RD Sharma Solutions Class 9 Circles Ex 16.1

RD Sharma Solutions Class 9 Chapter 16 Ex 16.1

Q1) Fill in the blanks:

(i) All points lying inside/outside a circle are called _____ points/_____ points.

(ii) Circle having the same centre and different radii are called _____ circles.

(iii) A point whose distance from the center of a circle is greater than its radius lies in _____ of the circle.

(iv) A continuous piece of a circle is _____ of the circle.

(v) The longest chord of a circle is a _____ of the circle.

(vi) An arc is a _____ when its ends are the ends of a diameter.

(vii) Segment of a circle is a region between an arc and _____ of the circle.

(viii) A circle divides the plane, on which it lies, in _____ parts.

Solution:

(i) Interior/Exterior

(ii) Concentric

(iii) The Exterior

(iv) Arc

(v) Diameter

(vi) Semi circle

(vii) Center

(viii) Three

Q2) Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.

(ii) Line segment joining the center to any point on the circle is a radius of the circle,

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A circle has only finite number of equal chords.

(v) A chord of a circle, which is twice as long as its radius is the diameter of the circle.

(vi) Sector is the region between the chord and its corresponding arc.

(vii) The degree measure of an arc is the complement of the central angle containing the arc.

(viii) The degree measure of a semi-circle is 180^0.

Solution:

(i) T

(ii) T

(iii) T

(iv) F

(v) T

(vi) T

(vii) F

(viii) T

RD Sharma Solutions Class 9 Chapter 16 Ex 16.2

Q1) The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Solution:

Given that,

Radius of circle (OA) = 8cm

Chord (AB) = 12cm

Draw $OC \perp AB$

We know that

The perpendicular from centre to chord bisects the chord

$\therefore AC = BC = \frac{12}{2} = 6cm$

Now in ΔOCA, by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$=> 6^2 + OC^2 = 8^2$

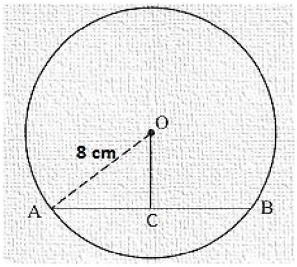
$=> 36 + OC^2 = 64$

$=> OC^2 = 64-36$

$=> OC^2 = 28$

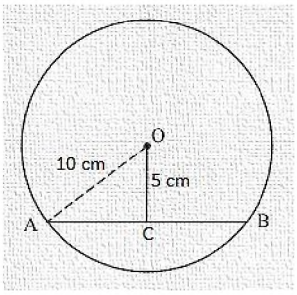
$=> OC = \sqrt{28}$

$=> OC = 5.291cm$



Q2) Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Solution:



Given that,

Distance (OC) = 5cm

Radius of the circle (OA) = 10cm

In ΔOCA, by Pythagoras theorem

$OC^2 + AC^2 = OA^2$

$=> 5^2 + AC^2 = 10^2$

$=> 25 + AC^2 = 100$

$=> AC^2 = 100 - 25$

$=> AC^2 = 75$

$=> AC = \sqrt{75}$

$=> AC = 8.66cm$

We know that, the perpendicular from the centre to chord bisects the chord

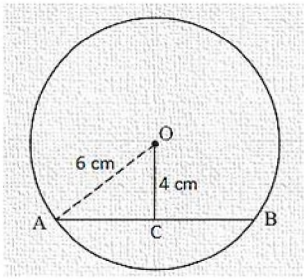
Therefore, AC = BC = 8.66cm

Then the chord AB = 8.66 + 8.66

= 17.32cm

Q3) Find the length of a chord which is at a distance of 4 cm from the centre of a circle of radius 6 cm.

Solution:



Given that,

Radius of the circle (OA) = 6cm

Distance (OC) = 4cm

In ΔOCA, by Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$\Rightarrow AC^2 + 4^2 = 6^2$

$\Rightarrow AC^2 = 36 - 16$

$\Rightarrow AC^2 = 20$

$\Rightarrow AC = \sqrt{20}$

$\Rightarrow AC = 4.47cm$

We know that the perpendicular distance from centre to chord bisects the chord.

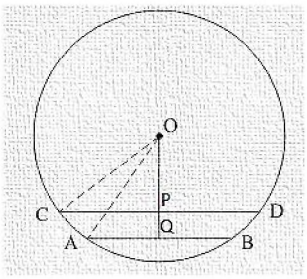
AC = BC = 4.47cm

Then AB = 4.47 + 4.47

= 8.94cm

Q4) Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution:



Construction: Draw $OP \perp CD$

Chord AB = 5cm

Chord CD = 11cm

Distance PQ = 3cm

Let OP = x cm

And OC = OA = r cm

We know that the perpendicular from centre to chord bisects it.

$\therefore CP = PD = \frac{11}{2} cm$

And $AQ = BQ = \frac{5}{2} cm$

In ΔOCP, by Pythagoras theorem

$OC^2 = OP^2 + CP^2$

$\Rightarrow r^2 = x^2 + (\frac{11}{2})^2 \ldots\ldots (i)$

In ΔOQA, by Pythagoras theorem

$OA^2 = OQ^2 + AQ^2$

$\Rightarrow r^2 = (x+3)^2 + (\frac{5}{2})^2 \ldots\ldots (ii)$

Compare equation (i) and (ii)

$\Rightarrow (x+3)^2 + (\frac{5}{2})^2 = x^2 + (\frac{11}{2})^2$

$\Rightarrow x^2 + 6x + 9 + \frac{25}{4} = x^2 + \frac{121}{4}$

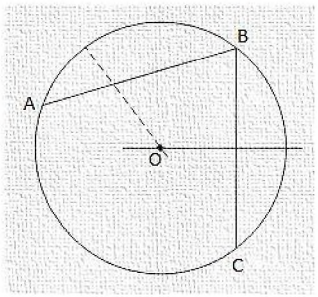
$\Rightarrow x^2 - x^2 + 6x = \frac{121}{4} - \frac{25}{4} - 9$

$\Rightarrow 6x = 15$

$\Rightarrow x = \frac{15}{6} = \frac{5}{2}$

Q5) Give a method to find the centre of a given circle.

Solution:



<u>Steps of Construction</u>:

(1) Take three points A, B and C on the given circle.

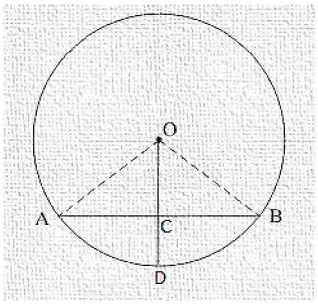
(2) Join AB and BC.

(3) Draw the perpendicular bisectors of the chord AB and BC which intersect each other at O.

(4) Point O will give the required circle because we know that, the Perpendicular bisectors of chord always pass through the centre.

Q6) Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Solution:



Given:

C is the mid-point of chord AB.

To prove: D is the mid-point of arc AB.

Proof:

In $\Delta OAC \; and \; \Delta OBC$

OA = OB [Radius of circle]

OC = OC [Common]

AC = BC [C is the mid-point of AB]

$Then \; \Delta OAC \cong \Delta OBC \qquad [By \; SSS \; condition]$

$\therefore \angle AOC = \angle BOC$

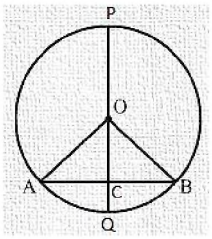
$\Rightarrow m\bar{AD} \cong m\bar{BD}$

$\Rightarrow \bar{AD} \cong \bar{BD}$

Hence, D is the mid-point of arc AB.

Q7) Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Solution:



Given:

PQ is a diameter of circle which bisects the chord AB at C.

To Prove: PQ bisects $\angle AOB$

Proof:

In $\angle AOC \ and \ \angle BOC$

OA = OB [Radius of circle]

OC = OC [Common]

AC = BC [Given]

$Then \ \Delta AOC \cong \Delta BOC \qquad [By \ SSS \ condition]$

$\angle AOC = \angle BOC \qquad [C.P.C.T]$

Hence PQ bisects $\angle AOB$.

Q8) Prove that two different circles cannot intersect each other at more than two points.

Solution:

Suppose two circles intersect in three points A, B, C.

Then A, B, C are non-collinear so a unique circle passes through these three points. This is contradiction to the face that two given circles are passing through A, B, C. Hence, two circles cannot intersect each other at more than two points.

Q9) A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.

Solution:

(1) Draw a line segment AB of 5cm.

(2) Draw the perpendicular bisectors of AB.

(3) With centre A and radius of 4cm, draw an arc which intersects the perpendicular bisector at point O. The point O will be the required centre.

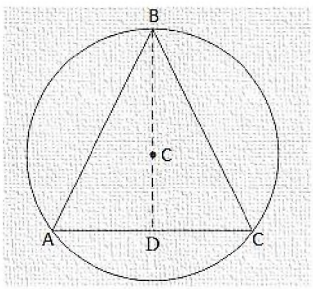
(4) Join OA.

(5) With centre O and radius OA, draw a circle.

No, we cannot draw a circle of radius 2cm passing through A and B because when we draw an arc of radius 2cm with centre A, the arc will not intersect the perpendicular bisector and we will not find the centre.

Q10) An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Solution:



Let ABC be an equilateral triangle of side 9cm and let AD is one of its median.

Let G be the centroid of ΔABC. Then AG : GD = 2 : 1

We know that in an equilateral triangle, centroid coincides with the circum centre.

Therefore, G is the centre of the circumference with circum radius GA.

Also G is the centre and GD is perpendicular to BC.

Therefore, In right triangle ADB, we have

$AB^2 = AD^2 + DB^2$

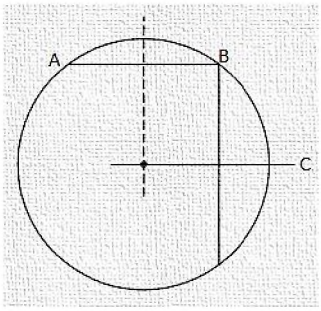
$\Rightarrow 9^2 = AD^2 + DB^2$

$$\Rightarrow AD = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2} cm$$

$$\therefore Radius = AG = \frac{2}{3} AD = 3\sqrt{3} cm$$

Q11) Given an arc of a circle, complete the circle.

Solution:



Steps of Construction:

(1) Take three points A, B and C on the given arc.

(2) Join AB and BC.

(3) Draw the perpendicular bisectors of chords AB and BC which intersect each other at point O. Then O will be the required centre of the required circle.

(4) Join OA.

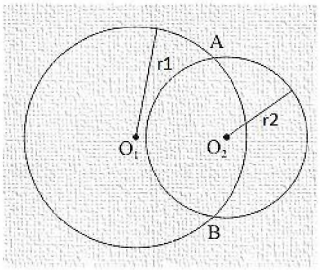
(5) With centre O and radius OA, complete the circle.

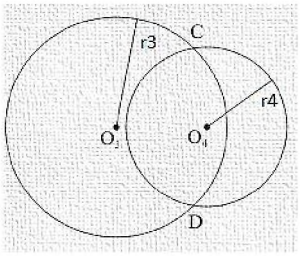
Q12) Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Solution:

Each pair of circles have 0, 1 or 2 points in common.

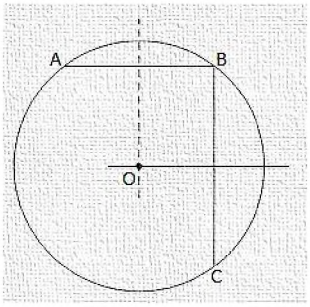
The maximum number of points in common is '2'.





Q13) Suppose you are given a circle. Give a construction to find its centre.

Solution:



Steps of Construction:

(1) Take three points A, B and C on the given circle.

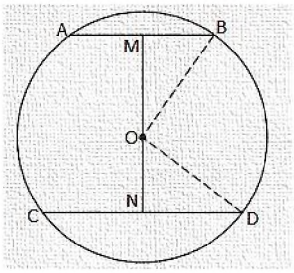
(2) Join AB and BC.

(3) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O.

(4) Point O will be the required centre of the circle because we know that the perpendicular bisector of the chord always passes through the centre.

Q14) Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Solution:



Draw $OM \perp AB \; and \; ON \perp CD.$

Join OB and OD.

$BM = \frac{AB}{2} = \frac{5}{2}$ [Perpendicular from the centre bisects the chord]

$ND = \frac{CD}{2} = \frac{11}{2}$

Let ON be x, so OM will be 6 – x.

ΔMOB

$OM^2 + MB^2 = OB^2$

$(6-x)^2 + (\frac{5}{2})^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2 \ldots\ldots (i)$

In ΔNOD

$ON^2 + ND^2 = OD^2$

$x^2 + (\frac{11}{2})^2 = OD^2$

$x^2 + \frac{121}{2} = OD^2 \ldots\ldots (ii)$

We have OB = OD. [Radii of same circle]

So, from equation (i) and (ii).

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{2}$

$\Rightarrow 12x = 36 + \frac{25}{4} - \frac{121}{4}$

$= \frac{144+25-121}{4} = \frac{48}{4} = 12$

x = 1

From equation (ii)

$(1)^2 + (\frac{121}{4}) = OD^2$

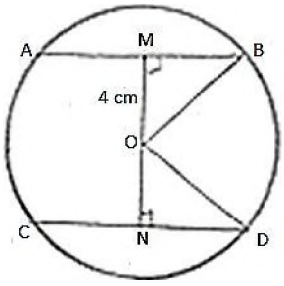
$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$

$OD = \frac{5\sqrt{5}}{2}$

So, the radius of the circle is found to be $\frac{5\sqrt{5}}{2} cm$

Q15) The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?

Solution:



Distance of smaller chord AB from centre of circle = 4cm, OM = 4cm

$MB = \frac{AB}{2} = \frac{6}{2} = 3cm$

$In\ \Delta OMB$

$OM^2 + MB^2 = OB^2$

$4^2 + 9^2 = OB^2$

$16 + 9 = OB^2$

$OB = \sqrt{25}$

OB = 5cm

$In\ \Delta OND$

OD = OB = 5cm [Radii of same circle]

$ND = \frac{CD}{2} = \frac{8}{2} = 4cm$

$ON^2 + ND^2 = OD^2$

$ON^2 + 4^2 = 5^2$

$ON^2 = 25 - 16$

$ON = \sqrt{9}$

ON = 3cm

So, the distance of bigger chord from the circle is 3cm.

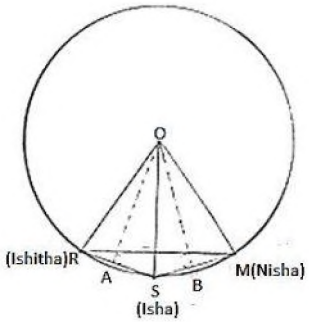
RD Sharma Solutions Class 9 Circles Ex 16.3

RD Sharma Solutions Class 9 Chapter 16 Ex 16.3

Q1) Three girls Ishita, Isha end Nishe are playing a game by standing on a circle of radius 20m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24m each, what is the distance between Ishita and Nisha.

Solution:

Let R, S and M be the position of Ishita, Isha and Nisha respectively.



$AR = AS = \frac{24}{2} = 12cm$

OR = OS = OM = 20 cm [Radii of circle]

$In\ \Delta OAR,$

$OA^2 + AR^2 = OR^2$

$OA^2 + 12^2 = 20^2$

$OA^2 = 400 - 144 = 256m^2$

OA = 16m

We know that, in an isosceles triangle altitude divides the base.

So in $\Delta RSM,\ \angle RCS = 90^0\ and\ RC = CM$

Area of $\Delta ORS = \frac{1}{2} \times OA \times RS$

$\Rightarrow \frac{1}{2} \times RC \times OS = \frac{1}{2} \times 16 \times 24$

$\Rightarrow RC \times 20 = 16 \times 24$

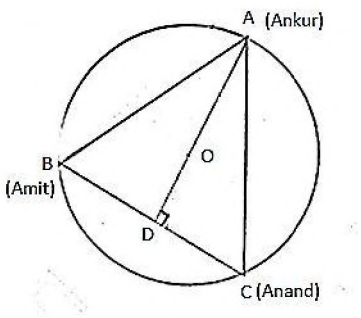
$\Rightarrow RC = 19.2$

$\Rightarrow RM = 2(19.2) = 38.4m$

So, the distance between Ishita and Nisha is 38.4m.

Q2) A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Solution:



Given that AB = BC = CA

So, ABC is an equilateral triangle

OA (radius) = 40m

Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ABC.

We also know that median intersect each other at the ratio 2 : 1.

As AD is the median of equilateral triangle ABC, we can write:

$\frac{OA}{OD} = \frac{2}{1}$

$\Rightarrow \frac{4\ OM}{OD} = \frac{2}{1}$

$\Rightarrow OD = 20m$

Therefore, AD = OA + OD = (40 + 20) m

= 60 m

$In\ \Delta ADC$

By using Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$AC^2 = 60^2 + (\frac{AC}{2})^2$

$AC^2 = 3600 + \frac{AC^2}{4}$

$\Rightarrow \frac{3}{4}AC^2 = 3600$

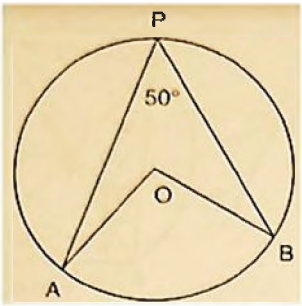
$\Rightarrow AC^2 = 4800$

$\Rightarrow AC = 40\sqrt{3}m$

So, length of string of each phone will be $40\sqrt{3}m$..

RD Sharma Solutions Class 9 Chapter 16 Ex 16.4

Q1) In figure 16.120, O is the centre of the circle. If $\angle APB = 50^0,\ find\ \angle AOB\ and\ \angle OAB.$



Solution:

$\angle APB = 50^0$

By degree measure theorem

$\angle AOB = 2\angle APB$
$\Rightarrow \angle APB = 2 \times 50^0 = 100^0$
$since\ OA = OB \qquad [Radius\ of\ circle]$
$Then\ \angle OAB = \angle OBA \qquad [Angles\ opposite\ to\ equal\ sides]$
$Let\ \angle OAB = x$
$In\ \Delta OAB,\ by\ angle\ sum\ property$
$\angle OAB + \angle OBA + \angle AOB = 180^0$

=>x + x + 100^0 = 180^0

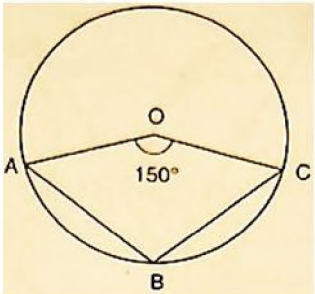
=>2x = 180^0 – 100^0

=>2x = 80^0

=>x = 40^0

$\angle OAB = \angle OBA = 40^0$

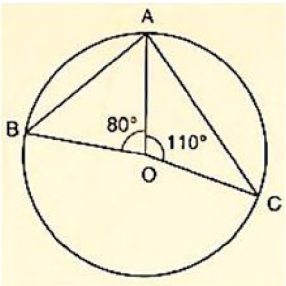
Q2) In figure 16.121, it is given that O is the centre of the circle and $\angle AOC = 150^0$. Find $\angle ABC$.



Solution:

$\angle AOC = 150^0$
$\therefore \angle AOC + reflex\ \angle AOC = 360^0 \qquad [Complex\ angle]$
$\Rightarrow 150^0 + reflex\ \angle AOC = 360^0$
$\Rightarrow reflex\ \angle AOC = 360^0 - 150^0$
$\Rightarrow reflex\ \angle AOC = 210^0$
$\Rightarrow 2\angle ABC = 210^0 \qquad [By\ degree\ measure\ theorem]$
$\Rightarrow \angle ABC = \frac{210^0}{2} = 105^0$

Q3) In figure 16.22, O is the centre of the circle. Find $\angle BAC$.



Solution:

We have $\angle AOB = 80^0$

And $\angle AOC = 110^0$

Therefore, $\angle AOB + \angle AOC + \angle BOC = 360^0 \qquad [Complete\ angle]$

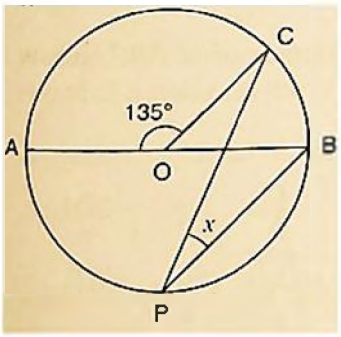
$\Rightarrow 80^0 + 100^0 + \angle BOC = 360^0$
$\Rightarrow \angle BOC = 360^0 - 80^0 - 110^0$
$\Rightarrow \angle BOC = 170^0$

By degree measure theorem

$\angle BOC = 2\angle BAC$
$\Rightarrow 170^0 = 2\angle BAC$
$\Rightarrow \angle BAC = \frac{170^0}{2} = 85^0$

Q4) If O is the centre of the circle, find the value of x in each of the following figures.

(i)

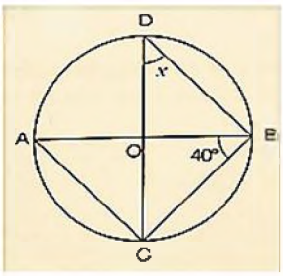


Solution:

$\angle AOC = 135^0$
$\therefore \angle AOC + \angle BOC = 180^0 \quad [Linear\ pair\ of\ angles]$
$\Rightarrow 135^0 + \angle BOC = 180^0$
$\Rightarrow \angle BOC = 180^0 - 135^0$
$\Rightarrow \angle BOC = 45^0$

By degree measure theorem
$\angle BOC = 2\angle CPB$
$\Rightarrow 45^0 = 2x$
$\Rightarrow x = \frac{45^0}{2} = 22\frac{1}{2}^0$

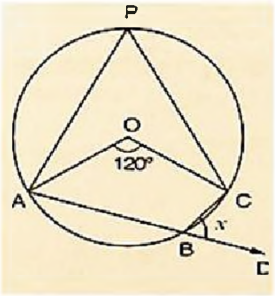
(ii)



Solution:

We have
$\angle ABC = 40^0$
$\angle ACB = 90^0 \quad [Angle\ in\ semicircle]$
In ΔABC, *by angle sum property*
$\angle CAB + \angle ACB + \angle ABC = 180^0$
$\Rightarrow \angle CAB + 90^0 + 40^0 = 180^0$
$\Rightarrow \angle CAB = 180^0 - 90^0 - 40^0$
$\Rightarrow \angle CAB = 50^0$
Now,
$\angle CDB = \angle CAB \quad [Angle\ is\ same\ in\ segment]$
$\Rightarrow x = 50^0$

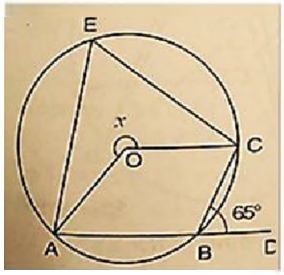
(iii)



Solution:

We have
$\angle AOC = 120^0$
By degree measure theorem.
$\angle AOC = 2\angle APC$
$\Rightarrow 120^0 = 2\angle APC$
$\Rightarrow \angle APC = \frac{120^0}{2} = 60^0$
$\angle APC + \angle ABC = 180^0$ [*Opposite angles of cyclic quadrilaterals*]
$\Rightarrow 60^0 + \angle ABC = 180^0$
$\Rightarrow \angle ABC = 180^0 - 60^0$
$\Rightarrow \angle ABC = 120^0$
$\therefore \angle ABC + \angle DBC = 180^0$ [*Linear pair of angles*]
$\Rightarrow 120 + x = 180^0$
$\Rightarrow x = 180^0 - 120^0 = 60^0$

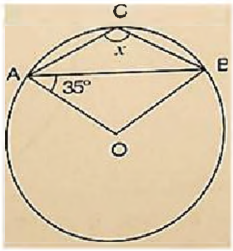
(iv)



Solution:

We have
$\angle CBD = 65^0$
$\therefore \angle ABC + \angle CBD = 180^0$ [*Linear pair of angles*]
$\Rightarrow \angle ABC = 65^0 = 180^0 \Rightarrow \angle ABC = 180^0 - 65^0 = 115^0$
$\therefore reflex\ \angle AOC = 2\angle ABC$ [*By degree measure theorem*]
$\Rightarrow x = 2 \times 115^0$
$\Rightarrow x = 230^0$

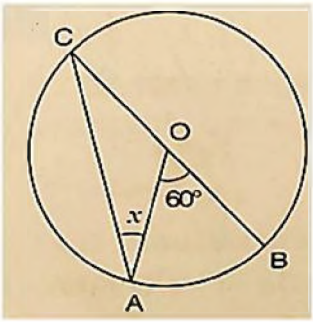
(v)



Solution:

$We\ have$
$\angle OAB = 35^0$
$Then,\ \angle OBA = \angle OAB = 35^0 \quad [Angles\ opposite\ to\ equal\ radii]$
$In\ \Delta AOB,\ by\ angle\ sum\ property$
$\Rightarrow \angle AOB + \angle OAB + \angle OBA = 180^0$
$\Rightarrow \angle AOB + 35^0 + 35^0 = 180^0$
$\Rightarrow \angle AOB = 180^0 - 35^0 - 35^0 = 110^0$
$\therefore \angle AOB + reflex\angle AOB = 360^0 \quad [Complex\ angle]$
$\Rightarrow 110^0 + reflex\angle AOB = 360^0$
$\Rightarrow reflex\angle AOB = 360^0 - 110^0 = 250^0$
$By\ degree\ measure\ theorem\ reflex\angle AOB = 2\angle ACB$
$\Rightarrow 250^0 = 2x$
$\Rightarrow x = \frac{250^0}{2} = 125^0$

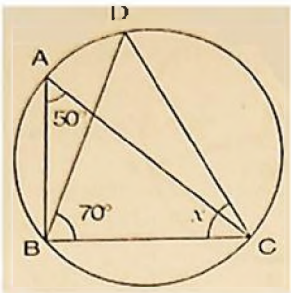
(vi)



Solution:

$We\ have$
$\angle AOB = 60^0$
$By\ degree\ measure\ theorem\ reflex$
$\angle AOB = 2\angle ACB$
$\Rightarrow 60^0 = 2\angle ACB$
$\Rightarrow \angle ACB = \frac{60^0}{2} = 30^0 \quad [Angles\ opposite\ to\ equal\ radii]$
$\Rightarrow x = 30^0.$

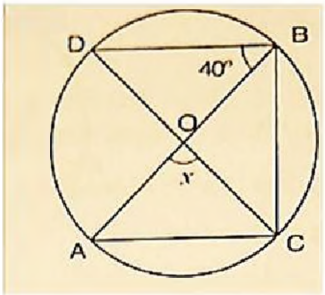
(vii)



Solution:

$We\ have$
$\angle BAC = 50^0\ and\ \angle DBC = 70^0$
$\therefore \angle BDC = \angle BAC = 50^0 \quad [Angle\ in\ same\ segment]$
$In\ \Delta BDC,\ by\ angle\ sum\ property$
$\angle BDC + \angle BCD + \angle DBC = 180^0$
$\Rightarrow 50^0 + x + 70^0 = 180^0$
$\Rightarrow x = 180^0 - 50^0 - 70^0 = 60^0$

(viii)



Solution:

We have,

$\angle DBO = 40^0$ *and* $\angle DBC = 90^0$ [*Angle in a semi circle*]

$\Rightarrow \angle DBO + \angle OBC = 90^0$

$\Rightarrow 40^0 + \angle OBC = 90^0$

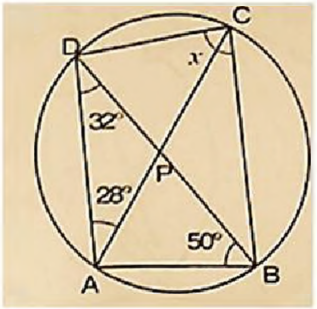
$\Rightarrow \angle OBC = 90^0 - 40^0 = 50^0$

By degree measure theorem

$\angle AOC = 2\angle OBC$

$\Rightarrow x = 2 \times 50^0 = 100^0$

(ix)



Solution:

In ΔDAB, *by angle sum property*

$\angle ADB + \angle DAB + \angle ABD = 180^0$

$\Rightarrow 32^0 + \angle DAB + 50^0 = 180^0$

$\Rightarrow \angle DAB = 180^0 - 32^0 - 50^0$

$\Rightarrow \angle DAB = 98^0$

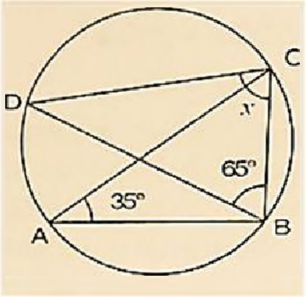
Now,

$\angle OAB + \angle DCB = 180^0$ [*Opposite angles of cyclic quadrilateral*]

$\Rightarrow 98^0 + x = 180^0$

$\Rightarrow x = 180^0 - 98^0 = 82^0$

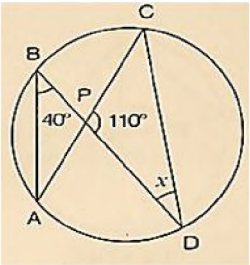
(x)



Solution:

$We\ have,$
$\angle BAC = 35^0$
$\angle BDC = \angle BAC = 35^0 \qquad [Angle\ in\ same\ segment]$
$In\ \Delta BCD,\ by\ angle\ sum\ property$
$\angle BDC + \angle BCD + \angle DBC = 180^0$
$\Rightarrow 35^0 + x + 65^0 = 180^0$
$\Rightarrow x = 180^0 - 35^0 - 65^0 = 80^0$

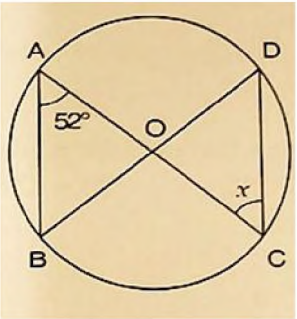
(xi)



Solution:

$We\ have,$
$\angle ABD = 40^0$
$\angle ACD = \angle ABD = 40^0 \qquad [Angle\ in\ same\ segment]$
$In\ \Delta PCD,\ by\ angle\ sum\ property$
$\angle PCD + \angle CPO + \angle PDC = 180^0$
$\Rightarrow 40^0 + 110^0 + x = 180^0$
$\Rightarrow x = 180^0 - 150^0$
$\Rightarrow x = 30^0$

(xii)



Solution:

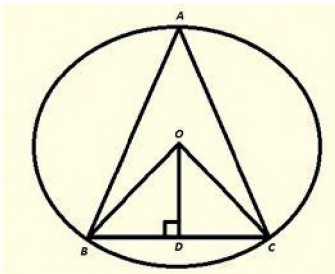
$Given\ that,$
$\angle BAC = 52^0$
$Then\ \angle BDC = \angle BAC = 52^0 \qquad [Angle\ in\ same\ segment]$
$Since\ OD = OC$
$Then\ \angle ODC = \angle OCD \qquad [Opposite\ angle\ to\ equal\ radii]$
$\Rightarrow x = 52^0$

Q5) O is the circumference of the triangle ABC and Odis perpendicular on BC. Prove that $\angle BOD = \angle A$.

Solution:

Given O is the circum centre of triangle ABC and $OD \perp BC$

To prove $\angle BOD = 2\angle A$



Proof:

In

$\Delta OBD \ and \ \Delta OCD$

$\angle ODB = \angle ODC \qquad [Each \ 90^0]$

$OB = OC \qquad [Radius \ of \ circle]$

$OD = OD \qquad [Common]$

$Then \ \Delta OBD \cong \Delta OCD$ *[By RHS Condition].*

$\therefore \angle BOD = \angle COD \ldots\ldots (i) \quad [PCT].$

$By \ degree \ measure \ theorem$

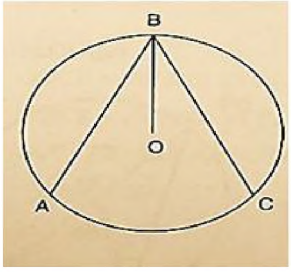
$\angle BOC = 2\angle BAC$

$\Rightarrow 2\angle BOD = 2\angle BAC \quad [By \ using \ (i)]$

$\Rightarrow \angle BOD = \angle BAC$

.

Q6) In figure 16.135, O is the centre of the circle, BO is the bisector of $\angle ABC$. Show that AB = AC.



Solution:

Given, BO is the bisector of $\angle ABC$

To prove AB = BC

Proof:

Since, BO is the bisector of $\angle ABC.$

Then, $\angle ABO = \angle CBO \ldots\ldots (i)$

Since, OB = OA [Radius of circle]

Then, $\angle ABO = \angle DAB \ldots\ldots (ii)$ [opposite angles to equal sides]

Since OB = OC [Radius of circle]

Then, $\angle OAB = \angle OCB \ldots\ldots (iii)$ [opposite angles to equal sides]

Compare equations (i), (ii) and (iii)

$\angle OAB = \angle OCB \ldots\ldots (iv)$

$In \ \Delta OAB \ and \ \Delta OCB$

$\angle OAB = \angle OCB$ [From (iv)]

$\angle OBA = \angle OBC$ [Given]

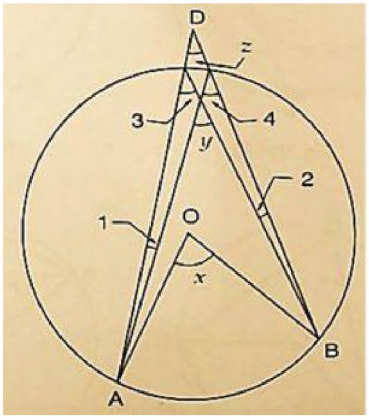
OB = OB [Common]

Then,

$\Delta OAB \cong \Delta OCB \qquad [By \ AAS \ condition]$

$\therefore AB = BC \qquad [CPCT]$

Q7) In figure 16.136, O is the centre of the circle, then prove that $\angle x = \angle y + \angle z$.



Solution:

We have,

$\angle 3 = \angle 4 \quad [Angles\ in\ same\ segment]$

$\therefore \angle x = 2\angle 3 \quad [By\ degree\ measure\ theorem]$

$\Rightarrow \angle x = \angle 3 + \angle 3 \Rightarrow \angle x = \angle 3 + \angle 4 \ldots\ldots (i) \quad [\angle 3 = \ angle 4]$

$But\ \angle y = \angle 3 + \angle 1 \quad [By\ exterior\ angle\ property]$

$\Rightarrow \angle 3 = \angle y - \angle 1 \ldots\ldots (ii)$

$from\ (i)\ and\ (ii)$

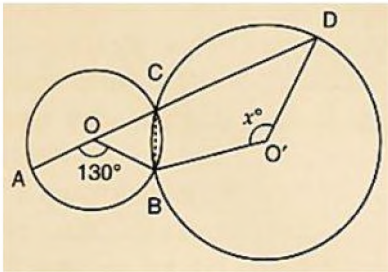
$\angle x = \angle y - \angle 1 + \angle 4$

$\Rightarrow \angle x = \angle y + \angle 4 - \angle 1$

$\Rightarrow \angle x = \angle y + \angle z + \angle 1 - \angle 1 \quad [By\ exterior\ angle\ property]$

$\Rightarrow \angle x = \angle y + \angle z$

Q8) In figure 16.137, O and O' are centers of two circles intersecting at B and C. ACD is a straight line, find x.



Solution:

$By\ degree\ measure\ theorem$

$\angle AOB = 2\angle ACB$

$\Rightarrow 130^0 = 2\angle ACB \Rightarrow \angle ACB = \frac{130^0}{2} = 65^0$

$\therefore \angle ACB + \angle BCD = 180^0 \quad [Linera\ pair\ of\ angles]$

$\Rightarrow 65^0 + \angle BCD = 180^0$

$\Rightarrow \angle BCD = 180^0 - 65^0 = 115^0$

$By\ degree\ measure\ theorem$

$reflex\angle BOD = 2\angle BCD$

$\Rightarrow reflex\angle BOD = 2 \times 115^0 = 230^0$

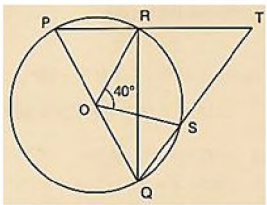
$Now,\ reflex\angle BOD + \angle BO'D = 360^0 \quad [Complex\ angle]$

$\Rightarrow 230^0 + x = 360^0$

$\Rightarrow x = 360^0 - 230^0$

$\therefore x = 130^0$

Q9) In figure 16.138, O is the centre of a circle and PQ is a diameter. If $\angle ROS = 40^0,\ find\ \angle RTS$..



Solution:

Since PQ is diameter

Then,

$\angle PRQ = 90^0 \quad [Angle\ in\ semi\ circle]$
$\therefore \angle PRQ + \angle TRQ = 180^0 \quad [Linear\ pair\ of\ angle]$
$90^0 + \angle TRQ = 180^0$
$\angle TRQ = 180^0 - 90^0 = 90^0.$

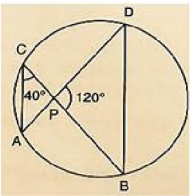
By degree measure theorem

$\angle ROS = 2\angle RQS$
$\Rightarrow 40^0 = 2\angle RQS$
$\Rightarrow \angle RQS = \frac{40^0}{2} = 20^0$

In ΔRQT, by Angle sum property

$\angle RQT + \angle QRT + \angle RTS = 180^0$
$\Rightarrow 20^0 + 90^0 + \angle RTS = 180^0$
$\Rightarrow \angle RTS = 180^0 - 20^0 - 90^0 = 70^0$

Q10) In figure 16.139, if $\angle ACB = 40^0,\ \angle DPB = 120^0,\ find\ \angle CBD.$



Solution:

We have,

$\angle ACB = 40^0;\ \angle DPB = 120^0$

$\therefore \angle APB = \angle DCB = 40^0 \quad [Angle\ in\ same\ segment]$
$In\ \Delta POB,\ by\ angle\ sum\ property$
$\angle PDB + \angle PBD + \angle BPD = 180^0$
$\Rightarrow 40^0 + \angle PBD + 120^0 = 180^0$
$\Rightarrow \angle PBD = 180^0 - 40^0 - 120^0$
$\Rightarrow \angle PBD = 20^0$
$\therefore \angle CBD = 20^0$

Q11) A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

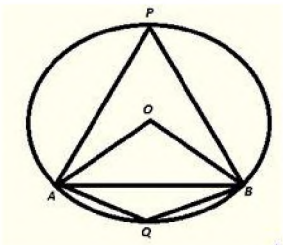
Solution:

We have,

Radius OA = Chord AB

=>OA = OB = AB

Then triangle OAB is an equilateral triangle.



$\therefore \angle AOB = 60^0 \quad [one\ angle\ of\ equilateral\ triangle]$

By degree measure theorem

$\angle AOB = 2\angle APB$
$\Rightarrow 60^0 = 2\angle APB$
$\Rightarrow \angle APB = \frac{60^0}{2} = 30^0$
$Now,\ \angle APB + \angle AQB = 180^0 \qquad [opposite\ angles\ of\ cyclic\ quadrilateral]$
$\Rightarrow 30^0 + \angle AQB = 180^0$
$\Rightarrow \angle AQB = 180^0 - 30^0 = 150^0.$

Therefore, Angle by chord AB at minor arc = 150^0

Angle by chord AB at major arc = 30^0