Real Numbers Ex 2.1

RD Sharma Solutions Class 9 Chapter 2 Ex 2.1

Level 1

1. Simplify the following:

(i) $3(a^4 b^3)^{10} \times 5 (a^2 b^2)^3$

Solution:

$= 3(a^{40} b^{30}) \times 5 (a^6 b^6)$

$= 15 (a^{46} b^{36})$

(ii) $(2x^{-2} y^3)^3$

Solution:

= $(2^3 x^{-2\times3} y^{3\times3})$

= $8x^{-6} y^9$

(iii) $\frac{(4\times10^7)(6\times10^{-5})}{8\times10^4}$

Solution:

$\frac{(4\times10^7)(6\times10^{-5})}{8\times10^4}$

= $\frac{(24\times10^7\times10^{-5})}{8\times10^4}$

= $\frac{(24\times10^{7-5})}{8\times10^4}$

= $\frac{(24\times10^2)}{8\times10^4}$

= $\frac{(3\times10^2)}{10^4}$

= $\frac{3}{100}$

(iv) $\frac{4ab^2(-5ab^3)}{10a^2b^2}$

Solution:

= $\frac{-20a^2b^5}{10a^2b^2}$

= $-2b^3$

(v) $(\frac{x^2y^2}{a^2b^3})^n$

Solution:

= $\frac{x^{2n}y^{2n}}{a^{2n}b^{3n}}$

(vi) $\frac{(a^{3n-9})^6}{a^{2n-4}}$

Solution:

= $\frac{a^{18n-54}}{a^{2n-4}}$

= $a^{18n-2n-54+4}$

= a^{16n-50}

2. If a = 3 and b = -2, find the values of:

(i) $a^a + b^b$

(ii) $a^b + b^a$

(iii) $a^b + b^a$

Solution:

(i) We have,

$a^a + b^b$

= $3^3 + (-2)^{-2}$

= $3^3 + (-\frac{1}{2})^2$

= $27 + \frac{1}{4}$

= $\frac{109}{4}$

(ii) $a^b + b^a$

= $3^{-2} + (-2)^3$

= $(\frac{1}{3})^2 + (-2)^3$

= $\frac{1}{9} - 8$

= $-\frac{71}{9}$

(iii) We have,

$a^b + b^a$

= $(3 + (-2))^{3(-2)}$

= $(3 - 2))^{-6}$

= 1^{-6}

= 1

3.Prove that:

(i) $(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$ **= 1**

(ii) $(\frac{x^a}{x^{-b}})^{a^2-ab+b^2} \times (\frac{x^b}{x^{-c}})^{b^2-bc+c^2} \times (\frac{x^c}{x^{-a}})^{c^2-ca+a^2}$ **=** $x^{2(a^3+b^3+c^3)}$

(III) $(\frac{x^a}{x^b})^c \times (\frac{x^b}{x^c})^a \times (\frac{x^c}{x^a})^b$ **= 1**

Solution:

(i) To prove

$(\frac{x^a}{x^b})^{a^2+ab+b^2} \times (\frac{x^b}{x^c})^{b^2+bc+c^2} \times (\frac{x^c}{x^a})^{c^2+ca+a^2}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{x^{a^3+a^2b+ab^2}}{x^{a^2b+ab^2+b^3}} \times \frac{x^{b^3+b^2c+bc^2}}{x^{b^2c+bc^2+c^3}} \times \frac{x^{c^3+c^2a+ca^2}}{x^{c^2a+ca^2+a^3}}$

$x^{a^3+a^2b+ab^2-(b^3+a^2b+ab^2)} \times x^{b^3+b^2c+bc^2-(c^3+b^2c+bc^2)} \times x^{c^3+c^2a+ca^2-(a^3+c^2a+ca^2)}$

$x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3}$

$x^{a^3-b^3+b^3-c^3+c^3-a^3}$

x^0

1

Or,

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$(\frac{x^a}{x^{-b}})^{a^2-ab+b^2} \times (\frac{x^b}{x^{-c}})^{b^2-bc+c^2} \times (\frac{x^c}{x^{-a}})^{c^2-ca+a^2}$ = $x^{2(a^3+b^3+c^3)}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$x^{(a+b)(a^2-ab+b^2)} \times x^{(b+c)(b^2-bc+c^2)} \times x^{(c+a)(c^2-ca+a^2)}$

$x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3}$

$x^{a^3+b^3+b^3+c^3+c^3+a^3}$

$x^{2(a^3+b^3+c^3)}$

Therefore, LHS = RHS

Hence proved

(iii) To prove,

$(\frac{x^a}{x^b})^c \times (\frac{x^b}{x^c})^a \times (\frac{x^c}{x^a})^b$ =1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$(\frac{x^{ac}}{x^{bc}}) \times (\frac{x^{ba}}{x^{ca}}) \times (\frac{x^{bc}}{x^{ab}})$

$x^{ac-bc} \times x^{ba-ca} \times x^{bc-ab}$

$x^{ac-bc+ba-ca+bc-ab}$

x^0

1

Therefore, LHS = RHS

Hence proved

4. Prove that:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ **= 1**

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Solution:

(i) $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{1}{1+\frac{x^a}{x^b}} + \frac{1}{1+\frac{x^b}{x^a}}$

$\frac{x^b}{x^b+x^a} + \frac{x^a}{x^a+x^b}$

$\frac{x^b+x^a}{x^a+x^b}$

1

Therefore, LHS = RHS

Hence proved

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-c}+x^{a-c}}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$

$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a}$

$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$

1

Therefore, LHS = RHS

Hence proved

5.Prove that:

(i) $\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ **= abc**

(II) $(a^{-1}+b^{-1})^{-1}$

Solution:

(i) To prove,

$\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ = abc

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{a+b+c}{\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}}$

$\frac{a+b+c}{\frac{a+b+c}{abc}}$

abc

Therefore, LHS = RHS

Hence proved

(ii) To prove,

$(a^{-1}+b^{-1})^{-1} = \frac{ab}{a+b}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{1}{(a^{-1}+b^{-1})}$

$\frac{1}{(\frac{1}{a}+\frac{1}{b})}$

$\frac{1}{(\frac{a+b}{ab})}$

$\frac{ab}{a+b}$

Therefore, LHS = RHS

Hence proved

6. If abc = 1, show that $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}$ = 1

Solution:

To prove,

$\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$\frac{1}{1+a+\frac{1}{b}}+\frac{1}{1+b+\frac{1}{c}}+\frac{1}{1+c+\frac{1}{a}}$

$\frac{b}{b+ab+1}+\frac{c}{c+bc+1}+\frac{a}{a+ac+1}$ ....(1)

We know abc = 1

c = $\frac{1}{ab}$

By substituting the value c in equation (1), we get

$\frac{b}{b+ab+1}+\frac{\frac{1}{ab}}{\frac{1}{ab}+b(\frac{1}{ab})+1}+\frac{a}{a+a(\frac{1}{ab})+1}$

$\frac{b}{b+ab+1}+\frac{\frac{1}{ab}\times ab}{1+b+ab}+\frac{ab}{1+ab+b}$

$\frac{b}{b+ab+1}+\frac{1}{1+b+ab}+\frac{ab}{1+ab+b}$

$\frac{1+ab+b}{b+ab+1}$

1

Therefore, LHS = RHS

Hence proved

7. Simplify:

(i) $\frac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

Solution:

= $\frac{3^n\times 9^n\times 9}{\frac{3^n}{3}\times\frac{9^n}{9}}$

= 9 x 3 x 9

= 243

(ii) $\frac{(5\times 25^{n+1})(25\times 5^{2n})}{(5\times 5^{2n+3})-(25)^{n+1}}$

Solution:

= $\frac{(5\times 25^n\times 25)-(25\times 25^n)}{(5\times 25^n\times 125)(25^n\times 25)}$

= $\frac{25^n\times 25(5-1)}{25^n\times 25(25-1)}$

= $\frac{4}{24}$

= $\frac{1}{6}$

(iii) $\frac{(5^{n+3})-(6\times 5^{n+1})}{(9\times 5^n)-(2^2\times 5^n)}$

Solution:

= $\frac{(5^{n+3})-(6\times 5^{n+1})}{(9\times 5^n)-(2^2\times 5^n)}$

= $\frac{(5^n\times 5^3)-(6\times 5^n\times 5)}{(9\times 5^n)-(2^2\times 5^n)}$

= $\frac{5^n(125-30)}{5^n(9-4)}$

= $\frac{95}{5}$

= 19

(iv) $\frac{(6\times8^{n+1})+(16\times2^{3n-2})}{(10\times2^{3n+1})-7\times(8)^{n}}$

Solution:

= $\frac{(6\times8^{n}\times8)+(16\times8^{n}\times\frac{1}{4})}{(10\times8^{n}\times2)-(7\times(8)^{n})}$

= $\frac{8^{n}(48+4)}{8^{n}(20-7)}$

= $\frac{52}{13}$

= 4

Level 2

8. Solve the following equations for x:

(i) 7^{2x+3} **= 1**

(ii) $2^{x+1}=4^{x-3}$

(iii) $2^{5x+3}=8^{x+3}$

(iv) $4^{2x}=\frac{1}{32}$

(v) $4^{x-1}\times(0.5)^{3-2x}=(\frac{1}{8})^{x}$

(vi) 2^{3x-7} **= 256**

Solution:

(i) We have,

=> 7^{2x+3} = 1

=> 7^{2x+3} = 7^0

=> 2x + 3 = 0

=> 2x = -3

=> x = $-\frac{3}{2}$

(ii) We have,

$2^{x+1}=4^{x-3}$

$2^{x+1}=2^{2x-6}$

x + 1 = 2x − 6

x = 7

(iii) We have,

$2^{5x+3}=8^{x+3}$

$2^{5x+3}=2^{3x+9}$

5x + 3 = 3x + 9

2x = 6

x = 3

(iv) We have,

$4^{2x}=\frac{1}{32}$

$2^{4x}=\frac{1}{2^5}$

$2^{4x}=2^{-5}$

4x = -5

x = $\frac{-5}{4}$

(v) We have,

$4^{x-1}\times(0.5)^{3-2x}=(\frac{1}{8})^{x}$

$2^{2x-2}\times(\frac{1}{2})^{3-2x}=(\frac{1}{2})^{3x}$

$2^{2x-2}\times2^{2x-3}=(\frac{1}{2})^{3x}$

$2^{2x-2+2x-3}=(\frac{1}{2})^{3x}$

$2^{4x-5}=2^{-3x}$

4x-5 = -3x

7x = 5

x = $\frac{5}{7}$

(vi) 2^{3x-7} = 256

2^{3x-7} = 2^8

3x − 7 = 8

3x = 15

x = 5

9. Solve the following equations for x:

(i) $2^{2x}-2^{x+3}+2^4=0$

(ii) $3^{2x+4}+1=2\times3^{x+2}$

Solution:

(i) We have,

=> $2^{2x}-2^{x+3}+2^4=0$

=> $2^{2x}+2^4=2^x.2^3$

=> Let $2^x=y$

=> $y^2+2^4=y\times2^3$

=> $y^2-8y+16=0$

=> $y^2-4y-4y+16=0$

=> y(y-4) -4(y-4) = 0

=> y = 4

=> $x^2=2^2$

=> x = 2

(ii) We have,

$3^{2x+4}+1=2\times3^{x+2}$

$(3^{x+2})^2+1=2\times3^{x+2}$

Let 3^{x+2} = y

$y^2+1=2y$

$y^2-2y+1=0$

$y^2-y-y+1=0$

$y(y-1)-1(y-1)=0$

$(y-1)(y-1)=0$

y = 1

10. If 49392 = $a^4b^2c^3$, find the values of a, b and c, where a, b and c, where a, b, and c are different positive primes.

Solution:

Taking out the LCM , the factors are $2^4,3^2and7^3$

$a^4b^2c^3$ = $2^4,3^2and7^3$

a = 2, b = 3 and c = 7 [Since, a, b and c are primes]

11. If 1176 = $2^a\times3^b\times7^c$, Find a, b, and c.

Solution:

Given that 2, 3 and 7 are factors of 1176.

Taking out the LCM of 1176, we get

$2^3\times3^1\times7^2$ = $2^a\times3^b\times7^c$

By comparing, we get

a = 3, b = 1 and c = 2.

12. Given 4725 = $3^a\times5^b\times7^c$, find

(i) The integral values of a, b and c

Solution:

Taking out the LCM of 4725, we get

$3^3 \times 5^2 \times 7^1 = 3^a \times 5^b \times 7^c$

By comparing, we get

a = 3, b = 2 and c = 1.

(ii) The value of $2^{-a} \times 3^b \times 7^c$

Solution:

\(2^{-a} \times 3^{b}\times 7^{c} = [latex]2^{-3}\times 3^{2}\times 7^{1}\)\(2^{-3}\times 3^{2}\times 7^{1}\]">
[latex]2^{-3}\times 3^{2}\times 7^{1}\) = $\frac{1}{8} \times 9 \times 7$

$\frac{63}{8}$

13. If a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}, prove that $a^{q-r}b^{r-p}c^{p-q}$ = 1

Solution:

Given,

a = xy^{p-1}, b = xy^{q-1} and c = xy^{r-1}

To prove, $a^{q-r}b^{r-p}c^{p-q}$ = 1

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

= $a^{q-r}b^{r-p}c^{p-q}$ ......(i)

By substituting the value of a, b and c in equation (i), we get

= $(xy^{p-1})^{q-r}(xy^{q-1})^{r-p}(xy^{r-1})^{p-q}$

= $xy^{pq-pr-q+r}xy^{qr-pq-r+p}xy^{rp-rq-p+q}$

= $xy^{pq-pr-q+r+qr-pq-r+p+rp-rq-p+q}$

= xy^0

= 1

Real Numbers Ex 2.2

RD Sharma Solutions Class 9 Chapter 2 Ex 2.2

Level 1

1. Assuming that x,y,z are positive real numbers, simplify each of the following

(i)

$\left(\sqrt{(x^{-3})}\right)^5$

$\left(\sqrt{(x^{-3})}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$

$\left(\frac{1}{x^{\frac{3}{2}}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$

$\left(\sqrt{(x^{-3})}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$

(ii)

$\sqrt{x^3y^{-2}}$

$\sqrt{x^3y^{-2}} = \sqrt{\frac{x^3}{y^2}}$

$= \left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$

$= \frac{x^{3\times\frac{1}{2}}}{y^{2\times\frac{1}{2}}}$

$\frac{x^{\frac{3}{2}}}{y}$

$\sqrt{x^3y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$

(iii)

$\left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2$

$= \left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2 = \left(\frac{1}{x^{\frac{2}{3}}y^{\frac{1}{2}}}\right)^2$

$= \left(\frac{1}{x^{\frac{2}{3}\times 2}y^{\frac{1}{2}\times 2}}\right)$

$= \frac{1}{x^{\frac{4}{3}}y}$

(iv)

$(\sqrt{x})^{-\frac{2}{3}}\sqrt{y^4} \div \sqrt{xy^{\frac{1}{2}}}$

$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}}\left(y^2\right) \div \sqrt{xy^{\frac{1}{2}}}$

$= \frac{x^{\frac{1}{2}\times\frac{2}{3}}y^2}{\left(xy^{\frac{1}{2}}\right)^{\frac{1}{2}}}$

$= \frac{x^{-\frac{1}{3}}y^2}{x^{\frac{1}{2}}y^{-\frac{1}{2}\times\frac{1}{2}}}$

$= \left(x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}\right) \times \left(y^2 \times y^{\frac{1}{4}}\right)$

$= \left(x^{-\frac{1}{3}-\frac{1}{2}}\right)\left(y^{2+\frac{1}{4}}\right)$

$= \left(x^{\frac{-2-3}{6}}\right)\left(y^{\frac{8+1}{4}}\right)$

$= \left(x^{-\frac{5}{6}}\right)\left(y^{-\frac{9}{4}}\right)$

$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$

(v)

$\sqrt[5]{243x^{10}y^5z^{10}}$

$= \left(243x^{10}y^5z^{10}\right)^{\frac{1}{5}}$

$= (243)^{\frac{1}{5}}x^{\frac{10}{5}}y^{\frac{5}{5}}z^{\frac{10}{5}}$

$= \left(3^5\right)^{\frac{1}{5}}x^2yz^2$

$= 3x^2yz^2$

(vi)

$\left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$

$=\left(\frac{y^{10}}{x^{4}}\right)^{\frac{5}{4}}$

$=\left(\frac{y^{10\times\frac{5}{4}}}{x^{4\times\frac{5}{4}}}\right)$

$=\left(\frac{y^{\frac{25}{2}}}{x^{5}}\right)$

(vii)

$\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^{5}\left(\frac{6}{7}\right)^{2}$

$=\left(\sqrt{\frac{2}{3}}\right)^{5}\left(\frac{6}{7}\right)^{\frac{4}{2}}$

$=\left(\frac{2}{3}\right)^{\frac{5}{2}}\left(\frac{6}{7}\right)^{\frac{4}{2}}$

$=\left(\frac{2^5}{3^5}\right)^{\frac{1}{2}}\left(\frac{6^4}{7^4}\right)^{\frac{1}{2}}$

$=\left(\frac{2^5}{3^5}\times\frac{6^4}{7^4}\right)^{\frac{1}{2}}$

$=\left(\frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3}\times\frac{6\times6\times6\times6}{7\times7\times7\times7}\right)$

$=\left(\frac{512}{7203}\right)$

2. Simplify

(i)

$\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$

$=(16)^{-\frac{1}{5}\times\frac{5}{2}}$

$=(16)^{-\frac{1}{2}}$

$=\left(4^{2}\right)^{-\frac{1}{2}}$

$=\left(4^{2\times-\frac{1}{2}}\right)$

$=\left(4^{-1}\right)$

$=\frac{1}{4}$

(ii) $\sqrt[5]{(32)^{-3}}$

$\sqrt[5]{(32)^{-3}}$

$=\left[\left(2^{5}\right)^{-3}\right]^{\frac{1}{5}}$

$=\left(2^{-15}\right)^{\frac{1}{5}}$

$=2^{-3}$

$=\frac{1}{2^3}$

$=\frac{1}{8}$

(iii)

$$\sqrt[3]{(343)^{-2}}$$

$$= \left[(343)^{-2}\right]^{\frac{1}{3}}$$

$$= (343)^{-2\times\frac{1}{3}}$$

$$= \left(7^3\right)^{-\frac{2}{3}}$$

$$= \left(7^{-2}\right)$$

$$= \left(\frac{1}{7^2}\right)$$

$$= \left(\frac{1}{49}\right)$$

(iv)

$$(0.001)^{\frac{1}{3}}$$

$$= \left(\frac{1}{1000}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{10^3}\right)^{\frac{1}{3}}$$

$$= \left(\frac{1^{\frac{1}{3}}}{\left(10^3\right)^{\frac{1}{3}}}\right)$$

$$\frac{1}{10^{3\times\frac{1}{3}}}$$

$$= \frac{1}{10} = 0.1$$

(v)

$$\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

$$= \frac{\left(\left(5^2\right)\right)^{\frac{3}{2}}\times\left(\left(3^5\right)\right)^{\frac{3}{5}}}{\left(\left(4^2\right)\right)^{\frac{5}{4}}\times\left(\left(4^2\right)\right)^{\frac{4}{3}}}$$

$$= \frac{5^{2\times\frac{3}{2}}\times3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3\times3^3}{2^5\times2^4}$$

$$= \frac{125\times27}{32\times16}$$

$$= \frac{3375}{512}$$

(vi) $\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$

$$= \frac{\left(\frac{\sqrt{2}}{5}\right)^8}{\left(\frac{\sqrt{2}}{5}\right)^{13}}$$

$$= \left(\frac{\sqrt{2}}{5}\right)^{8-13}$$

$$= \left(\frac{\sqrt{2}}{5}\right)^{-5}$$

$$= \frac{\left(2^{\frac{1}{2}}\right)^{-5}}{(5)^{-5}}$$

$$= \frac{\left(2^{\frac{1}{2}\times-5}\right)}{(5)^{-5}}$$

$$= \frac{1}{2^{\frac{5}{2}}} \times \frac{5^5}{1}$$

$$= \frac{5^5}{2^{\frac{5}{2}}}$$

$$= \frac{3125}{4\sqrt{2}}$$

(vii)

$$\left[\frac{5^{-1}\times7^{2}}{5^{2}\times7^{-4}}\right]^{\frac{7}{2}}\times\left[\frac{5^{-2}\times7^{3}}{5^{3}\times7^{-5}}\right]^{\frac{-5}{2}}$$

$$=\frac{(5^{-1}\times7^{2})^{\frac{7}{2}}}{\left(5^{2}\times7^{-4}\right)^{\frac{7}{2}}}\times\frac{(5^{-2}\times7^{3})^{\frac{-5}{2}}}{(5^{3}\times7^{-5})^{\frac{-5}{2}}}$$

$$=\frac{(5^{-1})^{\frac{7}{2}}\times(7^{2})^{\frac{7}{2}}}{(5^{2})^{\frac{7}{2}}\times\left(7^{-4}\right)^{\frac{7}{2}}}\times\frac{(5^{-2})^{\frac{-5}{2}}\times(7^{3})^{\frac{-5}{2}}}{(5^{3})^{\frac{-5}{2}}\times\left(7^{-5}\right)^{\frac{-5}{2}}}$$

$$=\frac{5^{-\frac{7}{2}}\times7^{7}}{5^{7}\times7^{-14}}\times\frac{5^{5}\times7^{-\frac{15}{2}}}{5^{-\frac{15}{2}}\times7^{-\frac{25}{2}}}$$

$$=\frac{7^{7+\frac{7}{14}}}{5^{7+\frac{7}{2}}}\times\frac{5^{5+\frac{15}{2}}}{7^{\frac{15}{2}+\frac{25}{2}}}$$

$$=\frac{7^{21}}{5^{\frac{21}{2}}}\times\frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}}$$

$$=\frac{7^{21}}{7^{20}}\times\frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}}$$

$$=7^{21-20}\times5^{\frac{25}{2}-\frac{21}{2}}$$

$$=7^{1}\times5^{\frac{4}{2}}$$

$$=7^{1}\times5^{2}$$

$$=7\times25$$

$$=175$$

3. Prove that

(i) $\left(\sqrt{3\times5^{-3}}\div\sqrt[3]{3^{-1}}\sqrt{5}\right)\times\sqrt[5]{3\times5^{6}}=\frac{3}{5}$

$$\left(\sqrt{3\times5^{-3}}\div\sqrt[3]{3^{-1}}\sqrt{5}\right)\times\sqrt[5]{3\times5^{6}}$$

$$=\left(\left(3\times5^{-3}\right)^{\frac{1}{2}}\div\left(3^{-1}\right)^{\frac{1}{3}}(5)^{\frac{1}{2}}\right)\times\left(3\times5^{6}\right)^{\frac{1}{6}}$$

$$=\left((3)^{\frac{1}{2}}\left(5^{-3}\right)^{\frac{1}{2}}\div\left(3^{-1}\right)^{\frac{1}{3}}(5)^{\frac{1}{2}}\right)\times\left(3\times5^{6}\right)^{\frac{1}{6}}$$

$$=\left((3)^{\frac{1}{2}}(5)^{\frac{-3}{2}}\div(3)^{\frac{-1}{3}}(5)^{\frac{1}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)^{\frac{6}{6}}\right)$$

$$=\left((3)^{\frac{1}{2}-\left(-\frac{1}{3}\right)}\times(5)^{-\frac{3}{2}-\frac{1}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{3+2}{6}}\times(5)^{-\frac{4}{2}}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{5}{6}}\times(5)^{-2}\right)\times\left((3)^{\frac{1}{6}}\times(5)\right)$$

$$=\left((3)^{\frac{5}{6}+\frac{1}{6}}\times(5)^{-2+1}\right)$$

$$=\left((3)^{\frac{6}{6}}\times(5)^{-1}\right)$$

$$=\left((3)^{1}\times(5)^{-1}\right)$$

$$=\left((3)\times(5)^{-1}\right)$$

$$=\left((3)\times\left(\frac{1}{5}\right)\right)$$

$$=\left(\frac{3}{5}\right)$$

(ii)

$$9^{\frac{3}{2}}-3\times5^{0}-\left(\frac{1}{81}\right)^{-\frac{1}{2}}$$

$$=\left(3^{2}\right)^{\frac{3}{2}}-3-\left(\frac{1}{9^{2}}\right)^{-\frac{1}{2}}$$

$$=3^{2\times\frac{3}{2}}-3-\left(9^{-2}\right)^{-\frac{1}{2}}$$

$$=3^{3}-3-(9)^{-2\times-\frac{1}{2}}$$

$$=27-3-9$$

$$=15$$

(iii)

$$\frac{1}{4}^{2}-3\times 8^{\frac{2}{3}}\times 4^{0}+\left(\frac{9}{16}\right)^{-\frac{1}{2}}$$

$$=\left(\frac{1}{2^{2}}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^{2}}{4^{2}}\right)^{-\frac{1}{2}}$$

$$=\left(2^{-2}\right)^{-2}-3\times 8^{\frac{2}{3}}\times 1+\left(\frac{3^{2\times-\frac{1}{2}}}{4^{2\times-\frac{1}{2}}}\right)$$

$$=2^{4}-3\times 2^{3\times\frac{2}{3}}+\frac{4}{3}$$

$$=16-3\times 2^{2}+\frac{4}{3}$$

$$=16-3\times 4+\frac{4}{3}$$

$$=16-12+\frac{4}{3}$$

$$=\frac{12+4}{3}$$

$$=\frac{16}{3}$$

(iv)

$$\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}}\times 5^{\frac{3}{5}}}\div\frac{4^{\frac{4}{3}}\times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}}\times 6}$$

$$=\frac{2^{\frac{1}{2}}\times 3^{\frac{1}{3}}\times\left(2^{2}\right)^{\frac{1}{4}}\left(2^{2}\right)^{-\frac{3}{5}}\times(2\times 3)}{(2\times 5)^{-\frac{1}{5}}\times 5^{\frac{3}{5}}\times 3^{\frac{4}{3}}\times 5^{-\frac{7}{5}}}$$

$$=\frac{2^{\frac{1}{2}}\times 2^{\frac{1}{2}}\times\left(2^{2}\right)^{-\frac{6}{5}}\times 2^{1}\times 3^{\frac{1}{3}}\times 3}{2^{-\frac{1}{5}}\times 5^{-\frac{1}{5}}\times 5^{\frac{3}{5}}\times 3^{\frac{4}{3}}\times 5^{-\frac{7}{5}}}$$

$$=\frac{2^{\frac{1}{5}}\times 2^{\frac{1}{2}}\times 2^{\frac{1}{2}}\times 2^{-\frac{6}{5}}\times 2\times 3^{\frac{1}{3}}\times 3\times 3^{-\frac{4}{3}}}{5^{-\frac{1}{5}}\times 5^{\frac{3}{5}}\times 5^{-\frac{7}{5}}}$$

$$=\frac{(2)^{\frac{1}{2}+\frac{1}{2}-\frac{6}{5}+1+\frac{1}{5}}\times(3)^{\frac{1}{3}+1-\frac{4}{3}}}{5^{-\frac{1}{5}}\times 5^{\frac{3}{5}}\times 5^{-\frac{7}{5}}}$$

$$=\frac{(2)^{\frac{1}{5}+1-\frac{6}{5}+1}\times(3)^{1-\frac{3}{3}}}{5^{-\frac{5}{5}}}$$

$$=\frac{(2)^{\frac{1}{5}+2-\frac{6}{5}}\times(3)^{1-1}}{5^{-1}}$$

$$=\frac{(2)^{2-1}\times(3)^{1-1}}{5^{-1}}$$

$$=\frac{(2)^{1}\times(3)^{0}}{5^{-1}}$$

$$=2\times 1\times 5$$

$$=10$$

(v)

$$\sqrt{\frac{1}{4}}+(0.01)^{-\frac{1}{2}}-(27)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{(0.01)^{\frac{1}{2}}}-\left(3^{3}\right)^{\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{(0.1)^{2\times\frac{1}{2}}}-(3)^{3\times\frac{2}{3}}$$

$$=\frac{1}{2}+\frac{1}{(0.1)^{1}}-(3)^{2}$$

$$=\frac{1}{2}+\frac{1}{(0.1)}-9$$

$$=\frac{1}{2}+10-9$$

$$=\frac{1}{2}+1$$

$$=\frac{3}{2}$$

(vi)

$\frac{2^n+2^{n-1}}{2^{n+1}-2^n}$

$=\frac{2^n+2^n\times2^{-1}}{2^n\times2^1-2^n}$

$=\frac{2^n\left[1+2^{-1}\right]}{2^n[2-1]}$

$=\frac{1+\frac{1}{2}}{1}$

$=1+\frac{1}{2}$

$=\frac{3}{2}$

(vii)

$\left(\frac{64}{125}\right)^{-\frac{2}{3}}+\frac{1}{\left(\frac{256}{625}\right)^{-\frac{1}{4}}}+\left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)$

$=\left(\frac{125}{64}\right)^{\frac{2}{3}}+\frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}}+\left(\frac{5}{(64)^{\frac{1}{3}}}\right)$

$=\left(\frac{5^3}{4^3}\right)^{\frac{2}{3}}+\frac{1}{\left(\frac{4}{5}\right)}+\left(\frac{5}{(4^3)^{\frac{1}{3}}}\right)$

$=\left(\frac{5}{4}\right)^2+\frac{5}{4}+\frac{5}{4}$

$=\frac{25}{16}+\frac{10}{4}$

$=\frac{25}{16}+\frac{40}{16}$

$=\frac{26+40}{16}$

$=\frac{65}{16}$

(viii)

$\frac{3^{-3}\times6^2\times\sqrt{98}}{5^2\times\sqrt[3]{\frac{1}{25}}\times(15)^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times\sqrt{7\times7\times2}}{5^2\times\left(\frac{1}{25}\right)^{\frac{1}{3}}\times(15)^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}}{5^2\times\left(\frac{1}{5^{2\times\frac{1}{3}}}\right)\times\frac{1}{(15)^{\frac{4}{3}}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}}{5^2\times5^{-\frac{2}{3}}\times\frac{1}{(5\times3)^{\frac{4}{3}}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}}{5^2\times5^{-\frac{2}{3}}\times5^{\frac{4}{3}}\times3^{\frac{4}{3}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}}{\left(5^2\times5^{-\frac{2}{3}}\times5^{-\frac{4}{3}}\right)\times3^{-\frac{4}{3}}\times3^{\frac{1}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}\times3^{\frac{4}{3}}\times3^{\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}}$

$=\frac{3^{-3}\times36\times7\sqrt{2}\times3^{\frac{4}{3}}\times3^{\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}}$

$=\frac{3^{-3+\frac{4}{3}-\frac{1}{3}}\times36\times7\sqrt{2}}{(5)^0}$

$=3^{-3+\frac{3}{3}}\times36\times7\sqrt{2}$

$=3^{-3+1}\times36\times7\sqrt{2}$

$=3^{-2}\times36\times7\sqrt{2}$

$=\frac{1}{3^2}\times36\times7\sqrt{2}$

$=\frac{1}{9}\times36\times7\sqrt{2}$

$=4\times7\sqrt{2}$

$=28\sqrt{2}$

(ix)

$$\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+(-3)^1}$$

$$=\frac{1-\frac{1}{0.1}}{\frac{8}{3}\times\left(\frac{3}{2}\right)^3-3}$$

$$=\frac{1-10}{\frac{8}{3}\times\frac{3^3}{2^3}-3}$$

$$=\frac{-9}{3^2-3}$$

$$=\frac{-9}{9-3}$$

$$=\frac{-9}{6}$$

$$=-\frac{3}{2}$$

4. Show that

(i) $\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}}=1$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^a}{x^b}}+\frac{1}{1+\frac{x^b}{x^a}}$$

$$\frac{x^b}{x^b+x^a}+\frac{x^a}{x^a+x^b}$$

$$\frac{x^b+x^a}{x^a+x^b}$$

$$1$$

Therefore, LHS = RHS

Hence proved

(ii) $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}+\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$\frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^b+x^a+x^c}+\frac{x^c}{x^c+x^b+x^a}$$

$$\frac{x^a+x^b+x^c}{x^a+x^b+x^c}$$

$$1$$

Therefore, LHS = RHS

Hence proved

(ii)

$$\left[\left(\frac{x^{a(a-b)}}{x^{a(a+b)}}\right)\div\left(\frac{x^{b(b-a)}}{x^{b(b+a)}}\right)\right]^{a+b}$$

$$\left[\left(\frac{x^{a(a-b)}}{x^{a(a+b)}}\right)\div\left(\frac{x^{b(b-a)}}{x^{b(b+a)}}\right)\right]^{a+b}$$

$$=\left[\left(\frac{x^{a^2-ab}}{x^{a^2+ab}}\right)\div\left(\frac{x^{b^2-ab}}{x^{b^2+ab}}\right)\right]^{a+b}$$

$$=\left[x^{(a^2-ab)-(a^2-ab)}\div x^{(b^2-ab)-(b^2-ab)}\right]^{a+b}$$

$$=\left[x^{-2ab}\div x^{-2ab}\right]^{a+b}$$

$$=\left[x^{-2ab-(-2ab)}\right]^{a+b}$$

$$=\left[x^{-2ab+2ab}\right]^{a+b}$$

$$=\left[x^0\right]^{a+b}$$

$$=[1]^{a+b}$$

$$=1$$

(iii)

$$\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}}\left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}}\left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$$

$$= \left(x^{\frac{1}{(a-b)(a-c)}}\right)\left(x^{\frac{1}{(b-c)(b-a)}}\right)\left(x^{\frac{1}{(c-a)(c-b)}}\right)$$
$$= x^{\frac{1}{(a-b)(a-c)}+\frac{1}{(b-c)(b-a)}+\frac{1}{(c-a)(c-b)}}$$
$$= x^{\frac{1}{(a-b)(a-c)}+\frac{-1}{(b-c)(a-b)}+\frac{1}{(a-c)(b-c)}}$$
$$= x^{\frac{(b-c)}{(a-b)(a-c)(b-c)}+\frac{-(a-c)}{(b-c)(a-b)(a-c)}+\frac{(a-b)}{(a-c)(b-c)(a-b)}}$$
$$= x^{\frac{b-c-a+c+a-b}{(a-b)(a-c)(b-c)}}$$

$$= x^{\frac{0}{(a-b)(a-c)(b-c)}}$$
$$= x^0 = 1$$

(iv) $\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c} = 2\left(a^3+b^3+c^3\right)$

$$\left(\frac{x^{a^2+b^2}}{x^{ab}}\right)^{a+b}\left(\frac{x^{b^2+c^2}}{x^{bc}}\right)^{b+c}\left(\frac{x^{c^2+a^2}}{x^{ac}}\right)^{a+c}$$
$$= \left(x^{a^2+b^2-ab}\right)^{a+b}\left(x^{b^2+c^2-bc}\right)^{b+c}\left(x^{c^2+a^2-ac}\right)^{a+c}$$
$$= \left(x^{a+b\left(a^2+b^2-ab\right)}\right)\left(x^{b+c\left(b^2+c^2-bc\right)}\right)\left(x^{a+c\left(c^2+a^2-ac\right)}\right)$$
$$= \left(x^{a^3+ab^2-a^2b+ab^2+b^3-ab^2}\right)\left(x^{b^3+bc^2-b^2c+cb^2+c^3-bc^2}\right)\left(x^{ac^2+a^3-a^2c+c^3+a^2c-ac^2}\right)$$
$$= \left(x^{a^3+b^3}\right)\left(x^{b^3+c^3}\right)\left(x^{a^3+c^3}\right)$$
$$= \left(x^{a^3+b^3+b^3+c^3+a^3+c^3}\right)$$
$$= \left(x^{2a^3+2b^3+2c^3}\right)$$
$$= \left(x^{2\left(a^3+b^3+c^3\right)}\right)$$

(v) $\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a} = 1$

$$\left(x^{a-b}\right)^{a+b}\left(x^{b-c}\right)^{b+c}\left(x^{c-a}\right)^{c+a}$$
$$= x^{a^2-b^2}x^{b^2-c^2}x^{c^2-a^2}$$
$$= x^{a^2-b^2+b^2-c^2+c^2-a^2}$$
$$= x^0$$
$$= 1$$

(vi) $\left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}} = x$

$$\left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}}$$
$$= \left[\left(x^{\frac{a-a^{-1}}{a-1}}\right)\right]^{\frac{a}{a+1}}$$
$$= \left[\left(x^{\frac{a-a^{-1}}{a-1}}\right)\right]^{\frac{a}{a+1}}$$
$$= \left(x^{\frac{a\left(a-a^{-1}\right)}{a^2-1}}\right)$$
$$= \left(x^{\frac{a^2-a^{-1+1}}{a^2-1}}\right)$$
$$= \left(x^{\frac{a^2-a^0}{a^2-1}}\right)$$
$$= \left(x^{\frac{a^2-1}{a^2-1}}\right)$$
$$= x^1 = x$$

(vii) $\left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y}\left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z}\left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x} = 1$

$\left[\frac{a^{x+1}}{a^{y+1}}\right]^{x+y}\left[\frac{a^{y+2}}{a^{z+2}}\right]^{y+z}\left[\frac{a^{z+3}}{a^{x+3}}\right]^{z+x}$
$=\left[a^{(x+1)-(y+1)}\right]^{x+y}\left[a^{(y+2)-(z+2)}\right]^{y+z}\left[a^{(z+3)-(x+3)}\right]^{z+x}$
$=[a^{x-y}]^{x+y}[a^{y-z}]^{y+z}[a^{z-x}]^{z+x}$
$=\left[a^{x^2-y^2}\right]\left[a^{y^2-z^2}\right]\left[a^{z^2-x^2}\right]$
$=a^{x^2-y^2+y^2-z^2+z^2-x^2}=a^0$
$=1$

(viii) $\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a}=1$

$\left(\frac{3^a}{3^b}\right)^{a+b}\left(\frac{3^b}{3^c}\right)^{b+c}\left(\frac{3^c}{3^a}\right)^{c+a}$
$=\left(3^{a-b}\right)^{a+b}\left(3^{b-c}\right)^{b+c}\left(3^{c-a}\right)^{c+a}$
$=3^{a^2-b^2}\times3^{b^2-c^2}\times3^{c^2-a^2}$
$=3^{a^2-b^2+b^2-c^2+c^2-a^2}$
$=3^0=1$

Level 2

5. If $2^x=3^y=12^z$ *,show that* $\frac{1}{z}=\frac{1}{y}+\frac{2}{x}$

$2^x=3^y=(2\times3\times2)^z$
$2^x=3^y=\left(2^2\times3\right)^z$
$2^x=3^y=\left(2^{2z}\times3^z\right)$
$2^x=3^y=12^z=k$
$2=k^{\frac{1}{x}}$
$3=k^{\frac{1}{y}}$
$12=k^{\frac{1}{z}}$
$12=2\times3\times2$
$12=k^{\frac{1}{z}}=k^{\frac{1}{y}}\times k^{\frac{1}{x}}\times k^{\frac{1}{x}}$
$k^{\frac{1}{z}}=k^{\frac{2}{x}+\frac{1}{y}}$
$\frac{1}{z}=\frac{1}{y}+\frac{2}{x}$

6. If $2^x=3^y=6^{-z}$ *,show that* $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$2^x=3^y=6^{-z}$
$2^x=k$
$2=k^{\frac{1}{x}}$
$3^y=k$
$3=k^{\frac{1}{y}}$
$6^{-z}=k$
$k=\frac{1}{6^z}$
$6=k^{-\frac{1}{z}}$
$2\times3=6$
$k^{\frac{1}{x}}\times k^{\frac{1}{y}}=k^{-\frac{1}{z}}$
$\frac{1}{x}+\frac{1}{y}=-\frac{1}{z}\qquad[by\ equating\ exponents]$
$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

7. If $a^x=b^y=c^z$ *and* $b^2=ac$ *, then show that* $y=\frac{2zx}{z+x}$

$Let\ a^x = b^y = c^z = k$
$a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$
$Now,$
$b^2 = ac$
$\left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$
$k^{\frac{2}{y}} = k^{\frac{1}{x}+\frac{1}{z}}$
$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$
$\frac{2}{y} = \frac{x+z}{xz}$
$y = \frac{2xz}{x+z}$

8. If $3^x = 5^y = (75)^z$*, show that* $z = \frac{xy}{2x+y}$

$3^x = k$
$3 = k^{\frac{1}{x}}$
$5^y = k$
$5 = k^{\frac{1}{y}}$
$75^z = k$
$75 = k^{\frac{1}{z}}$
$3^1 \times 5^2 = 75^1$
$k^{\frac{1}{x}} \times k^{\frac{2}{y}} = k^{\frac{1}{z}}$
$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$
$\frac{y+2x}{xy} = \frac{1}{z}$
$z = \frac{xy}{2x+y}$

9. If $(27)^x = \frac{9}{3^x}$*, find x*

We have,

$(27)^x = \frac{9}{3^x}$
$\left(3^3\right)^x = \frac{9}{3^x}$
$3^{3x} = \frac{9}{3^x}$
$3^{3x} = \frac{3^2}{3^x}$
$3^{3x} = 3^{2-x}$
$3x = 2 - x \qquad [On\ equating\ exponents]$
$3x + x = 2$
$4x = 2$
$x = \frac{2}{4}$
$x = \frac{1}{2}$
$Here\ the\ value\ of\ x\ is\ \frac{1}{2}$

10. Find the values of x in each of the following

(i). $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

We have

$2^{5x} \div 2^x = \sqrt[5]{2^{20}}$
$= \frac{2^{5x}}{2^x} = \left(2^{20}\right)^{\frac{1}{5}}$
$= 2^{5x-x} = 2^{20\times\frac{1}{5}}$
$= 2^{4x} = 2^4$
$= 4x = 4 \qquad [On\ equating\ exponent]$
$x = 1$

Hence the value of x is 1

(ii). $\left(2^3\right)^4 = \left(2^2\right)^x$

We have

$\left(2^{3}\right)^{4}=\left(2^{2}\right)^{x}$
$=2^{3\times 4}=2^{2\times x}$
$12=2x$
$2x=12 \qquad [On\ equating\ exponents]$
$x=6$

Hence the value of x is 6

(iii). $\left(\frac{3}{5}\right)^{x}\left(\frac{5}{3}\right)^{2x}=\frac{125}{27}$

We have

$\left(\frac{3}{5}\right)^{x}\left(\frac{5}{3}\right)^{2x}=\frac{125}{27}$
$\Rightarrow \frac{(3)^{x}}{(5)^{x}}\frac{(5)^{2x}}{(3)^{2x}}=\frac{5^{3}}{3^{3}}$
$\Rightarrow \frac{5^{2x-x}}{3^{2x-x}}=\frac{5^{3}}{3^{3}}$
$\Rightarrow \frac{5^{x}}{3^{x}}=\frac{5^{3}}{3^{3}}$
$\Rightarrow \left(\frac{5}{3}\right)^{x}=\left(\frac{5}{3}\right)^{3}$
$x=3 \qquad [on\ equating\ exponents]$

Hence the value of x is 3

(iv)
$5^{x-2}\times 3^{2x-3}=135$

We have,

$5^{x-2}\times 3^{2x-3}=135$
$\Rightarrow 5^{x-2}\times 3^{2x-3}=5\times 27$
$\Rightarrow 5^{x-2}\times 3^{2x-3}=5^{1}\times 3^{3}$
$\Rightarrow x-2=1, 2x-3=3\ [On\ equating\ exponents]$
$\Rightarrow x=2+1, 2x=3+3$
$\Rightarrow x=3, 2x=6\Rightarrow x=3$

Hence the value of x is 3

(v). $2^{x-7}\times 5^{x-4}=1250$

We have

$2^{x-7}\times 5^{x-4}=1250$
$\Rightarrow 2^{x-7}\times 5^{x-4}=2\times 625$
$\Rightarrow 2^{x-7}\times 5^{x-4}=2\times 5^{4}$
$\Rightarrow x-7=1\Rightarrow x=8, x-4=4\Rightarrow x=8$

Hence the value of x is 8

(vi).

$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}}=\frac{1}{32}$

$\left(4^{\frac{1}{3}}\right)^{2x+\frac{1}{2}}=\frac{1}{32}$

$(4)^{\frac{1}{3}\left(2x+\frac{1}{2}\right)}=\frac{1}{32}$

$(4)^{\frac{1}{3}\left(2x+\frac{1}{2}\right)}=\frac{1}{2^{5}}$

$(4)^{\frac{2}{3}x+\frac{1}{6}}=\frac{1}{2^{5}}$

$\left(2^{2}\right)^{\frac{2}{3}x+\frac{1}{6}}=\frac{1}{2^{5}}$

$(2)^{2\left(\frac{2}{3}x+\frac{1}{6}\right)}=\frac{1}{2^{5}}$

$(2)^{\frac{4}{3}x+\frac{2}{6}}=\frac{1}{2^{5}}$

$(2)^{\frac{4}{3}x+\frac{1}{3}}=2^{-5}$

$\frac{4}{3}x+\frac{1}{3}=-5$

$4x+1=-15$
$4x=-15-1$
$4x=-16$
$x=\frac{-16}{4}$
$x=-4$

Hence the value of x is 4

(vii).
$5^{2x+3}=1$
$5^{2x+3}=1\times 5^0$
$2x+3=0 \qquad [By\ equating\ exponents]$
$2x=-3$
$x=\frac{-3}{2}$

Hence the value of x is $\frac{-3}{2}$

(viii).
$(13)^{\sqrt{x}}=4^4-3^4-6$
$(13)^{\sqrt{x}}=256-81-6$
$(13)^{\sqrt{x}}=256-87$
$(13)^{\sqrt{x}}=169$
$(13)^{\sqrt{x}}=13^2$
$\sqrt{x}=2 \qquad [By\ equating\ exponents]$
$(\sqrt{x})^2=(2)^2$
$x=4$

Hence the value of x is 4

(ix).
$\left(\sqrt{\frac{3}{5}}\right)^{x+1}=\frac{125}{27}$
$\left(\sqrt{\frac{3}{5}}\right)^{x+1}=\frac{5^3}{3^3}$
$\left(\sqrt{\frac{3}{5}}\right)^{x+1}=\left(\frac{5}{3}\right)^3$
$\left(\sqrt{\frac{3}{5}}\right)^{x+1}=\left(\frac{3}{5}\right)^{-3}$
$\left(\frac{3}{5}\right)^{\frac{1}{2}(x+1)}=\left(\frac{3}{5}\right)^{-3}$
$\frac{1}{2}(x+1)=-3$
$x+1=-6$
$x=-6-1$
$x=-7$

Hence the value of x is 7

11. If x = $2^{\frac{1}{3}}+2^{\frac{2}{3}}$*, show that* $x^3-6x=6$

$x^3-6x=6$

x = $2^{\frac{1}{3}}+2^{\frac{2}{3}}$

Putting cube on both the sides, we get

$x^3 = (2^{\frac{1}{3}}+2^{\frac{2}{3}})^3$

As we know, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$x^3 = (2^{\frac{1}{3}})^3+(2^{\frac{2}{3}})^3+3(2^{\frac{1}{3}})(2^{\frac{2}{3}})(2^{\frac{1}{3}}+2^{\frac{2}{3}})$
$x^3 = (2^{\frac{1}{3}})^3+(2^{\frac{2}{3}})^3+3(2^{\frac{1}{3}+\frac{2}{3}})(x)$
$x^3 = (2^{\frac{1}{3}})^3+(2^{\frac{2}{3}})^3+3(2)(x)$
$x^3 = 6 + 6x$
$x^3 - 6x = 6$

Hence proved

12. Determine $(8x)^x$, if $9^{x+2} = 240 + 9^x$.

$9^{x+2} = 240 + 9^x$
$9^x . 9^2 = 240 + 9^x$

Let 9^x be y

$81y = 240 + y$
$81y - y = 240$
$80y = 240$
$y = 3$

Since, y = 3

Then,

$9^x = 3$

$3^{2x} = 3$

Therefore, $x = \frac{1}{2}$

$(8x)^x = (8 \times \frac{1}{2})^{\frac{1}{2}}$

$= (4)^{\frac{1}{2}}$

= 2

Therefore $(8x)^x = 2$

13. If $3^{x+1} = 9^{x-2}$, find the value of 2^{1+x}

$3^{x+1} = 9^{x-2}$

$3^{x+1} = 3^{2x-4}$
$x + 1 = 2x - 4$
$x = 5$

Therefore the value of $2^{1+x} = 2^{1+5} = 2^6 = 64$

14. If $3^{4x} = (81)^{-1}$ and $(10)^{\frac{1}{y}} = 0.0001$, find the value of 2^{-x+4y}.

$3^{4x} = (81)^{-1}$ and $(10)^{\frac{1}{y}} = 0.0001$

$3^{4x} = (3)^{-4}$
$x = -1$

And, $(10)^{\frac{1}{y}} = 0.0001$

$(10)^{\frac{1}{y}} = (10)^{-4}$

$\frac{1}{y} = -4$
$y = \frac{1}{-4}$

To find the value of 2^{-x+4y}, we need to substitute the value of x and y

$2^{-x+4y} = 2^{1+4(\frac{1}{-4})} = 2^{1-1} = 2^0 = 1$

15. If $5^{3x} = 125$ and $10^y = 0.001$. Find x and y.

$5^{3x} = 125$ and $10^y = 0.001$

$5^{3x} = 5^3$
$x = 1$

Now,

$10^y = 0.001$
$10^y = 10^{-3}$
$y = -3$

Therefore, the value of x = 1 and the value of y = -3

16. Solve the following equations

(i)
$3^{x+1} = 27 \times 3^4$
$3^{x+1} = 3^3 \times 3^4$
$3^{x+1} = 3^{3+4}$
$x + 1 = 3 + 4 \qquad [By\ equating\ exponents]$
$x + 1 = 7$
$x = 7 - 1$
$x = 6$
(ii)
$4^{2x} = \left(\sqrt[3]{16}\right)^{-\frac{6}{y}} = \left(\sqrt{8}\right)^2$
$\left(2^2\right)^{2x} = \left(16^{\frac{1}{3}}\right)^{-\frac{6}{y}} = \left(\sqrt{8}\right)^2$
$2^{4x} = \left[\left(2^4\right)^{\frac{1}{3}}\right]^{-\frac{6}{y}} = \left(2^{\frac{3}{2}}\right)^2$
$2^{4x} = \left(2^{\frac{4}{3}}\right)^{-\frac{6}{y}} = \left(2^{\frac{3}{2}}\right)^2$
$2^{4x} = \left(2^{\frac{4}{3}}\right)^{-\frac{6}{y}} = 2^3$
$2^{4x} = 2^3$
$4x = 3 \qquad (By\ equating\ exponents)$
$x = \frac{3}{4}$
$2^{-\frac{8}{y}} = 2^3$
$-\frac{8}{y} = 3 \qquad (By\ equating\ exponents)$
$y = \frac{-8}{3}$
(iii).
$3^{x-1} \times 5^{2y-3} = 225$
$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$
$x - 1 = 2 \qquad [By\ equating\ exponents]$
$x = 3$
$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$
$2y - 3 = 2 \qquad [By\ equating\ exponents]$
$2y = 5$
$y = \frac{5}{2}$

(iv).
$8^{x+1} = 16^{y+2}\ and\ \left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$
$\left(2^3\right)^{x+1}\ and\ \left(2^{-1}\right)^{3+x} = \left(2^{-2}\right)^{3y}$
$3x + 3 = 4y + 8\ and\ -3 - x = -6y$
$3x + 3 = 4y + 8\ and\ 3 + x = 6y$
$3x + 3 = 4y + 8\ and\ y = \frac{3+x}{6}$

3x+3=4y+8—eq1

$y = \frac{3+x}{6}$—-eq2

Substitute eq2 in eq1

$3x + 3 = 4\left(\frac{3+x}{6}\right) + 8$

$3x + 3 = 2\left(\frac{3+x}{3}\right) + 8$

$3x + 3 = \left(\frac{6+2x}{3}\right) + \frac{24}{3}$

$3(3x + 3) = 6 + 2x + 24$
$9x + 9 = 30 + 2x$
$7x = 21$
$x = \frac{21}{7}$
$x = 3$
Putting value of x in eq2
$\frac{3+3}{6} = yy = 1$

(v).
$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$

$2^{2x-2} \times \left(\frac{5}{10}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$

$2^{2x-2} \times \left(\frac{1}{2}\right)^{3-2x} = 2^{-3x}$

$2^{2x-2} \times 2^{-3+2x} = 2^{-3x}$
$2x - 2 - 3 + 2x = -3x \qquad [By\ equating\ exponents]$
$4x + 3x = 5$
$7x = 5$
$x = \frac{5}{7}$

(vi).
$\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$

$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-(1-2x)} \frac{1}{2} = -1 + 2x \qquad [By\ equating\ exponents]$

$\frac{1}{2} + 1 = 2x$

$2x = \frac{3}{2}$

$x = \frac{3}{4}$

17. If a and b are distinct positive primes such that $\sqrt[3]{a^6b^{-4}} = a^xb^{2y}$*, find x and y*

$\sqrt[3]{a^6b^{-4}} = a^xb^{2y}$

$\left(a^6b^{-4}\right)^{\frac{1}{3}} = a^xb^{2y}$

$a^{\frac{6}{3}}b^{\frac{-4}{3}} = a^xb^{2y}$

$a^2b^{\frac{-4}{3}} = a^xb^{2y}$

$x = 2, 2y = \frac{-4}{3}$

$y = \frac{\frac{-4}{3}}{2}$

$y = -\frac{2}{3}$

18.If a and b are different positive primes such that

(i).
$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^xb^y$

find x and y

$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^xb^y$

$\left(a^{-1-2}b^{2+4}\right)^7 \div \left(a^{3+2}b^{-5-3}\right) = a^xb^y$

$\left(a^{-3}b^6\right)^7 \div \left(a^5b^{-8}\right) = a^xb^y$

$\left(a^{-21}b^{42}\right) \div \left(a^5b^{-8}\right) = a^xb^y$

$\left(a^{-21-5}b^{42+8}\right) = a^xb^y$

$\left(a^{-26}b^{50}\right) = a^xb^y$

$x = -26, y = 50$

(ii) $(a+b)^{-1}\left(a^{-1}+b^{-1}\right)=a^{x}b^{y},\ find\ x\ and\ y$

$(a+b)^{-1}\left(a^{-1}+b^{-1}\right)$
$=\left(\frac{1}{a+b}\right)\left(\frac{1}{a}+\frac{1}{b}\right)$
$=\left(\frac{1}{a+b}\right)\left(\frac{b+a}{ab}\right)$
$=\frac{1}{ab}$
$=(ab)^{-1}=a^{-1}b^{-1}$
$By\ equating\ exponents$
$x=-1,y=-1$
$Therefore\ x+y+2=-1-1+2=0$

19. If $2^{x}\times 3^{y}\times 5^{z}=2160$, find x,y and z. Hence compute the value of $3^{x}\times 2^{-y}\times 5^{-z}$

$2^{x}\times 3^{y}\times 5^{z}=2160$
$2^{x}\times 3^{y}\times 5^{z}=2^{4}\times 3^{3}\times 5^{1}$
$x=4,y=3,z=1$
$3^{x}\times 2^{-y}\times 5^{-z}=3^{4}\times 2^{-3}\times 5^{-1}$
$=\frac{3\times3\times3\times3}{2\times2\times2\times5}$
$=\frac{81}{40}$

20. If $1176=2^{a}\times 3^{b}\times 7^{c}$, find the values of a,b and c. Hence compute the value of $2^{a}\times 3^{b}\times 7^{-c}$ as a fraction

$1176=2^{a}\times 3^{b}\times 7^{c}$
$2^{3}\times 3^{1}\times 7^{2}=2^{a}\times 3^{b}\times 7^{c}$
$a=3,b=1,c=2$
$We\ have\ to\ find\ the\ value\ of\ 2^{a}\times 3^{b}\times 7^{-c}$
$2^{a}\times 3^{b}\times 7^{-c}=2^{3}\times 3^{1}\times 7^{-2}$
$=\frac{2\times2\times2\times3}{7\times7}$
$=\frac{24}{49}$

21. Simplify

(i)
$\left(\frac{x^{a+b}}{x^{c}}\right)^{a-b}\left(\frac{x^{b+c}}{x^{a}}\right)^{b-c}\left(\frac{x^{c+a}}{x^{b}}\right)^{c-a}$
$\left(x^{a+b-c}\right)^{a-b}\left(x^{b+c-a}\right)^{b-c}\left(x^{c+a-b}\right)^{c-a}$
$\left(x^{a^{2}-b^{2}-ca+cb}\right)\left(x^{b^{2}-c^{2}-ab+ac}\right)\left(x^{c^{2}-a^{2}-bc+ab}\right)$
$x^{a^{2}-b^{2}-ca+cb+b^{2}-c^{2}-ab+ac+c^{2}-a^{2}-bc+ab}$
$x^{0}=1$

(ii)
$\sqrt[lm]{\frac{x^{l}}{x^{m}}}\times\sqrt[mn]{\frac{x^{m}}{x^{n}}}\times\sqrt[nl]{\frac{x^{n}}{x^{l}}}$
$\sqrt[lm]{x^{l-m}}\times\sqrt[mn]{x^{m-n}}\times\sqrt[nl]{x^{n-l}}$
$\left(x^{l-m}\right)^{\frac{1}{lm}}\times\left(x^{m-n}\right)^{\frac{1}{mn}}\times\left(x^{n-l}\right)^{\frac{1}{nl}}$
$(x)^{\frac{l-m}{lm}}\times(x)^{\frac{m-n}{mn}}\times(x)^{\frac{n-l}{nl}}$
$(x)^{\frac{l-m}{lm}+\frac{m-n}{mn}+\frac{n-l}{nl}}$
$(x)^{n(\frac{l-m}{lm})+l(\frac{m-n}{mn})+m(\frac{n-l}{nl})}$
$(x)^{\frac{nl-mn+lm-nl+mn-ml}{mnl}}$
$(x)^{\frac{0}{mnl}}$
$x^{0}=1$

22. Show that

$$\frac{\left(a+\frac{1}{b}\right)^m \times \left(a-\frac{1}{b}\right)^n}{\left(b+\frac{1}{a}\right)^m \times \left(b-\frac{1}{a}\right)^n} = \left(\frac{a}{b}\right)^{m+n}$$

$$= \frac{\left(\frac{ab+1}{b}\right)^m \times \left(\frac{ab-1}{b}\right)^n}{\left(\frac{ab+1}{a}\right)^m \times \left(\frac{ab+1}{a}\right)^n}$$

$$= \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n$$

$$= \left(\frac{a}{b}\right)^{m+n}$$

Hence $LHS = RHS$

23.

(i). $If\ a = x^{m+n}y^l, b = x^{n+l}y^m$ *and* $c = x^{l+m}y^n$, *prove that* $a^{m-n}b^{n-l}c^{l-m} = 1$

$$\left(x^{m+n}y^l\right)^{m-n}\left(x^{n+l}y^m\right)^{n-l}\left(x^{l+m}y^n\right)^{l-m}$$

$$= \left(x^{(m+n)(m-n)}y^{l(m-n)}\right)\left(x^{(n+l)(n-l)}y^{m(n-l)}\right)\left(x^{(l+m)(l-m)}y^{n(l-m)}\right)$$

$$= \left(x^{m^2-n^2}y^{lm-ln}\right)\left(x^{n^2-l^2}y^{mn-ml}\right)\left(x^{l^2-m^2}y^{nl-nm}\right)$$

$$= x^{m^2-n^2+n^2-l^2+l^2-m^2}y^{lm-ln+mn-ml+nl-nm}$$

$$= x^0y^0$$

$$= 1$$

(ii). $If x = a^{m+n}, y = a^{n+l}$ *and* $z = a^{l+m}$, *prove that* $x^my^nz^l = x^ny^lz^m$

$$LHS = x^my^nz^l$$

$$\left(a^{m+n}\right)^m\left(a^{n+l}\right)^n\left(a^{l+m}\right)^l$$

$$= a^{m^2+nm} \times a^{n^2+ln} \times a^{l^2+ml}$$

$$= a^{n^2+nm} \times a^{l^2+ln} \times a^{m^2+ml}$$

$$= a^{(m+n)n}a^{(n+l)l}a^{(l+m)m}$$

$$= x^ny^lz^m$$