RD Sharma Solutions Class 9 Chapter 14 Ex 14.1

1) Three angles of a quadrilateral are respectively equal to 110^0, 50^0 and 40^0. Find its fourth angle.

Solution:

Given,

Three angles are 110^0, 50^0 and 40^0

Let the fourth angle be 'x'

We have,

Sum of all angles of a quadrilateral = 360^0

$110^0 + 50^0 + 40^0 = 360^0$

=> x = $360^0 - 200^0$

=>x = 160^0

Therefore, the required fourth angle is 160^0.

2) In a quadrilateral ABCD, the angles A, B, C and D are in the ratio of 1:2:4:5. Find the measure of each angles of the quadrilateral.

Solution:

Let the angles of the quadrilaterals be

A = x, B = 2x, C = 4x and D = 5x

Then,

A + B + C + D = 360^0

=> x + 2x + 4x + 5x = 360^0

=> 12x = 360^0

=> $x = \frac{360^0}{12}$

=> x = 30^0

Therefore, A = x = 30^0

B = 2x = 60^0

C = 4x = 120^0

D = 5x = 150^0

3) In a quadrilateral ABCD, CO and Do are the bisectors of $\angle C\ and \angle D$ respectively. Prove that $\angle COD = \frac{1}{2}(\angle A\ and \angle B)$.

Solution:

$In\ \Delta DOC$

$\angle 1 + \angle COD + \angle 2 = 180^0$ [Angle sum property of a triangle]

=> $\angle COD = 180 - (\angle 1 - \angle 2)$

=> $\angle COD = 180 - \angle 1 + \angle 2$

=> $\angle COD = 180 - [\frac{1}{2}LC + \frac{1}{2}LD]$ $[\because OC\ and\ Od\ are\ bisectors\ of\ LC\ and\ LD\ respectively]$

=> $\angle COD = 180 - \frac{1}{2}(LC + LD) \ldots .(i)$

In quadrilateral ABCD

$\angle A + \angle B + \angle C + \angle D = 360^0$ $[Angle\ sum\ property\ of\ quadrilateral]$

$\angle C + \angle D = 360^0 - (\angle A + \angle B)$....(ii)

Substituting (ii) in (i)

=>$\angle COD = 180 - \frac{1}{2}(360 - (\angle A + \angle B))$

=>$\angle COD = 180 - 180 + \frac{1}{2}(\angle A + \angle B))$

=>$\angle COD = \frac{1}{2}(\angle A + \angle B))$

4) The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Solution:

Let the common ratio between the angles is 't'

So the angles will be 3t, 5t, 9t and 13t respectively.

Since the sum of all interior angles of a quadrilateral is 360^0

Therefore, 3t + 5t + 9t + 13t = 360^0

=>30t = 360^0

=>t = 12^0

Hence, the angles are

3t = 3*12 = 36^0

5t = 5*12 = 60^0

9t = 9*12 = 108^0

13t = 13*12 = 156^0

RD Sharma Solutions Class 9 Chapter 14 Ex 14.2

Q1) Two opposite angles of a parallelogram are $(3x-2)^0$ and $(50-x)^0$. Find the measure of each angle of the parallelogram.

Solution:

We know that,

Opposite sides of a parallelogram are equal.

$(3x-2)^0 = (50-x)^0$

$\Rightarrow 3x + x = 50 + 2$

$\Rightarrow 4x = 52$

$\Rightarrow x = 13^0$

Therefore, $(3x-2)^0 = (3*13-2) = 37^0$

$(50-x)^0 = (50-13) = 37^0$

Adjacent angles of a parallelogram are supplementary.

$\therefore x + 37 = 180^0$

$\therefore x = 180^0 - 37^0 = 143^0$

Hence, four angles are : $37^0, 143^0, 37^0, 143^0$.

Q2) If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Solution:

Let the measure of the angle be x.

Therefore, the measure of the angle adjacent is $\frac{2x}{3}$

We know that the adjacent angle of a parallelogram is supplementary.

Hence, $x + \frac{2x}{3} = 180^0$

$2x + 3x = 540^0$

$\Rightarrow 5x = 540^0$

$\Rightarrow x = 108^0$

Adjacent angles are supplementary

$\Rightarrow x + 108^0 = 180^0$

$\Rightarrow x = 180^0 - 108^0 = 72^0$

$\Rightarrow x = 72^0$

Hence, four angles are $180^0, 72^0, 180^0, 72^0$

Q3) Find the measure of all the angles of a parallelogram, if one angle is 24^0 less than twice the smallest angle.

Solution:

$x + 2x - 24 = 180^0$

$\Rightarrow 3x - 24 = 180^0$

$\Rightarrow 3x = 108^0 + 24$

$\Rightarrow 3x = 204^0$

$\Rightarrow x = \frac{204}{3} = 68^0$

$\Rightarrow x = 68^0$

$\Rightarrow 2x - 24^0 = 2*68^0 - 24^0 = 112^0$

Hence, four angles are $68^0, 112^0, 68^0, 112^0$.

Q4) The perimeter of a parallelogram is 22cm. If the longer side measures 6.5cm what is the measure of the shorter side?

Solution:

Let the shorter side be 'x'.

Therefore, perimeter = x + 6.5 + 6.5 + x [Sum of all sides]

22 = 2 (x + 6.5)

11 = x + 6.5

=>x = 11 − 6.5 = 4.5cm

Therefore, shorter side = 4.5cm

Q5) In a parallelogram ABCD, $\angle D = 135^0$. Determine the measures of $\angle A$ and $\angle B$.

Solution:

In a parallelogram ABCD

Adjacent angles are supplementary

So, $\angle D + \angle C = 180^0$

$\angle C = 180^0 - 135^0$

$\angle C = 45^0$

In a parallelogram opposite sides are equal.

$\angle A = \angle C = 45^0$

$\angle B = \angle D = 135^0$

Q6) ABCD is a parallelogram in which $\angle A = 70^0$. Compute $\angle B,\ \angle C\ and\ \angle D.$

Solution:

In a parallelogram ABCD

$\angle A = 70^0$

$\angle A + \angle B = 180^0$ [Since, adjacent angles are supplementary]

$70^0 + \angle B = 180^0$ [$\because \angle$ A=70°]

$\angle B = 180^0 - 70^0$

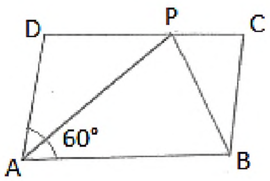
$\angle B = 110^0$

In a parallelogram opposite sides are equal.

$\angle A = \angle C = 70^0$

$\angle B = \angle D = 110^0$

Q7) In Figure 14.34, ABCD is a parallelogram in which $\angle A = 60^0$. If the bisectors of $\angle A,\ and\ \angle B$ meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Solution:

AP bisects $\angle A$

Then, $\angle DAP = \angle PAB = 30^0$

Adjacent angles are supplementary

Then, $\angle A + \angle B = 180^0$

$\angle B + 60^0 = 180^0$

$\angle B = 180^0 - 60^0$

$\angle B = 120^0$

BP bisects $\angle B$

Then, $\angle PBA = \angle PBC = 30^0$

$\angle PAB = \angle APD = 30^0$ [Alternate interior angles]

Therefore, AD = DP [Sides opposite to equal angles are in equal length]

Similarly

$\angle PBA = \angle BPC = 60^0$ [Alternate interior angles]

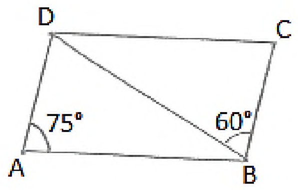
Therefore, PC = BC

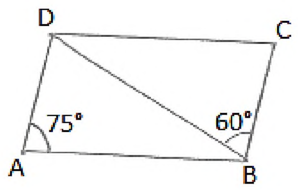
DC = DP + PC

DC = AD + BC [Since, DP = AD and PC = BC]

DC = 2AD [Since, AD = BC, opposite sides of a parallelogram are equal]

Q8) In figure 14.35, ABCD is a parallelogram in which $\angle DAB = 75^0\ and\ \angle DBC = 60^0$. Compute $\angle CDB,\ and\ \angle ADB.$





Solution:

To find $\angle CDB \text{ and } \angle ADB$

$\angle CBD = \angle ABD = 60^0$ [Alternate interior angle. AD||BC and BD is the transversal]

In $\angle BDC$

$\angle CBD + \angle C + \angle CDB = 180^0$ [Angle sum property]

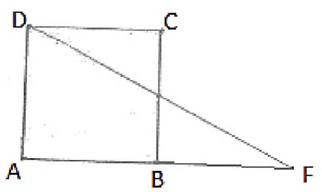
$\Rightarrow 60^0 + 75^0 + \angle CDB = 180^0$

$\Rightarrow \angle CDB = 180^0 - (60^0 + 75^0)$

$\Rightarrow \angle CDB = 45^0$

Hence, $\angle CDB = 45^0,\ \angle ADB = 60^0$

Q9) In figure 14.36, ABCD is a parallelogram and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that AF = 2AB.



Solution:

In $\Delta BEF \text{ and } \Delta CED$

$\angle BEF = \angle CED$ [Verified opposite angle]

BE = CE [Since, E is the mid-point of BC]

$\angle EBF = \angle ECD$ [Since, Alternate interior angles are equal]

$\therefore \Delta BEF \cong \Delta CED$ [ASA congruence]

$\therefore BF = CD \quad [CPCT]$

AF = AB + AF

AF = AB + AB

AF = 2AB.

Hence proved.

Q10) Which of the following statements are true (T) and which are false (F)?

(i) In a parallelogram, the diagonals are equal.

(ii) In a parallelogram, the diagonals bisect each other.

(iii) In a parallelogram, the diagonals intersect each other at right angles.

(iv) In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.

(v) If all the angles of a quadrilateral are equal, it is a parallelogram.

(vi) If three sides of a quadrilateral are equal, it is a parallelogram.

(vii) If three angles of a quadrilateral are equal, it is a parallelogram.

(viii) If all the sides of a quadrilateral are equal, it is a parallelogram.

Solution:

(i) False

(ii) True

(iii) False

(iv) False

(v) True

(vi) False

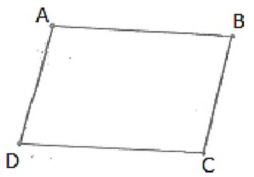
(vii) False

(viii) True

RD Sharma Solutions Class 9 Chapter 14 Ex 14.3

Q1) In a parallelogram ABCD, determine the sum of angles $\angle C\ and\ \angle D$.

Solution:



$\angle C\ and\ \angle D$ are consecutive interior angles on the same side of the transversal CD.

$\therefore \angle C + \angle D = 180^0$

Q2) In a parallelogram ABCD, If $\angle B = 135^0$, determine the measures of its other angles.

Solution:

Given $\angle B = 135^0$

ABCD is a parallelogram

$\therefore \angle A = \angle C,\ \angle B = \angle D \ and \ \angle A + \angle B = 180^0$

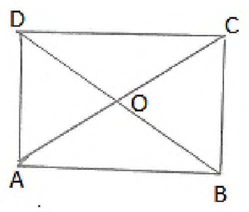
$\Rightarrow \angle A + 135^0 = 180^0$

$\Rightarrow \angle A = 45^0$

$\Rightarrow \angle A = \angle C = 45^0 \ and \ \angle B = \angle C = 135^0$

Q3) ABCD is a square. AC and BD intersect at O. State the measure of $\angle AOB$.

Solution:

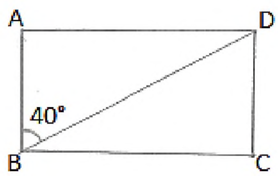


Since, diagonals of a square bisect each other at right angle.

$\therefore \angle AOB = 90^0$

Q4) ABCD is a rectangle with $\angle ABD = 40^0$. Determine $\angle DBC$

Solution:



We have,

$\angle ABC = 90^0$

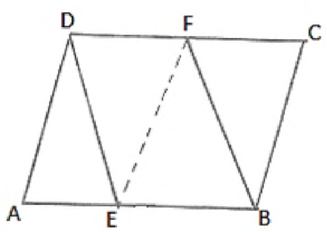
$\Rightarrow \angle ABD + \angle DBC = 90^0 \qquad [\because \angle ABD = 40^0]$

$\Rightarrow 40^0 + \angle DBC = 90^0$

$\therefore \angle DBC = 50^0$

Q5) The sides AB and CD of a parallelogram ABCD are bisected at E and F. Prove that EBFD is a parallelogram.

Solution:



Since ABCD is a parallelogram

$\therefore AB \parallel DC \ and \ AB = DC$

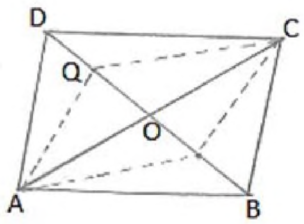
$\Rightarrow EB \parallel DF \ and \ \frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow EB \parallel DF \ and \ EB = DF$

EBFD is a parallelogram.

Q6) P and Q are the points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP. Prove also that AC bisects PQ.

Solution:



We know that,

Diagonals of a parallelogram bisect each other.

Therefore, OA = OC and OB = OD

Since P and Q are point of intersection of BD.

Therefore, BP = PQ = QD

Now, OB = OD are BP = QD

=>OB − BP = OD − QD

=>OP = OQ

Thus in quadrilateral APCQ, we have

OA = OC and OP = OQ

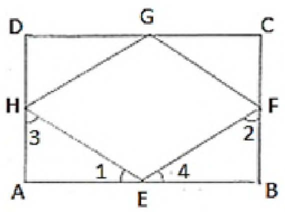
Diagonals of Quadrilateral APCQ bisect each other.

Therefore APCQ is a parallelogram.

Hence $AP \parallel CQ.$

Q7) ABCD is a square. E, F, G and H are points on AB, BC, CD and DA respectively, such that AE = BF = CG = DH. Prove that EFGH is a square.

Solution:



We have,

AE = BF = CG = DH = x (say)

BE = CF = DG = AH = y (say)

$In\ \Delta AEH\ and \Delta BEF,\ we\ have$

AE = BF

$\angle A = \angle B$

And AH = BE

So, by SAS congruency criterion, we have

$\Delta AEH \simeq \Delta BFE$

$\Rightarrow \angle 1 = \angle 2\ and\ \angle 3 = \angle 4$

But $\angle 1 + \angle 3 = 90^0\ and\ \angle 2 + \angle A = 90^0$

$\Rightarrow \angle 1 + \angle 3 + \angle 2 + \angle A = 90^0 + 90^0$

$\Rightarrow \angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^0$

$\Rightarrow 2(\angle 1 + \angle 4) = 180^0$

$\Rightarrow \angle 1 + \angle 4 = 90^0$

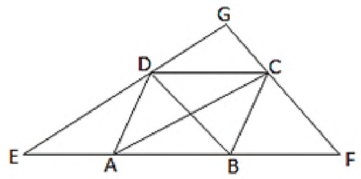
HEF=90^0

Similarly we have $\angle F = \angle G = \angle H = 90^0$

Hence, EFGH is a Square.

Q8) ABCD is a rhombus, EAFB is a straight line such that EA = AB = BF. Prove that ED and FC when produced meet at right angles.

Solution:



We know that the diagonals of a rhombus are perpendicular bisector of each other.

$\therefore OA = OC,\ OB = OD,\ and\ \angle AOD = \angle COD = 90^0$

And $\angle AOB = \angle COB = 90^0$

In ΔBDE, A and O are mid-points of BE and BD respectively.

$OA \parallel DE$

$OC \parallel DG$

In ΔCFA, B and O are mid-points of AF and AC respectively.

$OB \parallel CF$

$OD \parallel GC$

Thus, in quadrilateral DOGC, we have

$OC \parallel DG\ and\ OD \parallel GC$

=>DOCG is a parallelogram

$\angle DGC = \angle DOC$

$\angle DGC = 90^0$

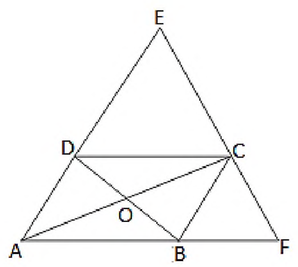
Q9) ABCD is a parallelogram, AD is produced to E so that DE = DC and EC produced meets AB produced in F. Prove that BF = BC.

Solution:

Draw a parallelogram ABCD with AC and BD intersecting at O.

Produce AD to E such that DE = DC

Join EC and produce it to meet AB produced at F.



In ΔDCE,

$\angle DCE = \angle DEC \ldots\ldots (i)$ [In a triangle, equal sides have equal angles]

$AB \parallel CD$ [Opposite sides of the parallelogram are parallel]

$\therefore AE \parallel CD$ [AB lies on AF]

$AF \parallel CD$ and EF is the Transversal.

$\angle DCE = \angle BFC \ldots\ldots (ii)$ [Pair of corresponding angles]

From (i) and (ii) we get

$\angle DEC = \angle BFC$

In ΔAFE,

$\angle AFE = \angle AEF$ [∠ DEC = ∠ BFC]

Therefore, AE = AF [In a triangle, equal angles have equal sides opposite to them]

=>AD + DE = AB + BF

=>BC + AB = AB + BF [Since, AD = BC, DE = CD and CD = AB, AB = DE]

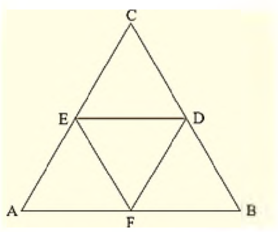
=> BC = BF

Hence proved.

RD Sharma Solutions Class 9 Chapter 14 Ex 14.4

Q1) In ΔABC, D, E and F are, respectively the mid points of BC, CA and AB. If the lengths of sides AB, BC and CA are 7cm, 8cm and 9cm, respectively, find the perimeter of ΔDEF.

Solution:



Given that,

AB = 7cm, BC = 8cm, AC = 9cm

In ΔABC,

F and E are the mid points of AB and AC.

$\therefore EF = \frac{1}{2}BC$

Similarly

$DF = \frac{1}{2}AC \; and \; DE = \frac{1}{2}AB$

Perimeter of ΔDEF = DE + EF + DF

=$\frac{1}{2}AB + \frac{1}{2}BC + \frac{1}{2}AC$

=$\frac{1}{2} * 7 + \frac{1}{2} * 8 + \frac{1}{2} * 9$

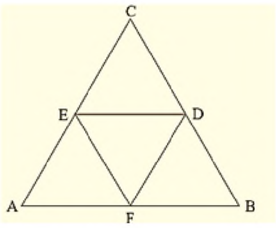
=3.5 + 4 + 4.5

=12cm

$\therefore Perimeter \; of \; \Delta DEF = 12cm$

Q2) In a ΔABC, $\angle A = 50^0$, $\angle B = 60^0$ and $\angle C = 70^0$. Find the measures of the angles of the triangle formed by joining the mid-points of the sides of this triangle.

Solution:



In ΔABC,

D and E are mid points of AB and BC.

By Mid point theorem,

$DE \parallel AC, \; DE = \frac{1}{2}AC$

F is the midpoint of AC

Then, $DE = \frac{1}{2}AC = CF$

In a Quadrilateral DECF

$DE \parallel AC, \; DE = CF$

Hence DECF is a parallelogram.

$\therefore \angle C = \angle D = 70^0$ [Opposite sides of a parallelogram]

Similarly

BEFD is a parallelogram, $\angle B = \angle F = 60^0$

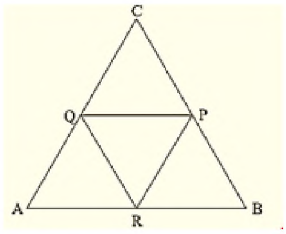
ADEF is a parallelogram, $\angle A = \angle E = 50^0$

$\therefore Angles \; of \; \Delta DEF \; are$

$\angle D = 70^0, \; \angle E = 50^0, \; \angle F = 60^0$

Q3) In a triangle, P, Q and R are the mid points of sides BC, CA and AB respectively. If AC = 21cm, BC = 29cm and AB = 30cm, find the perimeter of the quadrilateral ARPQ.

Solution:



In ΔABC,

R and P are mid points of AB and BC

$RP \parallel AC,\ RP = \frac{1}{2}AC \quad [By\ Midpoint\ Theorem]$

In a quadrilateral,

[A pair of side is parallel and equal]

$RP \parallel AQ,\ RP = AQ$

Therefore, RPQA is a parallelogram

$\Rightarrow AR = \frac{1}{2}AB = \frac{1}{2} * 30 = 15cm$

AR = QP = 15cm [Opposite sides are equal]

$\Rightarrow RP = \frac{1}{2}AC = \frac{1}{2} * 21 = 10.5cm$

RP = AQ = 10.5cm [Opposite sides are equal]

Now,

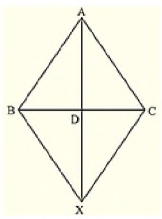
Perimeter of ARPQ = AR + QP + RP +AQ

= 15 +15 +10.5 +10.5

= 51cm

Q4) In a ΔABC median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Solution:



In a quadrilateral ABXC, we have

AD = DX [Given]

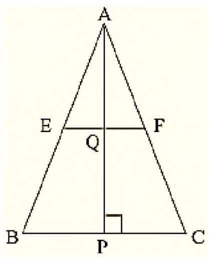
BD = DC [Given]

So, diagonals AX and BC bisect each other.

Therefore, ABXC is a parallelogram.

Q5) In a ΔABC, E and F are the mid-points of AC and AB respectively. The altitude AP to BC intersects FE at Q. Prove that AQ = QP.

Solution:



In a ΔABC

E and F are mid points of AB and AC

$\therefore EF \parallel FE,\ \frac{1}{2}BC = FE \quad [By\ mid\ point\ theorem]$

In ΔABP

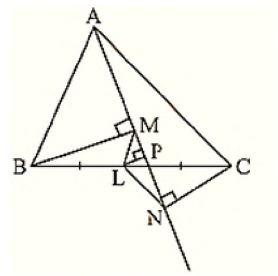
F is the mid-point of AB and $\therefore FQ \parallel BP \quad [\because EF \parallel BP]$

Therefore, Q is the mid-point of AP [By mid-point theorem]

Hence, AQ = QP.

Q6) In a ΔABC, BM and CN are perpendiculars from B and C respectively on any line passing through A. If L is the mid-point of BC, prove that ML = NL.

Solution:



Given that,

In $\Delta BLM \ and \ \Delta CLN$

$\angle BML = \angle CNL = 90^0$

BL = CL [L is the mid-point of BC]

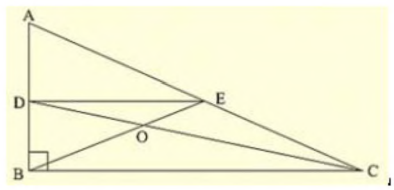
$\angle MLB = \angle NLC$ [Vertically opposite angle]

$\therefore \Delta BLM = \Delta CLN$

$\therefore LM = LN$ [corresponding parts of congruent triangles]

Q7)In figure 14.95, Triangle ABC is a right angled triangle at B. Given that AB = 9cm, AC = 15cm and D, E are the mid-points of the sides AB and AC respectively, calculate

(i) The length of BC ***(ii)*** $The \ area \ of \ \Delta ADE.$



Solution:

In $\Delta ABC, \ \angle B = 90^0$,

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$=> 15^2 = 9^2 + BC^2$

$=> BC = \sqrt{15^2 - 9^2}$

$=> BC = \sqrt{225 - 81}$

$=> BC = \sqrt{144}$ = 12cm

In ΔABC,

D and E are mid-points of AB and AC

$\therefore DE \parallel BC, \ DE = \frac{1}{2} BC \quad [By \ mid - point \ theorem]$

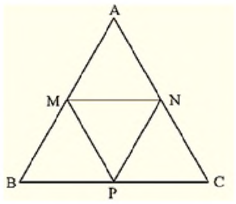
$AD = DB = \frac{AB}{2} = \frac{9}{2} = 4.5cm \quad [\because D \ is \ the \ mid - point \ of \ AB]]$

$Area \ of \ \Delta ADE = \frac{1}{2} * AD * DE$

$= \frac{1}{2} * 4.5 * 6$

$=13.5cm^2$

Q8) In figure 14.96, M, N and P are mid-points of AB, AC and BC respectively. If MN = 3cm, NP = 3.5cm and MP = 2.5cm, calculate BC, AB and AC.



Solution:

Given MN = 3cm, NP = 3.5cm and MP = 2.5cm.

To find BC, AB and AC

In ΔABC

M and N are mid-points of AB and AC

$\therefore MN = \frac{1}{2}BC,\ MN \parallel BC \qquad [By\ mid-point\ theorem]$

$\Rightarrow 3 = \frac{1}{2}BC$

$\Rightarrow 3 * 2 = BC$

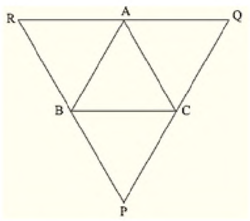
$\Rightarrow BC = 6cm$

Similarly

AC = 2MP = 2 (2.5) = 5cm

AB = 2 NP = 2 (3.5) = 7cm

Q9) ABC is a triangle and through A, B, C lines are drawn parallel to BC, CA and AB respectively intersecting at P, Q and R. Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.



Solution:

Clearly ABCQ and ARBC are parallelograms.

Therefore, BC = AQ and BC = AR

=>AQ = AR

=>A is the mid-point of QR

Similarly B and C are the mid points of PR and PQ respectively.

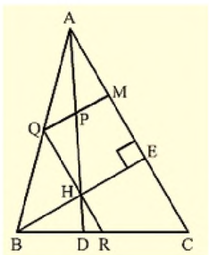
$\therefore AB = \frac{1}{2}PQ,\ BC = \frac{1}{2}QR,\ CA = \frac{1}{2}PR$

=>PQ = 2AB, QR = 2BC and PR = 2CA

=>PQ + QR + RP = 2 (AB + BC + CA)

=>Perimeter of ΔPQR = 2 (perimeter of ΔABC)

Q10) In figure 14.97, $BE \perp AC$, AD is any line from A to BC intersecting BE in H. P, Q and R are respectively the mid-points of AH, AB and BC. Prove that $\angle PQR = 90^0$



Solution:

Given,

$BE \perp AC$ and P, Q and R are respectively mid-point of AH, AB and BC.

To prove: $\angle PQR = 90^0$

Proof: In ΔABC, Q and R are mid-points of AB and BC respectively.

$\therefore QR \parallel AC \ldots.. (i)$

In ΔABH, Q and P are the mid-points of AB and AH respectively

$\therefore QP \parallel BH \ldots. (ii)$

But, $BE \perp AC$

Therefore, from equation (i) and equation (ii) we have,

$QP \perp QR$

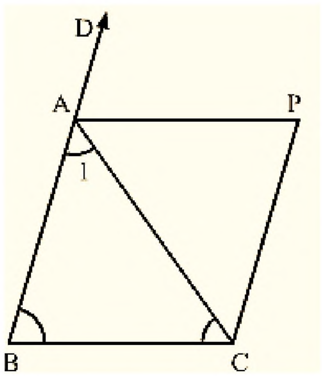
=>$\angle PQR = 90^0$

Hence Proved.

Q11) In figure 14.98, AB=AC and CP||BA and AP is the bisector of exterior $\angle CAD$ of ΔABC . Prove that

(i) $\angle PAC = \angle BCA.$

(ii) ABCP is a parallelogram.



Solution:

Given,

AB = AC and $CD \parallel BA$ and AP is the bisector of exterior $\angle CAD \ of \ \Delta ABC$

To prove:

(i) $\angle PAC = \angle BCA$

(ii) ABCP is a parallelogram.

Proof:

(i) We have,

AB=AC

=>$\angle ACB = \angle ABC$ [Opposite angles of equal sides of triangle are equal]

Now, $\angle CAD = \angle ABC + \angle ACB$

=>$\angle PAC + \angle PAD = 2\angle ACB$ $[\because \angle PAC = \angle PAD]$

=>$2\angle PAC = 2\angle ACB$

=>$\angle PAC = \angle ACB$

(ii) Now,

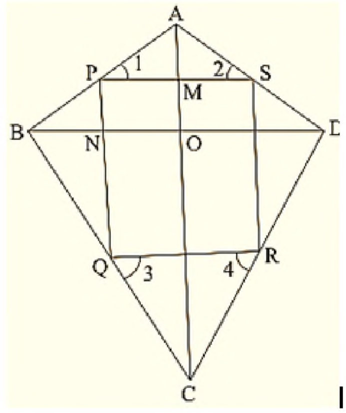
$\angle PAC = \angle BCA$

=>$AP \parallel BC$ and $CP \parallel BA$ [Given]

Therefore, ABCP is a parallelogram.

Q12) ABCD is a kite having AB=AD and BC=CD. Prove that the figure found by joining the mid points of the sides, in order, is a rectangle.

Solution:



Given,

A kite ABCD having AB=AD and BC=CD. P, Q, R, S are the mid-points of sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

To prove:

PQRS is a rectangle.

Proof:

In ΔABC, P and Q are the mid-points of AB and BC respectively.

$\therefore PQ \parallel AC \text{ and } PQ = \frac{1}{2}AC \ldots..(i)$

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore RS \parallel AC \text{ and } RS = \frac{1}{2}AC \ldots..(ii)$

From (i) and (ii) we have

$PQ \parallel RS \text{ and } PQ = RS$

Thus, in quadrilateral PQRS, a pair of opposite sides is equal and parallel. So, PQRS is a parallelogram. Now, we shall prove that one angle of parallelogram PQRS is a right angle.

Since AB=AD

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AD$

$\Rightarrow AP = AS \ldots.(iii) \quad [\because P \text{ and } S \text{ are mid points of } AB \text{ and } AD]$

$\Rightarrow \angle 1 = \angle 2 \ldots.(iv)$

Now, in $\Delta PBQ \text{ and } \Delta SDR, \text{ we have}$

PB = SD $\quad [\because AD = AB \Rightarrow \frac{1}{2}AD = \frac{1}{2}AB]$

BQ = DR [Since PB = SD]

And PQ = SR [Since, PQRS is a parallelogram]

So, by SSS criterion of congruence, we have

$\Delta PBQ \cong \Delta SDR$

$\Rightarrow \angle 3 = \angle 4 \quad [CPCT]$

Now, $\Rightarrow \angle 3 + \angle SPQ + \angle 2 = 180^0$

And $\angle 1 + \angle PSR + \angle 4 = 180^0$

$\therefore \angle 3 + \angle SPQ + \angle 2 = \angle 1 + \angle PSR + \angle 4$

$\Rightarrow \angle SPQ = \angle PSR \quad [\,\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4\,]$

Now, transversal PS cuts parallel lines SR and PQ at S and P respectively.

$\therefore \angle SPQ + \angle PSR = 180^0$

=>$2\angle SPQ = 180^0$

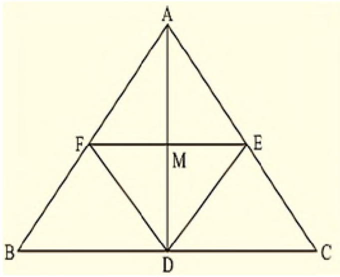
=>$\angle SPQ = 90^0 \quad [\because \angle PSR = \angle SPQ]$

Thus, PQRS is a parallelogram such that $\angle SPQ = 90^0$.

Hence, PQRS is a parallelogram.

Q13) Let ABC be an isosceles triangle in which AB=AC. If D, E, F be the mid points of the, sides BC,CA and AB respectively, show that the segment AD and EF bisect each other at right angles.

Solution:



Since D, E and F are mid-points of sides BC, CA and AB respectively.

$\therefore AB \parallel DE \text{ and } AC \parallel DF$

$\therefore AF \parallel DE \text{ and } AE \parallel DF$

ABDE is a parallelogram.

AF = DE and AE = DF

$\frac{1}{2}AB = DE \text{ and } \frac{1}{2}AC = DF$

DE = DF [Since, AB = AC]

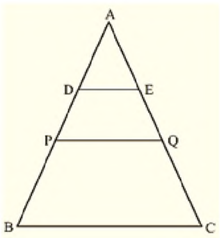
AE = AF = DE = DF

ABDF is a rhombus.

=>AD and FE bisect each other at right angle.

Q14) ABC is a triangle. D is a point on AB such that $AD = \frac{1}{4}AB$ and E is a point on AC such that $AE = \frac{1}{4}AC$. Prove that $DE = \frac{1}{4}BC$.

Solution:



Let P and Q be the mid-points of AB and AC respectively.

Then $PQ \parallel BC$

$PQ = \frac{1}{2}BC$.....(i)

In ΔAPQ, D and E are the mid-points of AP and AQ respectively.

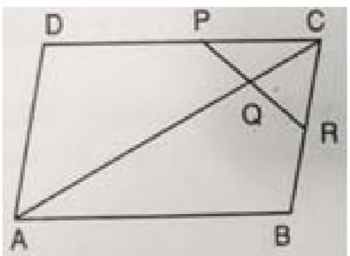
$\therefore DE \parallel PQ, \text{ and } DE = \frac{1}{2}PQ$.....(ii)

From (i) and (ii): $DE = \frac{1}{2}\ PQ = \frac{1}{2}\ (\frac{1}{2}BC)$

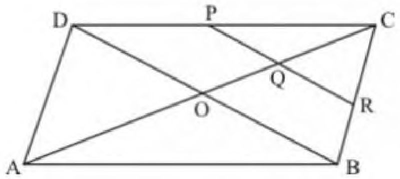
$\therefore DE = \frac{1}{4}BC$

Hence proved.

Q15) In Figure 14.99, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meets BC at R, prove that R is a mid-point of BC.



Solution:



Join B and D.

Suppose AC and BD intersect at O.

Then $OC = \frac{1}{2}AC$

Now,

$CQ = \frac{1}{4}AC$

$\Rightarrow CQ = \frac{1}{2}\left(\frac{1}{2}AC\right)$

= $\frac{1}{2}OC$

In ΔDCO, P and Q are mid points of DC and OC respectively.

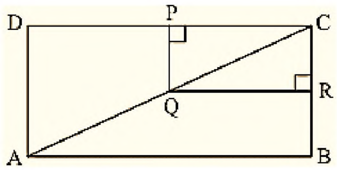
$\therefore PQ \parallel DO$

Also in ΔCOB, Q is the mid-point of OC and $QR \parallel OB$

Therefore, R is the mid-point of BC.

Q16) In figure 14.100, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that

(i) DP = PC ***(ii)*** $PR = \frac{1}{2}AC$



Solution:

(i) In ΔADC, Q is the mid-point of AC such that $PQ \parallel AD$

Therefore, P is the mid-point of DC.

=>DP = DC [Using mid-point theorem]

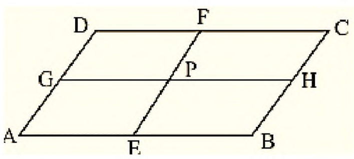
(ii) Similarly, R is the mid-point of BC

$\therefore PR = \frac{1}{2}BD$

$PR = \frac{1}{2}AC$ [Diagonal of rectangle are equal, BD = AC]

Q17) ABCD is a parallelogram; E and f are the mid-points of AB and CD respectively. GH is any line intersecting AD, EF and BC at G, P and H respectively. Prove that GP = PH.

Solution:



Since E and F are mid-points of AB and CD respectively

$AE = BE = \frac{1}{2}AB$

And $CF = DF = \frac{1}{2}CD$

But, AB = CD

$\frac{1}{2}AB = \frac{1}{2}CD$

=>BE = CF

Also, $BE \parallel CF$ $[\because AB \parallel CD]$

Therefore, BEFC is a parallelogram

$BC \parallel EF$ and BE = PH .....(i)

Now, $BC \parallel EF$

=>$AD \parallel EF$ $\quad [\because BC \parallel AD \ as \ ABCD \ is \ a \ parallelogram]$

Therefore, AEFD is a parallelogram.

=>AE = GP

But E is the mid-point of AB.

So, AE = BF

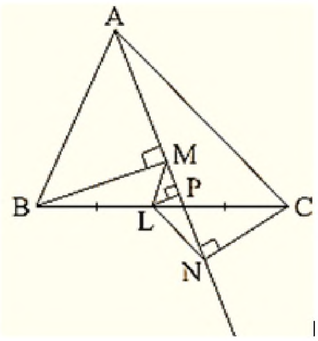
Therefore, GP = PH.

Q18) BM and CN are perpendiculars to a line passing through the vertex A of triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Solution:

To prove LM = LN

Draw LS as perpendicular to line MN.



Therefore, the lines BM, LS and CN being the same perpendiculars on line MN are parallel to each other.

According to intercept theorem,

If there are three or more parallel lines and the intercepts made by them on a transversal are equal, then the corresponding intercepts on any other transversal are also equal.

In the figure, MB, LS and NC are three parallel lines and the two transversal lines are MN and BC.

We have, BL = LC [As L is the given mid-point of BC]

Using the intercept theorem, we get

MS = SN .... (i)

Now in $\Delta MLS \ and \ \Delta LSN$

MS = SN using equation (i).

$\angle LSM = \angle LSN = 90^0 \quad [LS \perp MN]$

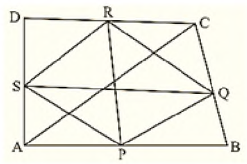
And SL = LS is common.

$\therefore \Delta MLS \cong \Delta LSN \quad [SAS \ Congruency \ Theorem]$

$\therefore LM = LN \quad [CPCT]$

Q19) Show that, the line segments joining the mid-points of opposite sides of a quadrilateral bisects each other.

Solution:



Let ABCD is a quadrilateral in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.

So, by using mid-point theorem we can say that

$SP \parallel BD \ and \ SP = \frac{1}{2}BD.....(i)$

Similarly in ΔBCD

$QR \parallel BD \ and \ QR = \frac{1}{2}BD.....(ii)$

From equations (i) and (ii), we have

$SP \parallel QR \ and \ SP = QR$

As in quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other.

So, SPQR is a parallelogram since the diagonals of a parallelogram bisect each other.

Hence PR and QS bisect each other.

Q20) Fill in the blanks to make the following statements correct:

(i) The triangle formed by joining the mid-points of the sides of an isosceles triangle is ___________.

(ii) The triangle formed by joining the mid-points of the sides of a right triangle is ___________.

(iii) The figure formed by joining the mid-points of consecutive sides of a quadrilateral is ___________.

Solution:

(i) Isosceles

(ii) Right triangle

(iii) Parallelogram