCMO :

$\angle ALO = \angle CMO$ (each right angle)

$AO = OC$ (diagonals of a parallelogram bisect each other)

$\angle LAO = \angle OCM$ (proved above)

So, ΔAOL is congruent to ΔCMO (SAS).

$\Rightarrow AL = CM$ [cpct]

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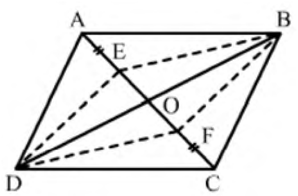
Question 28:

Points *E* and *F* lie on diagonal *AC* of a parallelogram ABCD such that *AE* = *CF*. What type of quadrilateral is *BFDE*?

ANSWER:



In the II^{gm} ABCD :
$AO = OC......(i)$ (diagonals of a parallelogram bisect each other)
$AE = CF.......(ii)$ (given)
Subtracting (ii) from (i) :
$AO - AE = OC - CF$
$EO = OF.........$ (iii)
In ΔDOE and ΔBOF :
$EO = OF$ (proved above)
$DO = OB$ (diagonals of a parallelogram bisect each other)
$\angle DOE = \angle BOF$ (vertically opposite angles)
By SAS congruence :
$\Delta DOE \cong \Delta BOF$
$\therefore DE = BF$ (c.p.c.t)
In ΔBOE and ΔDOF :
$EO = OF$ (proved above)
$DO = OB$ (diagonals of a parallelogram bisect each other)
$\angle DOF = \angle BOE$ (vertically opposite angles)
By SAS congruence :
$\Delta DOE \cong \Delta BOF$
$\therefore DF = BE$ (c.p.c.t)
Hence, the pair of opposite sides are equal. Thus, DEBF is a parallelogram.

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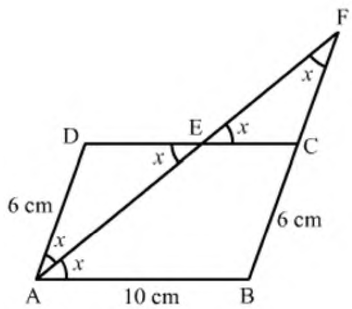
Question 29:

In a parallelogram *ABCD*, *AB* = 10 cm, *AD* = 6 cm. The bisector of ∠*A* meets *DC* in *E*, *AE* and *BC* produced meet at *F*. Find te length *CF*.

ANSWER:



AE is the bisector of $\angle A$.
$\therefore \angle DAE = \angle BAE = x$
$\angle BAE = \angle AED = x$ (alternate angles)
Since opposite angles in ΔADE are equal, ΔADE is an isosceles triangle.
$\therefore AD = DE = 6$ cm (sides opposite to equal angles)
$AB = CD = 10$ cm
$CD = DE + EC$
$\Rightarrow EC = CD - DE$
$\Rightarrow EC = 10 - 6 = 4$ cm
$\angle DEA = \angle CEF = x$ (vertically opposite angle)
$\angle EAD = \angle EFC = x$ (alternate angles)
Since opposite angles in ΔEFC are equal, ΔEFC is an isosceles triangle.
$\therefore CF = CE = 4$ cm (sides opposite to equal angles)
$\therefore CF = 4$ cm

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Question 1:

Which of the following statements are true for a rhombus?
(i) It has two pairs of parallel sides.
(ii) It has two pairs of equal sides.
(iii) It has only two pairs of equal sides.
(iv) Two of its angles are at right angles.
(v) Its diagonals bisect each other at right angles.
(vi) Its diagonals are equal and perpendicular.
(vii) It has all its sides of equal lengths.
(viii) It is a parallelogram.
(ix) It is a quadrilateral.
(x) It can be a square.
(xi) It is a square.

ANSWER:

(i) True
(ii) True
(iii) True
(iv) False
(v) True
(vi) False

Diagonals of a rhombus are perpendicular, but not equal.

(vii) True
(viii) True

It is a parallelogram because it has two pairs of parallel sides.

(ix) True

It is a quadrilateral because it has four sides.

(x) True

It can be a square if each of the angle is a right angle.

(xi) False

It is not a square because each of the angle is a right angle in a square.

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Question 2:

Fill in the blanks, in each of the following, so as to make the statement true:
(i) A rhombus is a parallelogram in which ......
(ii) A square is a rhombus in which ......
(iii) A rhombus has all its sides of ...... length.
(iv) The diagonals of a rhombus ...... each other at ...... angles.
(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a ......

ANSWER:

(i) A rhombus is a parallelogram in which adjacent sides are equal.
(ii) A square is a rhombus in which all angles are right angled.
(iii) A rhombus has all its sides of equal length.
(iv) The diagonals of a rhombus bisect each other at right angles.
(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a rhombus.

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Question 3:

The diagonals of a parallelogram are not perpendicular. Is it a rhombus? Why or why not?

ANSWER:

No, it is not a rhombus. This is because diagonals of a rhombus must be perpendicular.

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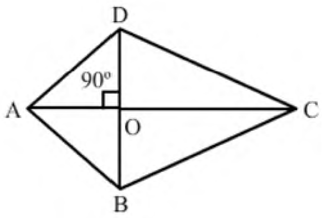
Question 4:

The diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? If your answer is 'No', draw a figure to justify your answer.

ANSWER:

No, it is not so.
Diagonals of a rhombus are perpendicular and bisect each other. Along with this, all of its sides are equal. In the figure given below, the diagonals are perpendicular to each other, but do not bisect each other.



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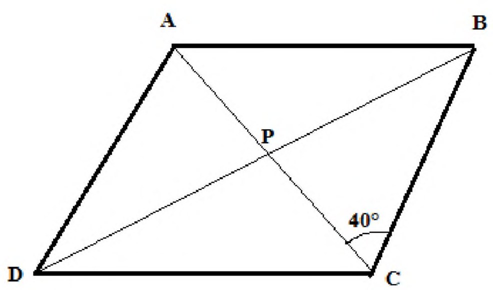
Question 5:

ABCD is a rhombus. If ∠*ACB* = 40°, find ∠*ADB*.

ANSWER:



In a rhombus, the diagonals are perpendicular.
$\therefore\ \angle \mathrm{BPC} = 90^\circ$
From Δ BPC, the sum of angles is 180°.
$\therefore\ \angle \mathrm{CB}P + \angle \mathrm{BP}C + \angle PBC = 180^\circ$
$\angle \mathrm{CB}P = 180^\circ - \angle \mathrm{BP}C - \angle PBC$
$\angle \mathrm{CBP} = 180^\circ - 40^\circ - 90^\circ = 50^\circ$
$\angle ADB = \angle \mathrm{CBP} = 50^\circ\ \ (alternate\ angle)$

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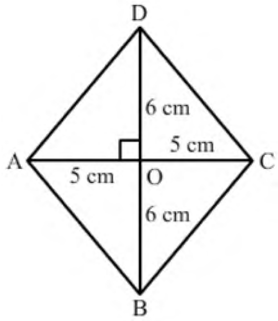
Question 6:

If the diagonals of a rhombus are 12 cm and 16cm, find the length of each side.

ANSWER:



All sides of a rhombus are equal in length.
The diagonals intersect at 90° and the sides of the rhombus form right triangles.
One leg of these right triangles is equal to 8 cm and the other is equal to 6 cm.
The sides of the triangle form the hypotenuse of these right triangles.
So, we get :
$\left(8^2+6^2\right)$ cm^2
$=(64+36)$ cm^2
$=100$ cm^2
The hypotneuse is the square root of $100\,\text{cm}^2$. This makes the hypotneuse equal to 10.
Thus, the side of the rhombus is equal to 10 cm.

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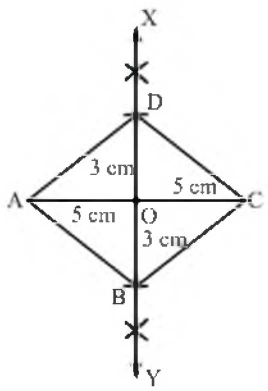
Question 7:

Construct a rhombus whose diagonals are of length 10 cm and 6 cm.

ANSWER:



1. Draw AC equal to 10 cm.
2. Draw XY, the right bisector of AC, meeting it at O.
3. With O as centre and radius equal to half of the length of the other diagonal, i.e. 3 cm, cut OB = OD = 3 cm.
4. Join AB, AD and CB, CD.

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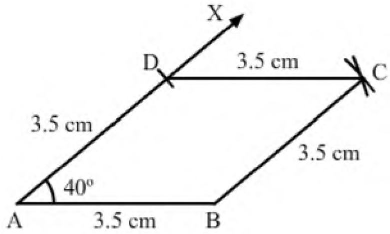
Question 8:

Draw a rhombus, having each side of length 3.5 cm and one of the angles as 40°.

ANSWER:



1. Draw a line segment AB of 3.5 cm.
2. Draw ∠BAX equal to 40˚.
3. With A as centre and the radius equal to AB, cut AD at 3.5 cm.
4. With D as centre, cut an arc of radius 3.5 cm.
5. With B as centre, cut an arc of radius 3.5 cm. This arc cuts the arc of step 4 at C.
6. Join DC and BC.

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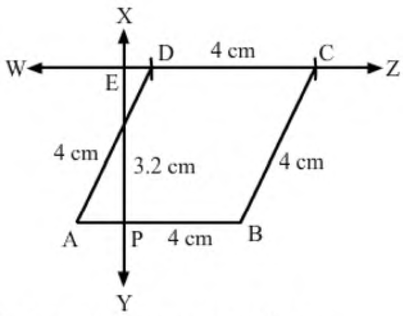
Question 9:

One side of a rhombus is of length 4 cm and the length of an altitude is 3.2 cm. Draw the rhombus.

ANSWER:



1. Draw a line segment AB of 4 cm.
2. Draw a perpendicular XY on AB, which intersects AB at P.
3. With P as centre, cut PE at 3.2 cm.
4. Draw a line WZ that passes through E. This line should be parallel to AB.
5. With A as centre, draw an arc of radius 4 cm that cuts WZ at D.
6. With D as centre and radius 4 cm, cut line DZ. Label it as point C.
4. Join AD and CB.

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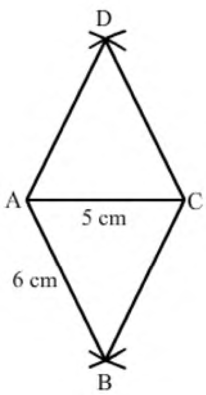
Question 10:

Draw a rhombus ABCD, if *AB* = 6 cm and *AC* = 5 cm.

ANSWER:



1. Draw a line segment AC of 5 cm.
2. With A as centre, draw an arc of radius 6 cm on each side of AC.

3. With C as centre, draw an arc of radius 6 cm on each side of AC. These arcs intersect the arcs of step 2 at B and D.
4. Join AB, AD, CD and CB.

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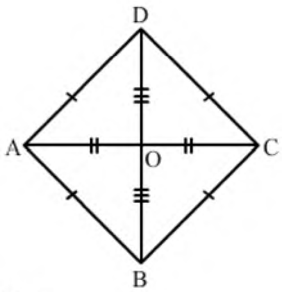
Question 11:

ABCD is a rhombus and its diagonals intersect at *O*.
(i) Is $\Delta BOC \cong \Delta DOC$? State the congruence condition used?
(ii) Also state, if $\angle BCO = \angle DCO$.

ANSWER:



(i) Yes

$$\begin{aligned}&\text{In } \Delta \text{BCO and } \Delta \text{DCO :}\\&\text{OC} = \text{OC (common)}\\&\text{BC} = \text{DC (all sides of a rhombus are equal)}\\&\text{BO} = \text{OD (diagonal} s \text{ of a rhomus bisect each other)}\\&\text{By SSS congruence :}\\&\Delta \text{BCO} \cong \Delta \text{DCO}\end{aligned}$$

(ii) Yes
By c.p.c.t:
$\angle BCO = \angle DCO$

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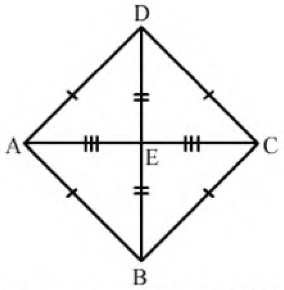
Question 12:

Show that each diagonal of a rhombus bisects the angle through which it passes.

ANSWER:



In $\Delta\, AED$ and $\Delta\, DEC$:

$AE = EC$ (diagonals bisect each other)

$AD = DC$ (sides are equal)

$DE = DE$ (common)

By SSS congruence :

$\Delta\, AED \cong \Delta\, CED$

$\angle ADE = \angle CDE$ (c. p. c. t)

Similarly, we can prove $\Delta\, AEB$ and $\Delta\, BEC$, $\Delta\, BEC$ and $\Delta\, DEC$, $\Delta\, AED$ and $\Delta\, AEB$ are congruent to each other.

Hence, diagonal of a rhombus bisects the angle through which it passes.

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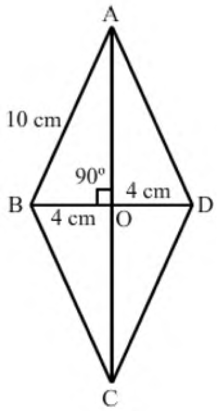
Question 13:

ABCD is a rhombus whose diagonals intersect at *O*. If *AB* = 10 cm, diagonal *BD* = 16 cm, find the length of diagonal *AC*.

ANSWER:



We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore\ BO = \frac{1}{2}BD = \left(\frac{1}{2} \times 16\right)$ cm

$= 8$ cm

$AB = 10$ cm and $\angle AOB = 90^\circ$

From right $\Delta\, OAB$:

$AB^2 = AO^2 + BO^2$

$\Rightarrow AO^2 = \left(AB^2 - BO^2\right)$

$\Rightarrow AO^2 = (10)^2 - (8)^2\ cm^2$

$\Rightarrow AO^2 = (100 - 64)\ cm^2 = 36\ cm^2$

$\Rightarrow AO = \sqrt{36}$ cm $= 6$cm

$\therefore\ AC = 2 \times AO = (2 \times 6)$ cm $= 12$ cm

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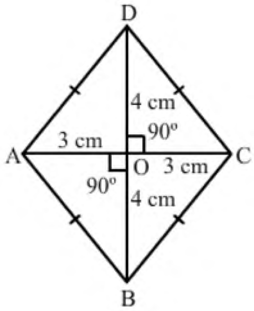
Question 14:

The diagonals of a quadrilateral are of lengths 6 cm and 8 cm. If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral?

ANSWER:



Let the given quadrilateral be ABCD in which diagonals AC is equal to 6 cm and BD is equal to 8 cm.

Also, it is given that the diagonals bisect each other at right angle, at point O.

$\therefore$ $AO = OC = \frac{1}{2}AC = 3$ cm

Also, $OB = OD = \frac{1}{2}BD = 4$ cm

In right $\triangle AOB$:

$AB^2 = OA^2 + OB^2$

$\Rightarrow AB^2 = (9 + 16)$ cm^2

$\Rightarrow AB^2 = 25$ cm^2

$\Rightarrow AB = 5$ cm

Thus, the length of each side of the quadrilateral is 5 cm.

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Question 1:

Which of the following statements are true for a rectangle?
(i) It has two pairs of equal sides.
(ii) It has all its sides of equal length.
(iii) Its diagonals are equal.
(iv) Its diagonals bisect each other.
(v) Its diagonals are perpendicular.
(vi) Its diagonals are perpendicular and bisect each other.
(vii) Its diagonals are equal and bisect each other.
(viii) Its diagonals are equal and perpendicular, and bisect each other.
(ix) All rectangles are squares.
(x) All rhombuses are parallelograms.
(xi) All squares are rhombuses and also rectangles.
(xii) All squares are not parallelograms.

ANSWER:

(i) True
(ii) False
(iii) True
(iv) True
(v) False
(vi) False
The diagonals are not perpendicular to each other.
(vii) True
(viii) False
The diagonals are not perpendicular to each other.
(ix) False
All sides are not equal.
(x) True
(xi) True
(xii) False
All squares are parallelogram.

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Question 2:

Which of the following statements are true for a square?
(i) It is a rectangle.
(ii) It has all its sides of equal length.
(iii) Its diagonals bisect each other at right angle.
(iv) Its diagonals are equal to its sides.

ANSWER:

(i) True
(ii) True
(iii) True
(iv) False
This is because the hypotenuse in any right angle triangle is always greater than its two sides.

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Question 3:

Fill in the blanks in each of the following, so as to make the statement true:
(i) A rectangle is a parallelogram in which .....
(ii) A square is a rhombus in which .....
(iii) A square is a rectangle in which .....

ANSWER:

(i) A rectangle is a parallelogram in which each angle is a right angle.

(ii) A square is a rhombus in which each angle is a right angle.

(iii) A square is a rectangle in which the adjacent sides are equal.

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Question 4:

A window frame has one diagonal longer than the other. Is the window frame a rectangle? Why or why not?

ANSWER:

No, since diagonals of a rectangle are equal.

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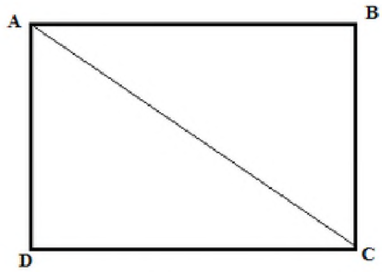
Question 5:

In a rectangle *ABCD*, prove that $\Delta ACB \cong \Delta CAD$.

ANSWER:



In ΔACB and ΔCAD:
AB = CD (rectangle property)

AD = BC (rectangle property)

AC (common side)

Hence, by SSS criterion, it is proved that $\Delta ACB \cong \Delta CAD$.

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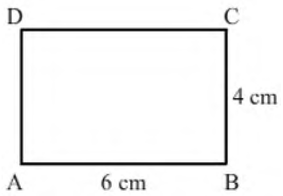
Question 6:

The sides of a rectangle are in the ratio 2 : 3, and its perimeter is 20 cm. Draw the rectangle.

ANSWER:



Let the side be x cm and y cm.
So, we have :
$2\left(x+y\right)=20$
Sides are in the ratio $2:3$.
$\therefore\ y=\frac{3x}{2}$
Putting the $value\ of\ y$:
$2\left(x+\frac{3x}{2}\right)=20$
$\frac{2x+3x}{2}=10$
$5x=20$
$x=4$
$\therefore\ y=\frac{3\times4}{2}=6$
Thus, sides of the rectangle will be 4 cm and 6 cm.
ABCD is the rectangle having side$s$ 4 cm and 6 cm.

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Question 7:

The sides of a rectangle are in the ratio 4 : 5. Find its sides if the perimeter is 90 cm.

ANSWER:

Let the side be x cm and $y\ cm$.
So, we have :
$2\,(x+y)=90$
Sides are in the ratio $4:5$.
$\therefore\ y=\frac{5x}{4}$
Putting the value of y :
$2\left(x+\frac{5x}{4}\right)=90$
$\frac{4x+5x}{4}=45$
$9x=180$
$x=20$
$\therefore\ y=\frac{5\times20}{4}=25$
Thus, the sides of the rectangle will be 20 cm and 25 cm.

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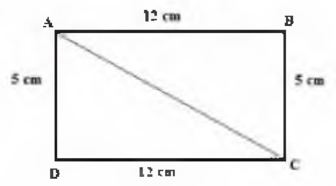
Question 8:

Find the length of the diagonal of a rectangle whose sides are 12 cm and 5 cm.

ANSWER:



Using Pythagoras theorem :
$AD^2+DC^2=AC^2$
$5^2+12^2=AC^2$
$25+14=AC^2$
$169=AC^2$
$AC=\sqrt{169}$
$=13$ cm
Thus, length of the diagonal is 13 cm.

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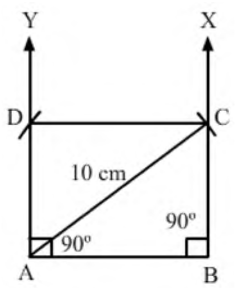
Question 9:

Draw a rectangle whose one side measures 8 cm and the length of each of whose diagonals is 10 cm.

ANSWER:

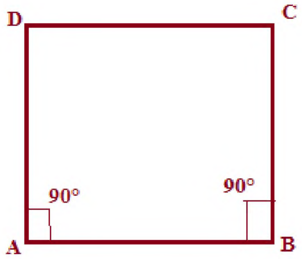
(i) Draw a side AB, equal to 8 cm.
(ii) With A as the centre, draw an arc of length 10 cm.
(iii) Draw ∠ABX = 90˚, which intersects the arc at C.
(iv)Draw ∠BAY = 90˚.
(v) With C as the centre, draw an arc of length 8 cm.
(vi) Join CD.
Thus, ABCD is the required rectangle.



Question 10:

Draw a square whose each side measures 4.8 cm.

ANSWER:



(i) Draw side $AB = 4.8$ cm.
(ii) From A, make an angle of 90° and cut it at 4.8 cm and mark it point D.
(iii) From B, make an angle of 90° and cut it at 4.8 cm and mark it point C.
(iv) Join C and D.
Thus, ABCD is the required square.

Question 11:

Identify all the quadrilaterals that have:
(i) Four sides of equal length
(ii) Four right angles

ANSWER:

(i) If all four sides are equal, then it can be either a square or a rhombus.

(ii) All four right angles, make it either a rectangle or a square.

Question 12:

Explain how a square is
(i) a quadrilateral?
(ii) a parallelogram?
(iii) a rhombus?
(iv) a rectangle?

ANSWER:

(i) Since a square has four sides, it is a quadrilateral.

(ii) Since the opposite sides are parallel and equal, it is a parallelogram.

(iii) Since the diagonals bisect each other and all the sides are equal, it is a rhombus.

(iv) Since the opposite sides are equal and all the angles are right angles, it is a rectangle.

Question 13:

Name the quadrilaterals whose diagonals:
(i) bisect each other
(ii) are perpendicular bisector of each other
(iii) are equal.

ANSWER:

(i) Rhombus, parallelogram, rectangle and square
(ii) Rhombus and square
(iii) Rectangle and square

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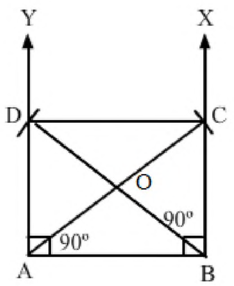
Question 14:

ABC is a right-angled trianle and *O* is the mid-point of the side opposite to the right angle. Explain why *O* is equidistant from *A*, *B* and *C*.

ANSWER:

(i) Construct a triangle ABC right angle at B.
(ii) Suppose O is the mid point of AC.
(iii) Complete the rectangle ABCD having AC as its diagonal.
Since diagonals of a rectangle are equal and they bisect each other, O is the midpoint of both AC and BD.
$\therefore$ OA = OB = OC



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Question 15:

A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

ANSWER:

(i) By measuring each angle - Each angle of a rectangle is 90°.

(ii) By measuring the length of the diagonals - Diagonals of a rectangle are equal.

(iii) By measuring the sides of rectangle - Each pair of opposite sides are equal.

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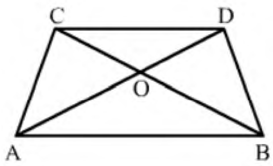
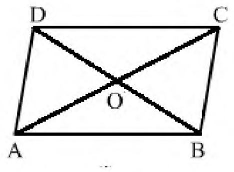
Question 1:

Given below is a parallelogram *ABCD*. Complete each statement along with the definition or property used.
(i) AD =
(ii) $\angle DCB$ =
(iii) OC =
(iv) $\angle DAB + \angle CDA$ =



ANSWER:

The correct figure is



(i)
$\text{AD} = \text{BC}$ (opposite sides of a parallelogram are equal)
(ii)
$\angle\text{DCB} = \angle\text{BAD}$ (opposite angles are equal)
(iii)
$\text{OC} = \text{OA}$ (diagonals of a prallelogram bisect each other)
(iv)
$\angle\text{DAB} + \angle\text{CDA} = 180^\circ$ (the sum of two adjacent angles of a parallelogram is 180^0)

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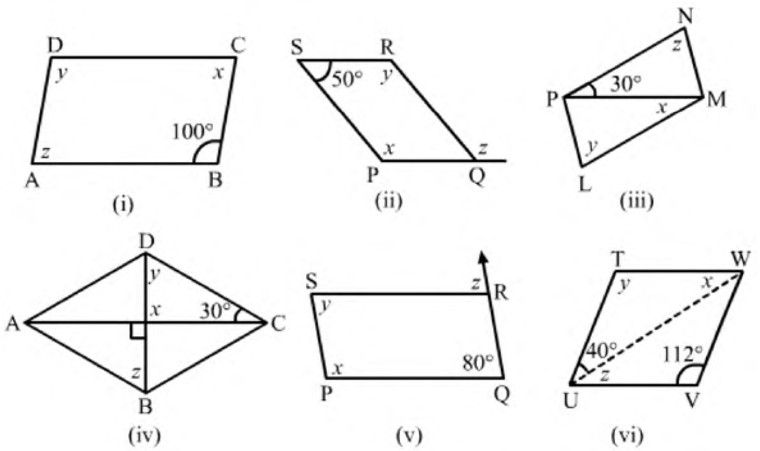
Question 2:

The following figures are parallelograms. Find the degree values of the unknowns *x*, *y*, *z*.



ANSWER:

(i)

Opposite angles of a parallelogram are same.

$\therefore$ $x = z$ and $y = 100^\circ$

Also, $y + z = 180^\circ$ (sum of adjacent angles of a quadrilateral is 180°)

$z + 100^\circ = 180^\circ$

$x = 180^\circ - 100^\circ$

$x = 80^\circ$

$\therefore$ $x = 80^\circ$, $y = 100^\circ$ and $z = 80^\circ$

(ii)

Opposite angles of a parallelogram are same.

$\therefore$ $x = y$ and $\angle RQP = 100^\circ$

$\angle PSR + \angle SRQ = 180^\circ$

$y + 50^\circ = 180^\circ$

$x = 180^\circ - 50^\circ$

$x = 130^\circ$

$\therefore$ $x = 130^\circ$, $y = 130^\circ$

Since y and z are alternate angles, $z = 130^\circ$.

(iii)

Sum of all angles in a triangle is $180°$.

$\therefore 30^\circ + 90^\circ + z = 180^\circ$

$z = 60^\circ$

Opposite angles are equal in parallelogram.

$\therefore$ $y = z = 60^\circ$

and $x = 30^\circ$ (alternate angles)

(iv) $x = 90^\circ$ (vertically opposite angle)

Sum of all angles in a triangle is $180°$.

$\therefore$ $y + 90^\circ + 30^\circ = 180^\circ$

$y = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

$y = z = 60^\circ$ (alternate angles)

(v)

Opposite angles are equal in *a* parallelogram.

$\therefore$ $y = 80^\circ$

$y + x = 180^\circ$

$x = 180^\circ - 100^\circ = 80^\circ$

$z = y = 80^\circ$ (alternate angles)

(vi)

$y = 112^\circ$ (opposite angles are equal in a parallelogram)

In Δ UTW :

$x + y + 40^\circ = 180^\circ$ (angle sum property of *a* triangle)

$x = 180^\circ - (112^\circ - 40^\circ) = 28^\circ$

Bottom left vertex $= 180^\circ - 112^\circ = 68^\circ$

$\therefore$ $z = x = 28^\circ$ (alternate angles)