Exercise 4.1: Triangles

Q.1) Fill in the blanks using the correct word given in brackets:

(1.) All circles are ____________ (congruent, similar)

(2.) All squares are ___________ (similar, congruent)

(3.) All ____________ triangles are similar (isosceles, equilaterals)

(4.) Two triangles are similar, if their corresponding angles are ____________ (proportional, equal)

(5.) Two triangles are similar, if their corresponding sides are ____________ (proportional, equal)

(6.) Two polygons of the same number of sides are similar, if (a) ____________their corresponding angles are and (b) ____________ their corresponding sides are (equal, proportional)

Q.2) Write the truth value (T/F) of each of the following statements:

(1.) Any two similar figures are congruent.

(2.) Any two congruent figures are similar.

(3.) Two polygons are similar, if their sides are proportional.

(4.) Two polygons are similar, if their corresponding sides are proportional.

(5.) Two triangles are similar if their corresponding sides are proportional.

(6.) Two triangles are similar if their corresponding angles are proportional.

Sol.1:

(1) Similar

(2) Similar

(3) Equilateral

(4) Equal

(5) Proportional

(6) a.) Equal, b.) Proportional.

Soln.2:

(1) False

(2) True

(3) False

(4) False

(5) True

(6) True

Exercise 4.2: Triangles

Q.1: In a ΔΔ ABC, D and E are points on the sides AB and AC respectively such that DE ‖$\|$ BC.

1.) If AD = 6 cm, DB = 9 cm and AE = 8 cm, Find AC.

2.) If ADDB=34 $\frac{AD}{DB} = \frac{3}{4}$ and AC = 15 cm, Find AE.

3.) If ADDB=23 $\frac{AD}{DB} = \frac{2}{3}$ and AC = 18 cm, Find AE.

4.) If AD = 4 cm, AE = 8 cm, DB = x – 4 cm and EC = 3x – 19, find x.

5.) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

6.) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

7.) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.

8.) If ADBD=45 $\frac{AD}{BD} = \frac{4}{5}$ and EC = 2.5 cm, Find AE.

9.) If AD = x cm, DB = x – 2 cm, AE = x + 2 cm, and EC = x – 1 cm, find the value of x.

10.) If AD = 8x – 7 cm, DB = 5x – 3 cm, AE = 4x – 3 cm, and EC = (3x – 1) cm, Find the value of x.

11.) If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1, and CE = 5x – 3, find the value of x.

12.) If AD = 2.5 cm, BD = 3.0 cm, and AE = 3.75 cm, find the length of AC.

Sol:

1) It is given that ΔΔ ABC AND DE ||∥ BC

We have to find AC,

Since, AD = 6 cm,

DB = 9 cm and AE = 15 cm.

AB = 15 cm.

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 69=8x $\frac{6}{9} = \frac{8}{x}$

6x = 72 cm

x = 72/6 cm

x = 12 cm

Hence, **AC = 12 + 8 = 20.**

2) It is given that ADBD=34 $\frac{AD}{BD} = \frac{3}{4}$ and AC = 15 cm

We have to find out AE,

Let, AE = x

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 34=x15–x $\frac{3}{4} = \frac{x}{15–x}$

45 – 3x = 4x

-3x – 4x = – 45

7x = 45

x = 45/7

x = 6.43 cm

3) It is given that ADBD=23 $\frac{AD}{BD} = \frac{2}{3}$ and AC = 18 cm

We have to find out AE,

Let, AE = x and CE = 18 – x

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 23=x18–x $\frac{2}{3} = \frac{x}{18-x}$

3x = 36 – 2x

5x = 36 cm

X = 36/5 cm

X = 7.2 cm

Hence, **AE = 7.2 cm**

4) It is given that AD = 4 cm, AE = 8 cm, DB = x – 4 and EC = 3x – 19

We have to find x,

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 4x–4=83x–19 $\frac{4}{x-4} = \frac{8}{3x-19}$

4(3x – 19) = 8(x – 4)

12x – 76 = 8(x – 4)

12x – 8x = – 32 + 76

4x = 44 cm

X = 11 cm

5) It is given that AD = 8 cm, AB = 12 cm, and AE = 12 cm.

We have to find CE,

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 84=12CE $\frac{8}{4} = \frac{12}{CE}$

8CE = 4 X 12 cm

CE = (4 X 12)/8 cm

CE = 48/8 cm

CE = 6 cm

6) It is given that AD = 4 cm, DB = 4.5 cm, AE = 8 cm

We have to find out AC

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 44.5=8AC $\frac{4}{4.5} = \frac{8}{AC}$

AC=4.5×84 $AC = \frac{4.5 \times 8}{4}$ cm

AC = 9 cm

7) It is given that AD = 2 cm, AB = 6 cm, and AC = 9 cm

We have to find out AE

DB = 6 – 2 = 4 cm

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 24=x9–x $\frac{2}{4} = \frac{x}{9-x}$

4x = 18 – 2x

6x = 18

X = 3 cm

8) It is given that ADBD=45 $\frac{AD}{BD} = \frac{4}{5}$ and EC = 2.5 cm

We have to find out AE

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 45=AE2.5 $\frac{4}{5} = \frac{AE}{2.5}$

AE = 4×2.55 $\frac{4\times2.5}{5}$ = 2 cm

9) It is given that AD = x, DB = x – 2, AE = x + 2 and EC = x – 1

We have to find the value of x

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, xx–2=x+2x–1 $\frac{x}{x-2} = \frac{x+2}{x-1}$

X(x – 1) = (x – 2)(x + 2)

$x^2 - x - x^2 + 4 = 0$

x =4

10) It is given that AD = 8x – 7, DB = 5x – 3, AER = 4x – 3 and EC = 3x -1

We have to find the value of x

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 8x–75x–3=4x–33x–1 $\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

(8x – 7)(3x – 1) = (5x – 3)(4x – 3)

$24x^2 - 29x + 7 = 20x^2 - 27x + 9$

$4x^2 - 2x - 2 = 0$

$2(2x^2 - x - 1) = 0$

$2x^2 – x – 1 = 0$

$2x^2 – 2x + x – 1 = 0$

$2x(x – 1) + 1(x – 1) = 0$

$(x – 1)(2x + 1) = 0$

X = 1 or x = -1/2

Since the side of triangle can never be negative

Therefore, x = 1.

11) It is given that AD = 4x – 3, BD = 3x – 1, AE = 8x – 7 and EC = 5x – 3

For finding the value of x

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 4x–33x–1=8x–75x–3 $\frac{4x–3}{3x–1} = \frac{8x–7}{5x–3}$

$(4x – 3)(5x – 3) = (3x – 1)(8x – 7)$

$4x(5x – 3) -3(5x – 3) = 3x(8x – 7) -1(8x – 7)$

$20x^2 – 12x – 15x + 9 = 24x^2 – 29x + 7$

$20x^2 -27x + 9 = 24^2 -29x + 7$

Then,

$-4x^2 + 2x + 2 = 0$

$4x^2 – 2x – 2 = 0$

$4x^2 – 4x + 2x – 2 = 0$

$4x(x – 1) + 2(x – 1) = 0$

$(4x + 2)(x – 1) = 0$

X = 1 or x = -2/4

Since, side of triangle can never be negative

Therefore x = 1

12) It is given that, AD = 2.5 cm, AE = 3.75 cm and BD = 3 cm

So, ADBD=AECE $\frac{AD}{BD} = \frac{AE}{CE}$ (using Thales Theorem)

Then, 2.53=3.75CE $\frac{2.5}{3} = \frac{3.75}{CE}$

2.5CE = 3.75 x 3

CE=3.75×32.5$CE = \frac{3.75 \times 3}{2.5}$ CE=11.252.5$CE = \frac{11.25}{2.5}$

CE = 4.5

Now, AC = 3.75 + 4.5

AC = 8.25 cm.

Q.2) In a ΔΔ ABC, D and E are points on the sides AB and AC respectively. For each of the following cases show that DE ‖|| BC.

1.) AB = 12 cm, AD = 8 cm, AE = 12 cm, and AC = 18 cm.

2.) AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm, and AE = 1.8 cm.

3.) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

4.) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm.

Sol:

1) It is given that D and R are the points on sides AB and AC.

We have to find that DE ‖|| BC.

Acc. To Thales Theorem,

ADDB=AECE $\frac{AD}{DB} = \frac{AE}{CE}$ 84=126 $\frac{8}{4} = \frac{12}{6}$

2 = 2 (LHS = RHS)

Hence, DE ‖|| BC.

2) It is given that D and E are the points on sides AB and AC

We need to prove that DE ‖|| BC

Acc. To Thales Theorem,

ADDB=AECE $\frac{AD}{DB} = \frac{AE}{CE}$ 1.44.2=1.85.4 $\frac{1.4}{4.2} = \frac{1.8}{5.4}$

13=13 $\frac{1}{3} = \frac{1}{3}$ (RHS)

Hence, DE ‖|| BC.

3) It is given that D and E are the points on sides AB and AC.

We need to prove DE ‖|| BC.

Acc. To Thales Theorem,

ADDB=AECE $\frac{AD}{DB} = \frac{AE}{CE}$

AD = AB – DB = 10.8 – 4.5 = 6.3

And,

EC = AC – AE = 4.8 – 2.8 = 2

Now,

6.34.5=2.82.0 $\frac{6.3}{4.5} = \frac{2.8}{2.0}$

Hence, DE ‖|| BC.

4) It is given that D and E are the points on sides AB and Ac.

We need to prove that DE ‖|| BC.

Acc. To Thales Theorem,

ADDB=AECE $\frac{AD}{DB} = \frac{AE}{CE}$ 5.79.5=3.35.5 $\frac{5.7}{9.5} = \frac{3.3}{5.5}$

35=35 $\frac{3}{5} = \frac{3}{5}$ (LHS = RHS)

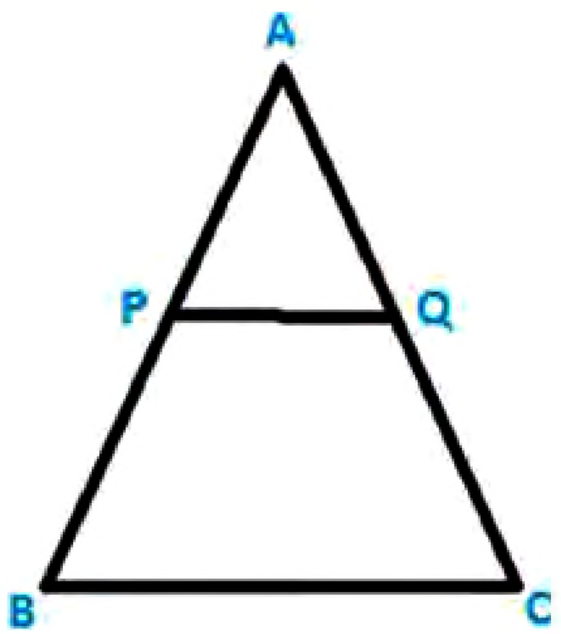
Hence, DE ∥|| BC.

Q.3) In a ΔΔ ABC, P and Q are the points on sides AB and AC respectively, such that PQ ∥|| BC. If AP = 2.4 cm, AQ = 2 cm, QC = 3 cm, and BC = 6 cm, Find AB and PQ.

Sol:

It is given that AP = 2.4 cm, AQ = 2 cm, QC = 3 cm, and BC = 6 cm.

We need to find AB and PQ.



Using Thales Theorem,

APPB=AQQC $\frac{AP}{PB} = \frac{AQ}{QC}$ 2.4PB=23 $\frac{2.4}{PB} = \frac{2}{3}$

2PB = 2.4 x 3 cm

PB=2.4×32 $PB = \frac{2.4 \times 3}{2}$ cm

PB = 3.6 cm

Now, AB = AP + PB

AB = 2.4 + 3.6

AB = 6 cm

Since, PQ || $||$ BC, AB is transversal, then,

ΔΔ APQ = ΔΔ ABC (by corresponding angles)

Since, PQ || $||$ BC, AC is transversal, then,

ΔΔ APQ = ΔΔ ABC (by corresponding angles)

In ΔΔ ABQ and ΔΔ ABC,

∠APQ=∠ABC $\angle APQ = \angle ABC$ ∠AQP=∠ACB $\angle AQP = \angle ACB$

Therefore, ΔΔ APQ = ΔΔ ABC (angle angle similarity)

Since, the corresponding sides of similar triangles are proportional,

Therefore, APAB=PQBC=AQAC $\frac{AP}{AB} = \frac{PQ}{BC} = \frac{AQ}{AC}$

APAB=PQBC $\frac{AP}{AB} = \frac{PQ}{BC}$ 2.46=PQ6 $\frac{2.4}{6} = \frac{PQ}{6}$

Therefore, PQ = 2.4 cm.

Q.4) In a ΔΔ ABC, D and E are points on AB and AC respectively, such that DE || $||$ BC. If AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm, and BC = 5 cm. Find BD and CE.

Sol: It is given that AD = 2.4 cm, AE = 3.2 cm, DE = 2 cm and BE = 5 cm.

We need to find BD and CE.

Since, DE || $||$ BC, AB is transversal, then,

∠APQ=∠ABC $\angle APQ = \angle ABC$

Since, DE || $||$ BC, AC is transversal, then,

∠AED=∠ACB $\angle AED = \angle ACB$

In ΔΔ ADE and ΔΔ ABC,

∠ADE=∠ABC $\angle ADE = \angle ABC$ ∠AED=∠ACB $\angle AED = \angle ACB$

So, ΔΔ ADE = ΔΔ ABC (angle angle similarity)

Since, the corresponding sides of similar triangles are proportional, then,

Therefore, ADAB=AEAC=DEBC $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

ADAB=DEBC $\frac{AD}{AB} = \frac{DE}{BC}$ 2.42.4+DB=25 $\frac{2.4}{2.4+DB} = \frac{2}{5}$

2.4 + DB = 6

DB = 6 – 2.4

DB = 3.6 cm

Similarly, AEAC=DEBC $\frac{AE}{AC} = \frac{DE}{BC}$

3.23.2+EC=25 $\frac{3.2}{3.2+EC} = \frac{2}{5}$

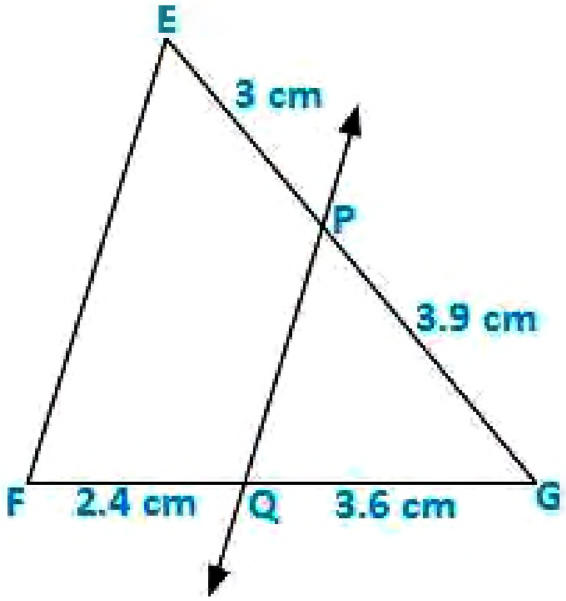
3.2 + EC = 8

EC = 8 – 3.2

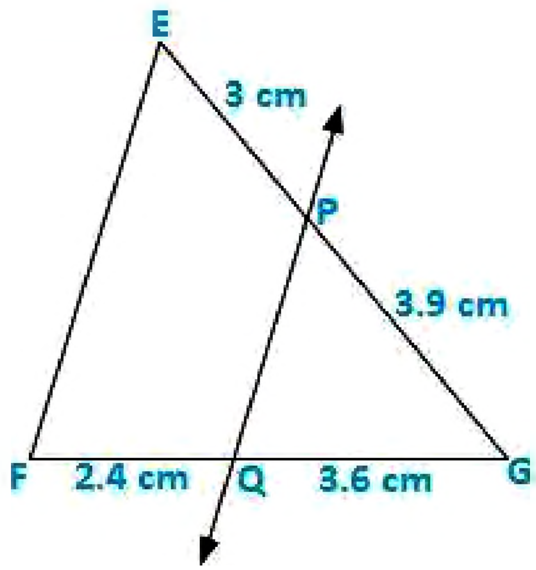
EC = 4.8 cm

Therefore, BD = 3.6 cm and CE = 4.8 cm.

Q.5) In figure given below, state PQ ∥ $\parallel$ EF.



Sol:



It is given that EP = 3 cm, PG = 3.9 cm, FQ = 3.6 cm and QG = 2.4 cm

We have to check that PQ ||‖ EF or not.

Acc. to Thales Theorem,

PGGE=GQFQ $\frac{PG}{GE} = \frac{GQ}{FQ}$

Now,

3.93≠3.62.4 $\frac{3.9}{3} \neq \frac{3.6}{2.4}$

As we can see it is not prortional.

So, PQ is not parallel to EF.

Q.6) M and N are the points on the sides PQ and PR respectively, of a Δ$\triangle$ PQR. For each of the following cases, state whether MN ‖$\parallel$ QR.

(i) PM = 4 cm, QM = 4.5 cm, PN = 4 cm, NR = 4.5 cm.

(ii) PQ = 1.28 cm, PR = 2.56 cm, PM = 0.16 cm, PN = 0.32 cm.

Sol:

(i) It is given that PM = 4 cm, QM = 4.5 cm, PN = 4 cm, and NR = 4.5 cm.

We have to check that MN ‖$\parallel$ QR or not.

Acc. to Thales Theorem,

PMQM=PNNR $\frac{PM}{QM} = \frac{PN}{NR}$ 44.5=44.5 $\frac{4}{4.5} = \frac{4}{4.5}$

Hence, MN ‖$\parallel$ QR.

(ii) It is given that PQ = 1.28 cm, PR = 2.56 cm, PM = 0.16 cm, and PN = 0.32 cm.

We have to check that MN ‖$\parallel$ QR or not.

Acc. to Thales Theorem,

PMQM=PNNR $\frac{PM}{QM} = \frac{PN}{NR}$

Now,

PMMQ=0.161.12 $\frac{PM}{MQ} = \frac{0.16}{1.12}$ = 1/7

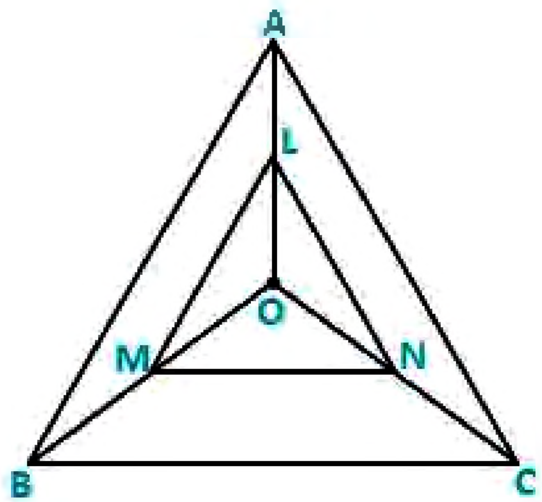
PNNR=0.322.24 $\frac{PN}{NR} = \frac{0.32}{2.24}$ = 1/7

Since,

0.161.12=0.322.24 $\frac{0.16}{1.12} = \frac{0.32}{2.24}$

Hence, MN ∥$||$ QR.

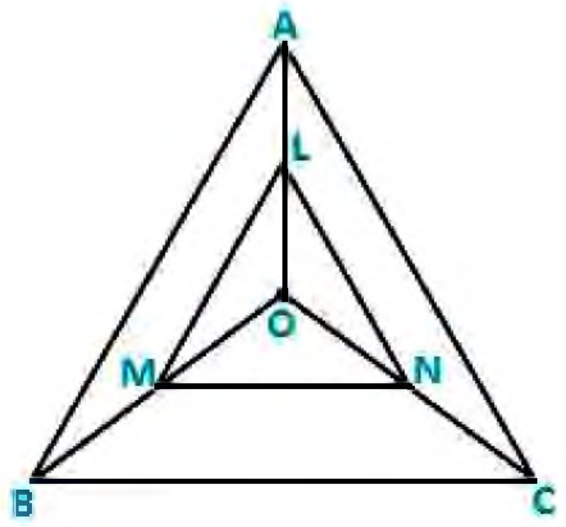
Q.7) In three line segments OA, OB, and OC, points L, M, N respectively are so chosen that LM ∥$||$ AB and MN ∥$||$ BC but neither of L, M, and N nor A, B, C are collinear. Show that LN ∥$||$ AC.



Sol:

In ΔΔ OAB, Since, LM ∥$||$ AB,

Then, OLLA=OMMB $\frac{OL}{LA} = \frac{OM}{MB}$ (using BPT)



In ΔΔ OBC, Since, MN ∥|| BC,

Then, OMMB=ONNC $\frac{OM}{MB} = \frac{ON}{NC}$ (using BPT)

Therefore, ONNC=OMMB $\frac{ON}{NC} = \frac{OM}{MB}$

From the above equations,

We get, OLLA=ONNC $\frac{OL}{LA} = \frac{ON}{NC}$

In a ΔΔ OCA,

OLLA=ONNC $\frac{OL}{LA} = \frac{ON}{NC}$

LN ∥|| AC (by converse BPT)

Q.8) If D and E are the points on sides AB and AC respectively of a ΔΔ ABC such that DE ∥|| BC and BD = CE. Prove that ΔΔ ABC is isosceles.

Sol:

It is given that in ΔΔ ABC, DE ∥|| BC and BD = CE.

We need to prove that ΔΔ ABC is isosceles.

Acc. to Thales Theorem,

ADBD=AEEC $\frac{AD}{BD} = \frac{AE}{EC}$

AD = AE

Now, BD = CE and AD = AE.

So, AD + BD = AE + CE.

Therefore, AB = AC.

Therefore, $\Delta\triangle$ ABC is isosceles.

Exercise 4.3: Triangles

Q.1) In a ΔΔ ABC, AD is the bisector of ∠∠ A , meeting side BC at D.

(i) if BD = 2.5 cm, AB = 5 cm, and AC = 4.2 cm, find DC.

(ii) if BD = 2 cm, AB = 5 cm, and DC = 3 cm, find AC.

(iii) if AB = 3.5 cm, AC = 4.2 cm, and DC = 2.8 cm, find BD.

(iv) if AB = 10 cm, AC = 14 cm, and BC = 6 cm, find BD and DC.

(v) if AC = 4.2 cm, DC = 6 cm, and BC = 10 cm, find AB.

(vi) if AB = 5.6 cm, BC = 6 cm, and DC = 3 cm, find BC.

(vii) if AB = 5.6 cm, BC = 6 cm, and BD = 3.2 cm, find AC.

(viii) if AB = 10 cm, AC = 6 cm, and BC = 12 cm, find BD and DC.

Sol:

(i) It is given that BD = 2.5 cm, AB = 5 cm, and AC = 4.2 cm.

In ΔΔ ABC, AD is the bisector of ∠∠ A , meeting side BC at D.

We need to find DC,

Since, AD is ∠∠ A bisector,

Then, ABAC=2.5DC $\frac{AB}{AC} = \frac{2.5}{DC}$

54.2=2.5DC $\frac{5}{4.2} = \frac{2.5}{DC}$

5DC = 4.2 x 2.5

DC = (4.2 x 2.5)/5

DC = 2.1

(ii) It is given that BD = 2 cm, AB = 5 cm, and DC = 3 cm

In ΔΔ ABC, AD is the bisector of ∠∠ A, meeting side BC at D

We need to find AC.

Since, AD is ∠∠ A bisector.

Therefore, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$ (since AD is the bisector of ∠∠ A and side BC)

Then, 5AC=23 $\frac{5}{AC} = \frac{2}{3}$

2AC = 5 x 3

AC = 15/2

AC = 7.5 cm

(iii) It is given that AB = 3.5 cm, AC = 4.2 cm, and DC = 2.8 cm

In ΔΔ ABC, AD is the bisector of ∠∠ A, meeting side BC at D

We need to find BD.

Since, AD is ∠∠ A bisector

Therefore, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$ (since, AD is the bisector of ∠∠ A and side BC)

Then, 3.54.2=BD2.8 $\frac{3.5}{4.2} = \frac{BD}{2.8}$

BD = (3.5 x 2.8)/4.2

BD = 7/3

BD = 2.3 cm

(iv) It is given that AB = 10 cm, AC = 14 cm, and BC = 6 cm

In ΔΔ ABC, AD is the bisector of ∠∠ A meeting side BC at D

We need to find BD and DC.

Since, AD is bisector of ∠∠ A

Therefore, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$ (AD is bisector of ∠∠ A and side BC)

Then, 1014=x6−x $\frac{10}{14} = \frac{x}{6-x}$

14x = 60 – 6x

20x = 60

x = 60/20

BD = 3 cm and DC = 3 cm.

(v) It is given that AC = 4.2 cm, DC = 6 cm, and BC = 10 cm.

In ΔΔ ABC, AD is the bisector of ∠∠ A, meeting side BC at D.

We need to find out AB,

Since, AD is the bisector of ∠∠ A

Therefore, ACAB=DCBD $\frac{AC}{AB} = \frac{DC}{BD}$

Then, 4.2AB=64 $\frac{4.2}{AB} = \frac{6}{4}$

6AB = 4.2 x 4

AB = (4.2 x 4)/6

AB = 16.8/6

AB = 2.8 cm

(vi) It is given that AB = 5.6 cm, BC = 6 cm, and DC = 3 cm

In ΔΔ ABC, AD is the bisector of ∠∠ A, meeting side BC at D

We need to find BC,

Since, AD is the ∠∠ A bisector

Therefore, ACAB=BDDC $\frac{AC}{AB} = \frac{BD}{DC}$

Then, 65.6=3DC $\frac{6}{5.6} = \frac{3}{DC}$

DC = 2.8 cm

And, BC = 2.8 + 3

BC = 5.8 cm

(vii) It is given that AB = 5.6 cm, BC = 6 cm, and BD = 3.2 cm

In ΔΔ ABC, AD is the bisector of ∠∠ A , meeting side BC at D

Therefore, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

5.6AC=3.22.8 $\frac{5.6}{AC} = \frac{3.2}{2.8}$ (DC = BC – BD)

AC = (5.6 x 2.8)/3.2

AC = 4.9 cm

(viii) It is given that AB = 10 cm, AC = 6 cm, and BC = 12 cm

In ΔΔ ABC, AD is the ∠∠ A bisector, meeting side BC at D.

We need to find BD and DC

Since, AD is bisector of ∠∠ A

So, ACAB=DCBD $\frac{AC}{AB} = \frac{DC}{BD}$

Let BD = x cm

Then,

610=12–xx $\frac{6}{10} = \frac{12-x}{x}$

6x = 120 – 10x

16x = 120

x = 120/16

x = 7.5

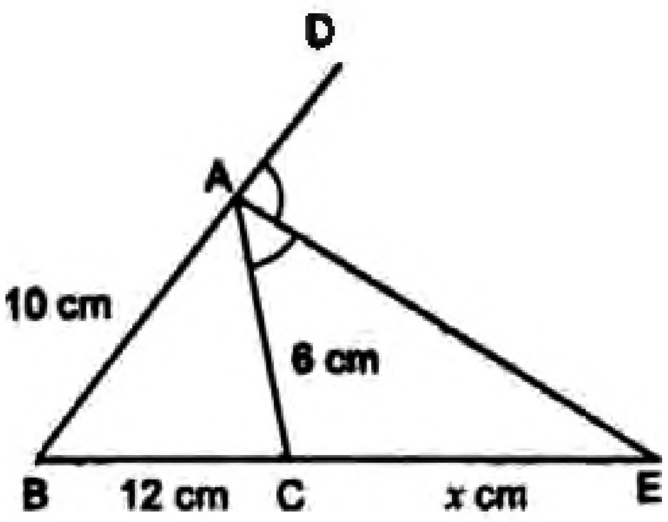
Now, DC = 12 – BD

DC = 12 – 7.5

DC = 4.5

BD = 7.5 cm and DC = 4.5 cm.

Q2.) AE is the bisector of the exterior ∠∠CAD meeting BC produced in E. If AB = 10 cm, AC = 6 cm, and BC = 12 cm, Find CE.



Sol:

It is given that AE is the bisector of the exterior ∠∠CAD

Meeting BC produced E and AB = 10 cm, AC = 6 cm, and BC = 12 cm.

Since AE is the bisector of the exterior ∠∠CAD.

So, BECE=ABAC $\frac{BE}{CE} = \frac{AB}{AC}$

12+xx = 10x $\frac{12+x}{x} = \frac{10}{x}$

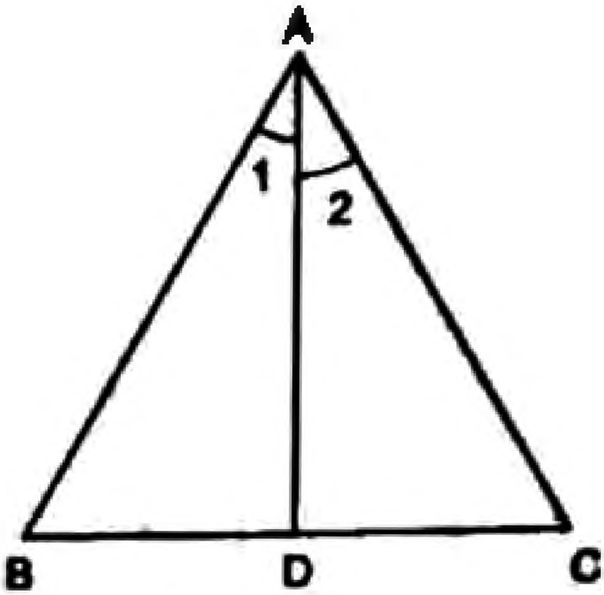
72 + 6x = 10x

4x = 72

x = 18

CE = 18 cm

Q.3) ΔΔ ABC is a triangle such that ABAC = BDDC $\frac{AB}{AC} = \frac{BD}{DC}$, ∠∠B = 70, ∠∠C = 50, find ∠∠BAD.



Sol:

It is given that in ΔΔ ABC, ABAC = BDDC $\frac{AB}{AC} = \frac{BD}{DC}$, ∠∠B = 70 and ∠∠C = 50

We need to find ∠∠ BAD

In ΔΔ ABC,

∠∠A = 180 – (70 + 50)

= 180 – 120

= 60

Since, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

Therefore, AD is the bisector of ∠∠A

Hence, ∠∠BAD = 60/2 = 30

Q.4) Check whether AD is the bisector of ∠∠A of ΔΔABC in each of the following :

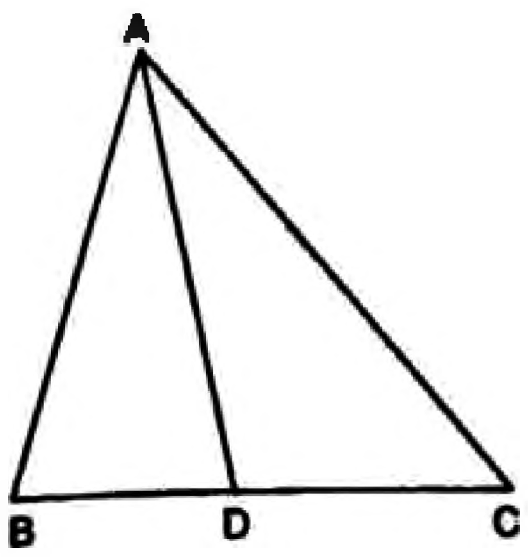
(i) AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm

(ii) AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm

(iii) AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24 cm

(iv) AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm

(v) AB = 5 cm, AC = 12 cm, BD = 2.5 cm and BC = 9 cm



Sol:

(i) It is given that AB = 5 cm, AC = 10 cm, BD = 1.5 cm and CD = 3.5 cm

We have to check whether AD is bisector of ∠∠ A

First we will check proportional ratio between sides.

Now,

ABAC=510=12 $\frac{AB}{AC}=\frac{5}{10}=\frac{1}{2}$ BDCD=1.53.5=37 $\frac{BD}{CD}=\frac{1.5}{3.5}=\frac{3}{7}$

Since, ABAC≠BDCD $\frac{AB}{AC}\neq\frac{BD}{CD}$

Hence, AD is not the bisector of ∠∠ A.

(ii) It is given that AB = 4 cm, AC = 6 cm, BD = 1.6 cm and CD = 2.4 cm.

We have to check whether AD is the bisector of ∠∠ A

First we will check proportional ratio between sides.

So, ABAC=BDDC $\frac{AB}{AC}=\frac{BD}{DC}$

46=1.62.4 $\frac{4}{6}=\frac{1.6}{2.4}$

23=23 $\frac{2}{3}=\frac{2}{3}$ (it is proportional)

Hence, AD is the bisector of ∠∠ A.

(iii) It is given that AB = 8 cm, AC = 24 cm, BD = 6 cm and BC = 24 cm.

We have to check whether AD is the bisector of ∠∠ A

First we will check proportional ratio between sides.

DC = BC – BD

DC = 24 – 6

DC = 18

So, ABAC=BDDC $\frac{AB}{AC}=\frac{BD}{DC}$

824=618 $\frac{8}{24}=\frac{6}{18}$

13=13 $\frac{1}{3} = \frac{1}{3}$ (it is proportional)

Hence, AD is the bisector of ∠∠ A.

(iv) It is given that AB = 6 cm, AC = 8 cm, BD = 1.5 cm and CD = 2 cm.

We have to check whether AD is the bisector of ∠∠ A

First, we will check proportional ratio between sides.

So, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

68=1.52 $\frac{6}{8} = \frac{1.5}{2}$

34=34 $\frac{3}{4} = \frac{3}{4}$ (it is proportional)

Hence, AD is the bisector of ∠∠ A.

(v) It is given that AB = 5 cm, AC = 12 cm, BD = 2.5 cm and BC = 9 cm.

We have to check whether AD is the bisector of ∠∠ A

First, we will check proportional ratio between sides.

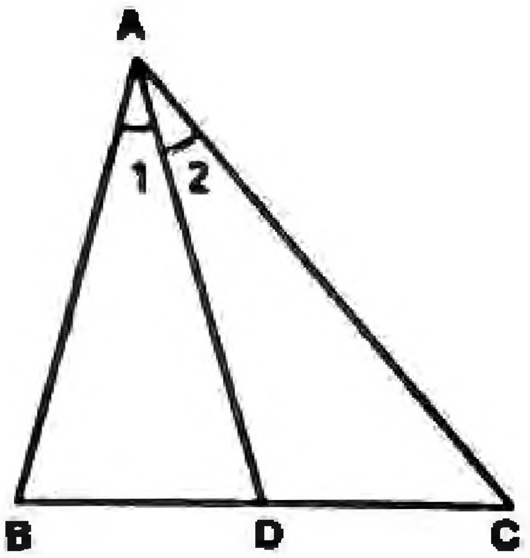
So, ABAC=512 $\frac{AB}{AC} = \frac{5}{12}$

BDCD=2.59=518 $\frac{BD}{CD} = \frac{2.5}{9} = \frac{5}{18}$

Since, ABAC≠BDCD $\frac{AB}{AC} \neq \frac{BD}{CD}$

Hence, AD is not the bisector of ∠∠ A.

Q.5) In fig. AD bisects ∠∠A, AB = 12 cm, AC = 20 cm, and BD = 5 cm, determine CD.



Soln.: It is given that AD bisects ∠∠ A

AB = 12 cm, AC = 20 cm, and BD = 5 cm.

We need to find CD.

Since AD is the bisector of ∠∠ A

then, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

1220=5DC $\frac{12}{20} = \frac{5}{DC}$

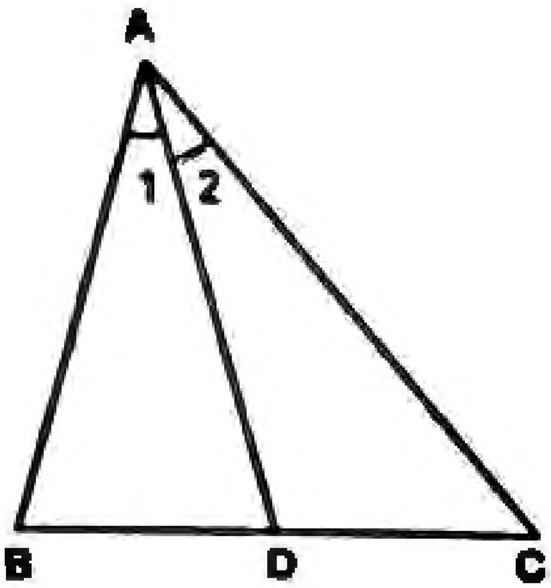
12 x DC = 20 x 5

DC = 100/12

DC = 8.33 cm

∴ CD = 8.33 cm.

Q6.) In ΔΔABC, if ∠∠1 = ∠∠2, prove that, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$



Sol: We need to prove that, ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

In ΔΔABC,

∠∠1 = ∠∠2

So, AD is the bisector of ∠∠A

Therefore,

ABAC=BDDC $\frac{AB}{AC} = \frac{BD}{DC}$

Q.7) D and E are the points on sides BC, CA and AB respectively. of a ΔΔABC such that AD bisects ∠∠A, BE bisects ∠∠B and CF bisects ∠∠C. If AB = 5 cm, BC = 8 cm, and CA = 4 cm, determine AF, CE, and BD.

Sol:

It is given that AB = 5 cm, BC = 8 cm and CA = 4 cm.

We need to find out, AF, CE and BD.

Since, AD is the bisector of ∠∠A

ABAC=BDCD $\frac{AB}{AC} = \frac{BD}{CD}$

Then,

54=BDBC–BD $\frac{5}{4} = \frac{BD}{BC–BD}$ 54=BD8–BD $\frac{5}{4} = \frac{BD}{8–BD}$

40 – 5BD = 4 BD

9BD = 40

So, BD = 40/9

Since, BE is the bisector of ∠∠ B

So, ABBC=AEEC $\frac{AB}{BC} = \frac{AE}{EC}$

ABBC=AC–ECEC $\frac{AB}{BC} = \frac{AC–EC}{EC}$ 58=4–CECE $\frac{5}{8} = \frac{4–CE}{CE}$

5CE = 32 – 8CE

5CE + 8CE = 32

13CE = 32

So, CE = 3213 $\frac{32}{13}$

Now, since, CF is the bisector of ∠∠C

So, BCCA=BFAF $\frac{BC}{CA} = \frac{BF}{AF}$

84=AB–AFAF $\frac{8}{4} = \frac{AB–AF}{AF}$ 84=5–AFAF $\frac{8}{4} = \frac{5–AF}{AF}$

8AF = 20 – 4AF

12AF = 20

So, 3AF = 5

AF = 5/3 cm, CE = 32/12 cm

and BD = 40/9 cm

Exercise 4.4: Triangles

Q1) In fig. (i) if AB||CD$AB \parallel CD$, find the value of x.

(ii) In fig. if AB||CD$AB \parallel CD$, find the value of x.

(iii) in fig. if AB||CD$AB \parallel CD$ .and OA = 3x – 19, OB = x – 4, OC = x- 3 and OD = 4, find x.

Sol:

(i) it is given that AB||CD$AB \parallel CD$

We have to find the value of x.

Diagonals of the parallelogram,

As we know, DOOA=COOB $\frac{DO}{OA} = \frac{CO}{OB}$

4x–24=2x+4x+1 $\frac{4x-2}{4} = \frac{2x+4}{x+1}$

4(2x + 4) = (4x – 2)(x +1)

8x + 16 = x(4x – 2) + 1(4x – 2)

8x + 16 = $4x^2$ – 2x + 4x – 2

$-4x^2 + 8x + 16 + 2 – 2x = 0$

$-4x^2 + 6x + 8 = 0$

$4x^2 – 6x – 18 = 0$

$4x^2 – 12x + 6x – 18 = 0$

$4x(x – 3) + 6(x – 3) = 0$

$(4x + 6)(x – 3) = 0$

X = – 6/4 or x = 3

(ii) it is given that AB||CD$AB \parallel CD$

We need to find the value of x.

Now, DOOA=COOB $\frac{DO}{OA} = \frac{CO}{OB}$

6x–52x+1=5x–33x–1 $\frac{6x–5}{2x+1} = \frac{5x–3}{3x–1}$

$(6x – 5)(3x – 1) = (2x + 1)(5x – 3)$

$3x(6x – 5) – 1(6x – 5) = 2x(5x – 3) + 1(5x – 3)$

$18x^2 – 10x^2 – 21x + 5 + x + 3 = 0$

$8x^2 – 16x – 4x + 8 = 0$

$8x(x – 2) – 4(x – 2) = 0$

$(8x – 4)(x – 2) = 0$

X = 4/8 = 1/2 or x = -2

X= 1/2

(iii) it is given that AB||CD$AB \parallel CD$

And OA = 3x – 19 OB = x – 4 OC = x – 3 and OD = 4

We need to find the value of x,

Now, Now, AOOC=BOOD $\frac{AO}{OC} = \frac{BO}{OD}$

3x–19x–3=x–44 $\frac{3x-19}{x-3} = \frac{x-4}{4}$

$4(3x - 19) = (x - 3)(x - 4)$

$12x - 76 = x(x - 4) - 3(x - 4)$

$12x - 76 = x^2 - 4x - 3x + 12$

$-x^2 + 7x - 12 + 12x - 76 = 0$

$-x^2 + 19x - 88 = 0$

$X^2 - 19x + 88 = 0$

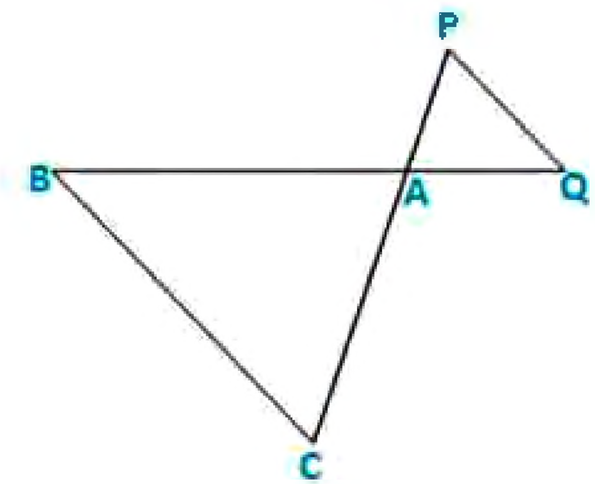
$X^2 - 11x - 8x + 88 = 0$

$X(x - 11) - 8(x - 11) = 0$

X = 11 or x = 8

Exercise 4.5: Triangles

Q1: In fig. given below ΔACB~ΔAPQ $\Delta ACB \sim \Delta APQ$. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, and AP = 2.8 cm find CA and AQ.



Sol: Given,

ΔACB~ΔAPQ $\Delta ACB \sim \Delta APQ$

BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, and AP = 2.8 cm

We need to find CA and AQ

Since, ΔACB~ΔAPQ$\Delta ACB \sim \Delta APQ$

BAAQ=CAAP=BCPQ$\frac{BA}{AQ} = \frac{CA}{AP} = \frac{BC}{PQ}$

Therefore, 6.5AQ=84$\frac{6.5}{AQ} = \frac{8}{4}$

AQ = 6.5x48$\frac{6.5x4}{8}$

AQ = 3.25 cm

Similarly, CAAP=BCPQ$\frac{CA}{AP} = \frac{BC}{PQ}$

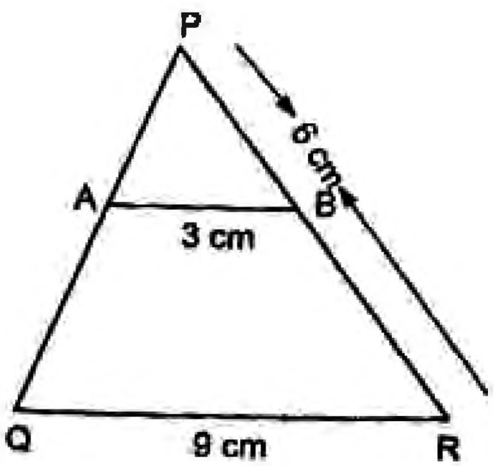
CA2.8=84$\frac{CA}{2.8} = \frac{8}{4}$

CA = 2.8 x 2

CA = 5.6 cm

Therefore, **CA = 5.6 cm and AQ = 3.25 cm.**

Q2: In fig. given, AB||QR$AB \parallel QR$, find the length of PB.



Sol: Given,

AB||PB$AB \parallel PB$

AB = 3 cm, QR = 9 cm and PR = 6 cm

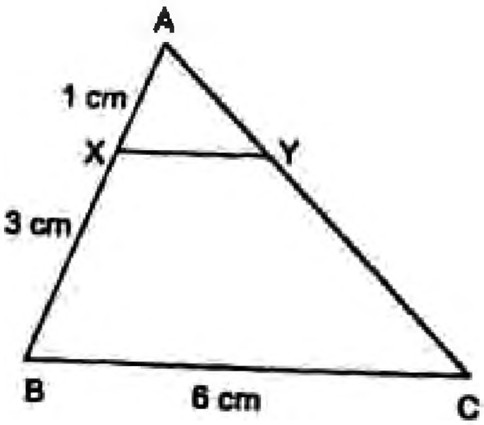
We need to find out PB,

Since, ABQR=PBPR $\frac{AB}{QR} = \frac{PB}{PR}$

i.e., 39=PB6 $\frac{3}{9} = \frac{PB}{6}$

PB = 2 cm

Q3.) In fig. given, XY ||BC$XY \parallel BC$. Find the length of XY.



Sol: Given,

XY ||BC$XY \parallel BC$

AX = 1 cm, XB = 3 cm, and BC = 6 cm

We need to find XY,

Since, ΔAXY ~ΔABC$\Delta AXY \sim \Delta ABC$

XYBC=AXAB $\frac{XY}{BC} = \frac{AX}{AB}$ (AB = AX + XB = 4)

XY6=14 $\frac{XY}{6} = \frac{1}{4}$ XY1=64 $\frac{XY}{1} = \frac{6}{4}$

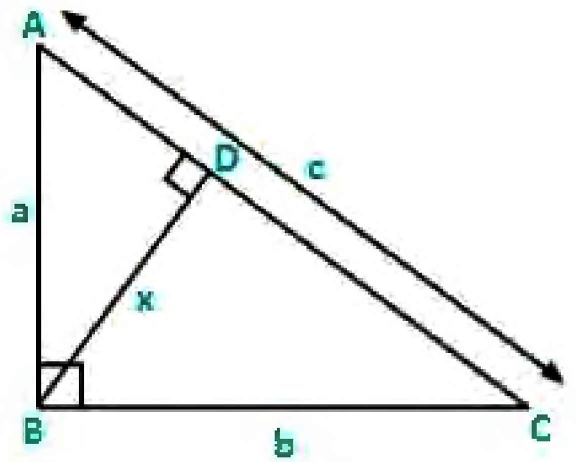
XY = 1.5 cm

Q4: In a right-angled triangle with sides a and b and hypotenuse c, the altitude drawn on the hypotenuse is x. Prove that ab = cx.

Sol:

Let the ΔABCΔABC be a right angle triangle having sides a and b and hypotenuse c. BD is the altitude drawn on the hypotenuse AC

We need to prove ab = cx



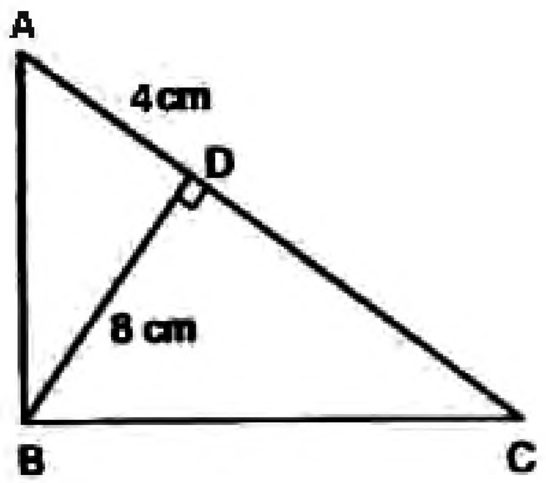
Since, the altitude is perpendicular on the hypotenuse, both the triangles are similar

ABBD=ACBC $\frac{AB}{BD} = \frac{AC}{BC}$ ax=cb $\frac{a}{x} = \frac{c}{b}$

xc = ab

∴ ab = cx

Q5) In fig. given, ∠ABC$\angle ABC$ = 90 and BD⊥AC$BD \perp AC$. If BD = 8 cm, and AD = 4 cm, find CD.



Sol:

Given,

∠ABC$\angle ABC$ = 90 and BD⊥AC$BD \perp AC$

When , BD = 8 cm, AD = 4 cm, we need to find CD.

Since, ABC is a right angled triangle and BD⊥AC$BD \perp AC$.

So, ΔDBA~ΔDCB$\Delta DBA \sim \Delta DCB$ (A-A similarity)

BDCD=ADBD $\frac{BD}{CD} = \frac{AD}{BD}$

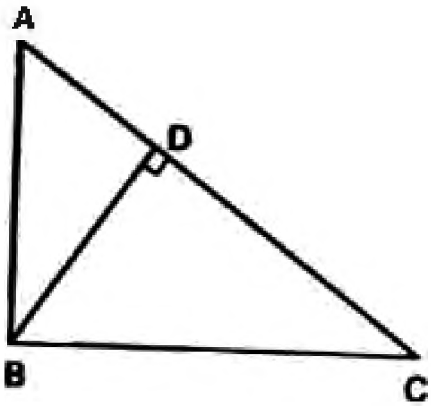
BD^2 = AD x DC

$(8)^2$ = 4 x DC

DC = 64/4 = 16 cm

∴ CD = 16 cm

Q6) In fig. given, ∠ABC$\angle ABC$ = 90 and BD⊥AC$BD \perp AC$. If AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, Find BC.



Sol:

Given: BD⊥AC$BD \perp AC$. AC = 5.7 cm, BD = 3.8 cm and CD = 5.4 cm, and ∠ABC $\angle ABC$ = 90.

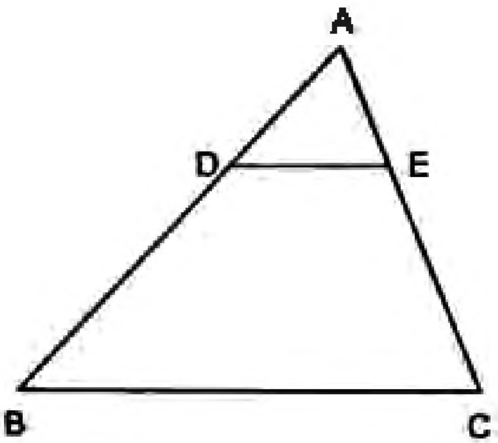
We need to find BC,

Since, ΔABC~ΔBDC$\Delta ABC \sim \Delta BDC$

ABBD=BCCD $\frac{AB}{BD} = \frac{BC}{CD}$ 5.73.8=BC5.4 $\frac{5.7}{3.8} = \frac{BC}{5.4}$ BC1=5.7x5.43.8 $\frac{BC}{1} = \frac{5.7x5.4}{3.8}$

BC = 8.1 cm

Q7) In the fig. given, DE∥BC$DE \parallel BC$ such that AE = (1/4)AC. If AB = 6 cm, find AD.



Sol:

Given, DE||BC$DE \parallel BC$ and AE = (1/4)AC and AB = 6 cm.

We need to find AD.
Since, ΔADE~ΔABC$\Delta ADE \sim \Delta ABC$

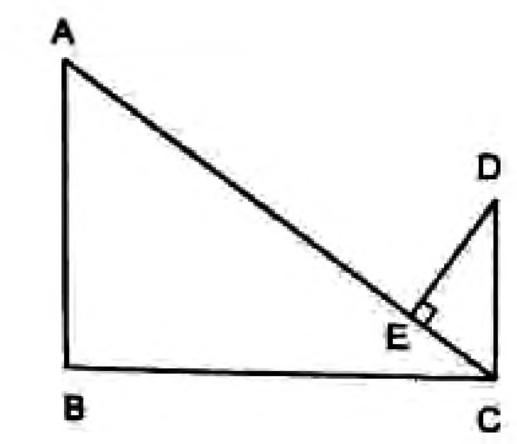
ADAB=AEAC $\frac{AD}{AB} = \frac{AE}{AC}$ AD6=14 $\frac{AD}{6} = \frac{1}{4}$

4 x AD = 6

AD = 6/4

AD = 1.5 cm

Q.8) In the fig. given, if AB⊥BC$AB \perp BC$, DC⊥BC$DC \perp BC$, and DE⊥AC $DE \perp AC$, prove that ΔCED~ΔABC$\Delta CED \sim \Delta ABC$



Sol:

Given, AB⊥BC$AB \perp BC$, DC⊥BC$DC \perp BC$, and DE⊥AC$DE \perp AC$

We need to prove that ΔCED~ΔABC$\Delta CED \sim \Delta ABC$

Now,

In ΔΔABC and ΔΔCED

∠B$\angle B$ = ∠E$\angle E$ = 90 (given)

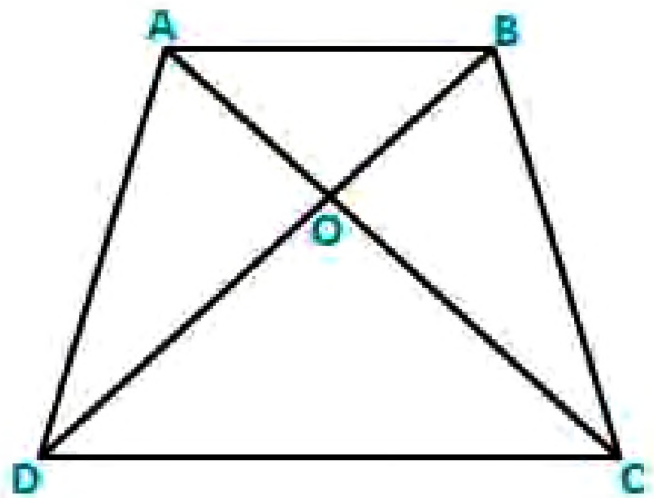
∠A$\angle A$ = ∠ECD$\angle ECD$ (alternate angles)

So, **ΔCED~ΔABC**$\Delta CED \sim \Delta ABC$ (A-A similarity)

Q.9) Diagonals AC and BD of a trapezium ABCD with AB||DC$AB \parallel DC$ **intersect each other at the point O. Using similarity criterion for two triangles, show that** **OAOC=OBOD**$\frac{OA}{OC} = \frac{OB}{OD}$

Sol: Given trapezium ABCD with AB||DC$AB \parallel DC$. OC is the point of intersection of AC and BD.

We need to prove OAOC=OBOD$\frac{OA}{OC} = \frac{OB}{OD}$



Now, in ΔΔAOB and ΔΔCOD

∠AOB$\angle AOB$ = ∠COD$\angle COD$ (VOA)

∠OAB$\angle OAB$ = ∠OCD$\angle OCD$ (alternate angles)

Therefore, ΔAOB~ΔCOD$\Delta AOB \sim \Delta COD$

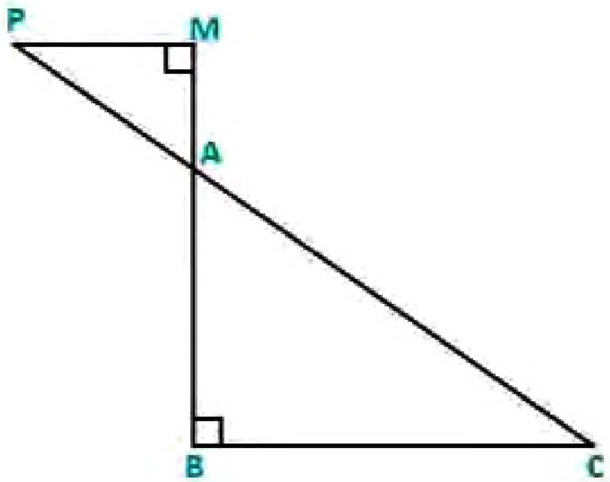
Therefore, **OAOC=OBOD**$\frac{OA}{OC} = \frac{OB}{OD}$ (corresponding sides are proportional)

Q.10) If ΔΔABC and ΔΔAMP are two right angled triangles, at angle B and M, repec. Such that ∠MAP $\angle MAP$ = ∠BAC $\angle BAC$. Prove that :

(i) ΔABC~ΔAMP $\Delta ABC \sim \Delta AMP$

(ii) CAPA=BCMP $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:



(i) Given ΔΔ ABC and ΔΔ AMP are the two right angled triangle.

∠MAP $\angle MAP$ = ∠BAC $\angle BAC$ (given)

∠AMP $\angle AMP$ = ∠B $\angle B$ = 90

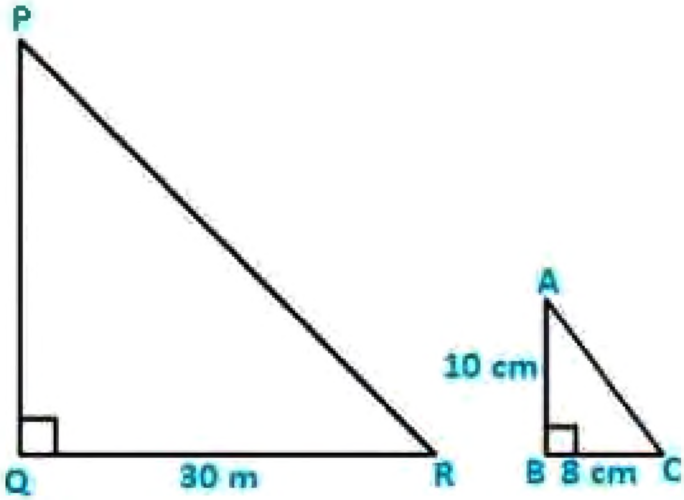
ΔABC~ΔAMP $\Delta ABC \sim \Delta AMP$ (A-A similarity)

(ii) ΔΔABC – ΔΔAMP

So, **CAPA=BCMP** $\frac{CA}{PA} = \frac{BC}{MP}$ (corresponding sides are proportional)

Q.11) A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.

Soln.: We need to find the height of PQ.



Now, ΔABC~ΔPQR$\Delta ABC \sim \Delta PQR$ (A-A similarity)

ABBC=PQQR $\frac{AB}{BC} = \frac{PQ}{QR}$ 108=PQ3000 $\frac{10}{8} = \frac{PQ}{3000}$

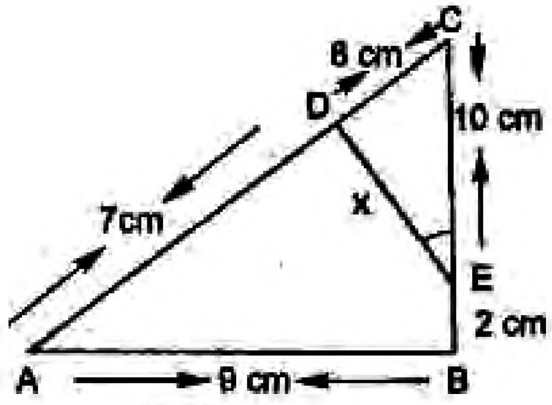
PQ = 3000x108 $\frac{3000x10}{8}$

PQ = 300008 $\frac{30000}{8}$

PQ = 3750100 $\frac{3750}{100}$

PQ = 37.5 m

Q.12) in fig. given, ∠A$\angle A$ = ∠CED$\angle CED$, prove that ΔCAB~ΔCED $\Delta CAB \sim \Delta CED$. Also find the value of x.



Sol:

Comparing ΔandCABΔCED$\Delta and CAB \Delta CED$

CACE=ABED $\frac{CA}{CE} = \frac{AB}{ED}$ (similar triangles have corresponding sides in the same proportions)

1510=9x $\frac{15}{10} = \frac{9}{x}$ x1=9x1015 $\frac{x}{1} = \frac{9x10}{15}$

x = 6 cm

Q13) The perimeters of two similar triangles are 25 cm and 15 cm, respect. If one side of the first triangle is 9 cm, what is the corresponding side of the other triangle?

Sol:

Given perimeter of two similar triangles are 25 cm, 15 cm and one side 9 cm

We need to find the other side.

Let the corresponding side of other triangle be x cm

Since ratio of perimeter = ratio of corresponding side

2515=9x $\frac{25}{15} = \frac{9}{x}$

25 x X = 9 x 15

X = 135/25

X = 5.4 cm

Q14) In ΔABCandΔDEF $\Delta ABC and \Delta DEF$, it is being given that AB = 5 cm, BC = 4 cm, CA = 4.2 cm, DE = 10 cm, EF = 8 cm, and FD = 8.4 cm. If AL⊥BC $AL \perp BC$, DM⊥EF $DM \perp EF$, find AL : Dm.

Sol:

Given AB = 5 cm, BC = 4 cm, CA = 4.2 cm, DE = 10 cm, EF = 8 cm, and FD = 8.4 cm

We need to find AL : DM

Since, both triangles are similar,

ABDE = BCEF = ACDF = 12 $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$

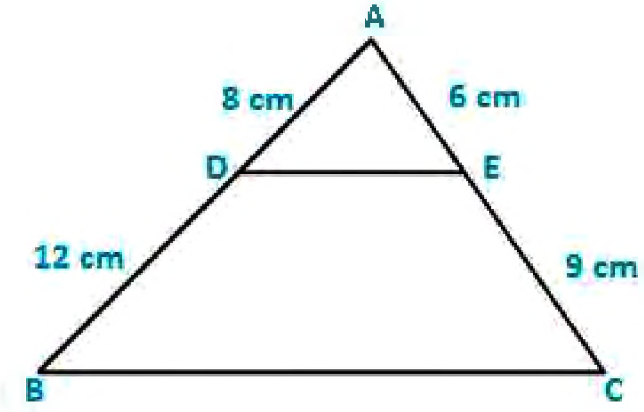
Here, we use the result that in similar triangle the ratio of corresponding altitude is same as the ratio of the corresponding sides.

Therefore, **AL : DM = 1 : 2**

Q.15) D and E are the points on the sides AB and AC respectively, of a ΔABC ΔABC such that AD = 8 cm, DB = 12 cm, AE = 6 cm, and CE = 9 cm. Prove that BC = 5/2 DE.

Sol: Given AD = 8 cm, AE = 6 cm, and CE = 9 cm

We need to prove that,



Since, ADAB=AEAC=25 $\frac{AD}{AB} = \frac{AE}{AC} = \frac{2}{5}$

Also, ΔADE~ΔABC $\Delta ADE \sim \Delta ABC$ (SAS similarity)

BCDE=ABAD $\frac{BC}{DE} = \frac{AB}{AD}$

BCDE=1(ADAB) $\frac{BC}{DE} = \frac{1}{(\frac{AD}{AB})}$ (ADAB=25 $\frac{AD}{AB} = \frac{2}{5}$)

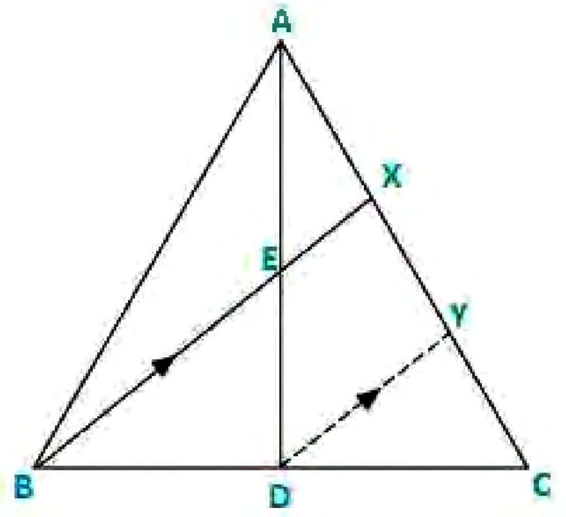
BCDE=52 $\frac{BC}{DE} = \frac{5}{2}$

BC = 5/2 DE

Q.16) D is the midpoint of side BC of a ΔABC ΔABC. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE: EX = 3 : 1

Soln.: ABC is a triangle in which D is the midpoint of BC, E is the midpoint of AD. BE produced meets AC at X.

We need to prove BE: EX = 3 : 1



In ΔΔ BCX and ΔΔDCY

∠∠CBX = ΔΔCBY (corresponding angles)

∠∠CXB = ΔΔCYD (corresponding angles)

ΔBCX~ΔDCY $\Delta BCX \sim \Delta DCY$ (angle-angle similarity)

We know that corresponding sides of similar sides of similar triangles are proportional

Thus, BCDC = BXDY = CXCY $\frac{BC}{DC} = \frac{BX}{DY} = \frac{CX}{CY}$

BXDY = BCDC $\frac{BX}{DY} = \frac{BC}{DC}$

BXDY = 2DCDC $\frac{BX}{DY} = \frac{2DC}{DC}$ (As D is the midpoint of BC)

BXDY = 21 $\frac{BX}{DY} = \frac{2}{1}$ ….(i)

In ΔΔAEX and ΔΔADY,

∠∠AEX = ΔΔADY (corresponding angles)

∠∠AXE = ΔΔAYD (corresponding angles)

ΔΔAEX – ΔΔADY (angle-angle similarity)

We know that corresponding sides of similar sides of similar triangles are proportional

Thus, AEAD = EXDY = AXAY $\frac{AE}{AD} = \frac{EX}{DY} = \frac{AX}{AY}$

EXDY =AEAD $\frac{EX}{DY} = \frac{AE}{AD}$

EXDY =AE2AE $\frac{EX}{DY} = \frac{AE}{2AE}$ (As D is the midpoint of BC)

EXDY =12 $\frac{EX}{DY} = \frac{1}{2}$...(ii)

Dividing eqn. (i) by eqn. (ii)

BXEX=41 $\frac{BX}{EX} = \frac{4}{1}$

BX = 4EX

BE + EX = 4EX

BE = 3EX

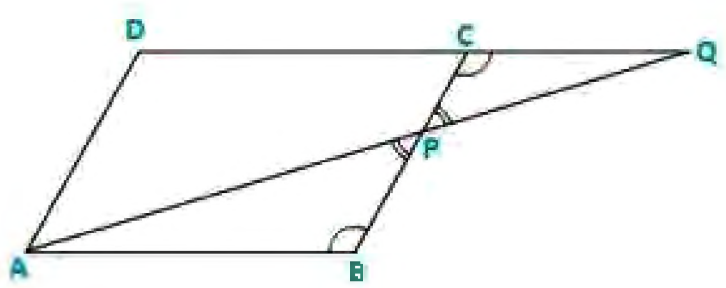
BE : EX = 3:1

Q.17) ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC.

Sol:

ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q.

We need to prove, the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC. We need to prove that BP x DQ = AB x BC



In ΔΔABP and ΔΔQCP,

∠∠ABP = ΔΔQCP (alternate angles as AB DC)

∠∠BPA = ΔΔQPC (VOA)

ΔABP ~ΔQCP $\Delta ABP \sim \Delta QCP$ ΔΔ (AA similarity)

We know that corresponding sides of similar triangles are proportional

Thus, AEAD = EXDY = AXAY $\frac{AE}{AD} = \frac{EX}{DY} = \frac{AX}{AY}$

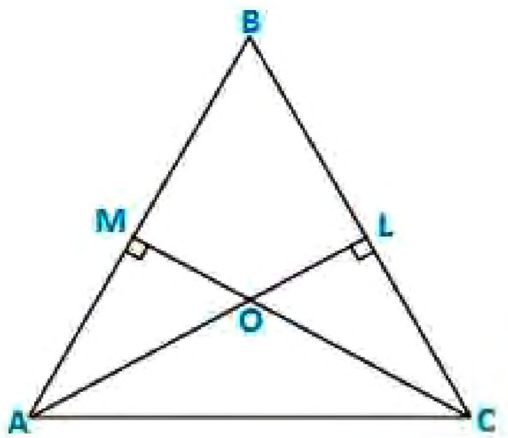
EXDY = AEAD $\frac{EX}{DY} = \frac{AE}{AD}$

Q.18) In ΔABCΔABC, AL and CM are the perpendiculars from the vertices A and C to BC and AB respect. If Al and CM intersec at O, prove that:

(i) ΔOMA~ΔOLC$\Delta OMA \sim \Delta OLC$

(ii) OAOC = OMOL $\frac{OA}{OC} = \frac{OM}{OL}$

Sol:



(i) in ΔΔOMA and ΔΔOLC,

∠∠AOM = ∠∠COL (VOA)

∠∠OMA = ∠∠OLC (90 each)

ΔOMA~ΔOLC$\Delta OMA \sim \Delta OLC$ (A-A similarity)

(ii) Since, ΔOMA~ΔOLC$\Delta OMA \sim \Delta OLC$ by A-A similarity, then

OMOL=OAOC=MALC $\frac{OM}{OL} = \frac{OA}{OC} = \frac{MA}{LC}$ (corresponding sides of similar triangles are proportional)

OAOC=OMOL $\frac{OA}{OC} = \frac{OM}{OL}$

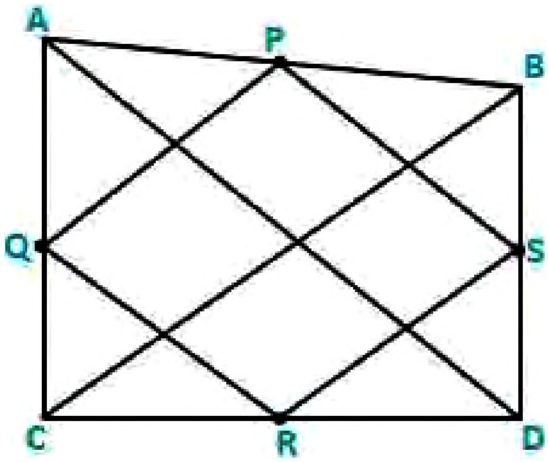
Q.19) ABCD is a quadrilateral in which AD = BC. If P,Q,R, S be the midpoints of AB, AC, CD and BD respect. Show that PQRS is a rhombus.

Soln.:

Given, ABCD is a quadrilateral in which AD = BC and P, Q, R, S are the mid points of AB, AC, CD, BD, respectively.

To prove,

PQRS is a rhombus



Proof,

In ΔΔABC, P and Q are the mid points of the sides B and AC respectively

By the midpoint theorem, we get,

PQ||BC$PQ \parallel BC$, PQ = 1/2 BC.

In ΔΔADC, Q and R are the mid points of the sides AC and DC respectively

By the mid point theorem, we get,

QR||AD$QR \parallel AD$ and QR = 1/2 AD = 1/2 BC (AD = BC)

In ΔΔBCD,

By the mid point theorem, we get,

RS||BC$RS \parallel BC$ and RS = 1/2 AD = 1/2 BC (AD = BC)

From above eqns.

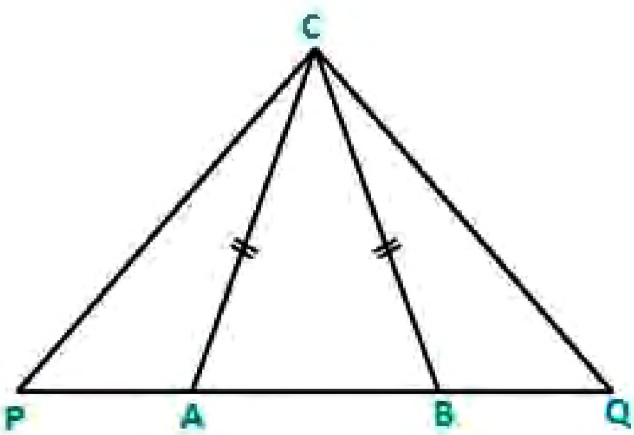
PQ = QR = RS

Thus, PQRS is a rhombus.

Q.20) In an isosceles ΔABCΔABC, the base AB is produced both ways to P and Q such that AP x BQ = AC^2 . Prove that ΔAPC~ΔBCQ$\Delta APC \sim \Delta BCQ$.

Sol: Given ΔΔABC is isosceles and AP x BQ = AC^2

We need to prove that ΔAPC~ΔBCQ$\Delta APC \sim \Delta BCQ$.



Given ΔΔABC is an isosceles triangle AC = BC.

Now, AP x BQ = AC^2 (given)

AP x BQ = AC x AC

APAC=ACBQ $\frac{AP}{AC} = \frac{AC}{BQ}$ APAC=BCBQ $\frac{AP}{AC} = \frac{BC}{BQ}$

Also, ∠∠CAB = ∠∠CBA (equal sides have angles opposite to them)

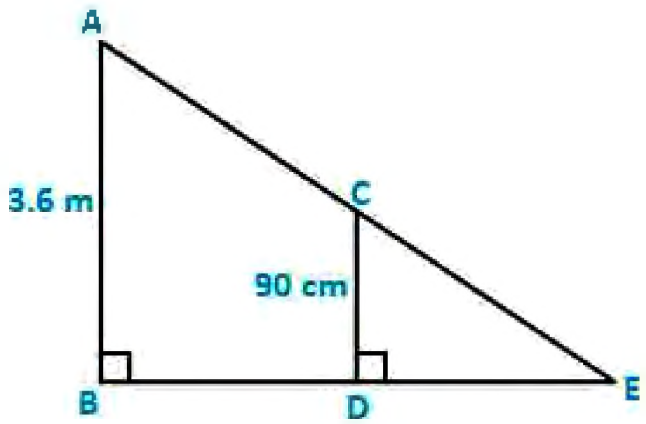
180 – CAP = 180 – CBQ

∠∠CAP = ∠∠CBQ

Hence, ΔAPC~ΔBCQ$\Delta APC \sim \Delta BCQ$ (SAS similarity)

Q.21) A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp is 3.6m above the ground, find the length of her shadow after 4 seconds.

Soln.: Given, girl's height = 90 cm, speed = 1.2m/sec and height of lamp = 3.6 m



We need to find the length of her shadow after 4 sec.

Let, AB be the lamp post and CD be the girl

Suppose DE is the length of her shadow

Let, DE = x

and BD = 1.2 x 4

BD = 4.8 m

Now, in ΔΔABE and ΔΔCDE we have,

∠∠B = ∠∠D

∠∠E = ∠∠E

So, by A-A similarity criterion,

ΔABE~ΔCDE$\Delta ABE \sim \Delta CDE$ BEDE=ABCD $\frac{BE}{DE} = \frac{AB}{CD}$

4.8+xx=3.60.9 $\frac{4.8+x}{x} = \frac{3.6}{0.9}$ = 4

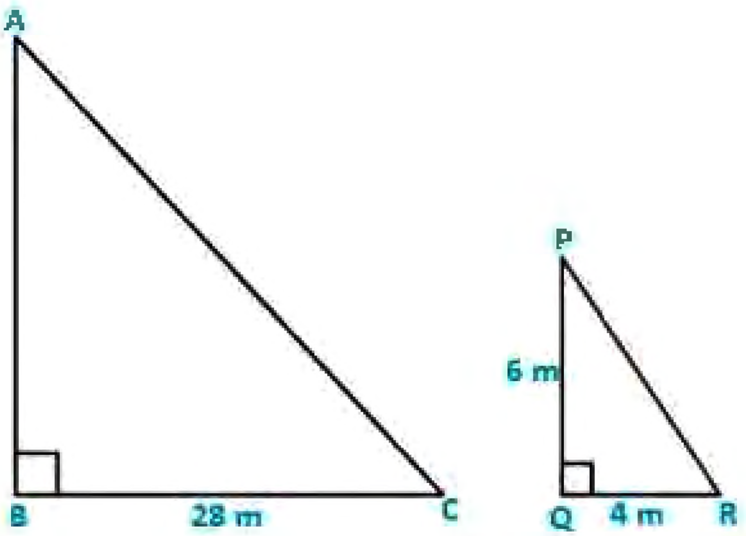
3x = 4.8

x = 1.6

hence, the length of her shadow after 4 sec. Is 1.6 m

Q.22) A vertical stick of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

Soln.: Given length of vertical stick = 6m



We need to find the height of the tower

Suppose AB is the height of the tower and BC is its shadow.

Now, ΔABC~ΔPCR$\Delta ABC \sim \Delta PCR$ (B = Q and A = P)

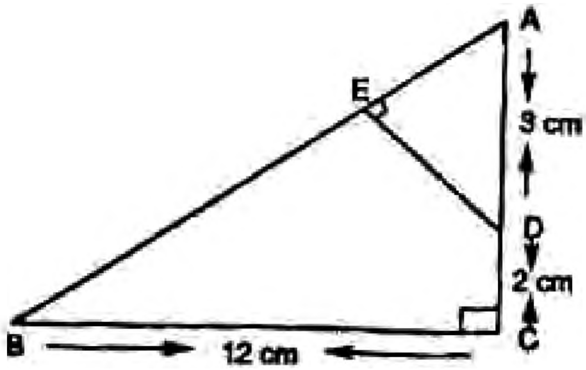
ABBC=PQQR $\frac{AB}{BC} = \frac{PQ}{QR}$ AB28=64 $\frac{AB}{28} = \frac{6}{4}$

AB = (28 x 6)/4

AB = 42m

Hence, the height of tower is 42m.

Q.23) In the fig. given, ΔΔABC is a right angled triangle at C and DE⊥AB$DE \perp AB$. Prove that ΔABC~ΔADE$\Delta ABC \sim \Delta ADE$.



Sol:

Given ΔΔACB is right angled triangle and C = 90

We need to prove that ΔABC~ΔADE$\Delta ABC \sim \Delta ADE$ and find the length of AE and DE.

ΔABC~ΔADE$\Delta ABC \sim \Delta ADE$

∠∠A = ∠∠A (common angle)

∠∠C = ∠∠E (90)

So, by A-A similarity criterion, we have

In ΔABC~ΔADE$\Delta ABC \sim \Delta ADE$

ABAD=BCDE=ACAE $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$ 133=12DE=5AE $\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$

Since, $AB^2 = AC^2 + BC^2$

$= 5^2 + 12^2$

$= 13^2$

∴ DE = 36/13 cm

and   AE = 15/13 cm

Q.25) In fig. given, we have AB||CD||EF$AB \parallel CD \parallel EF$. If AB = 6 cm, CD = x cm, EF = 10 cm, BD = 4 cm, and DE = y cm. Calculate the values of x and y.

14

Sol: Given AB CD EF.

AB = 6 cm, CD = x cm, and EF = 10 cm.

We need to calculate the values of x and y

In ΔΔADB and ΔΔDEF,

∠∠ADB = ∠∠EDF (VOA)

∠∠ABD = ∠∠DEF (alt. Interior angles)

EFAB=OEOB $\frac{EF}{AB} = \frac{OE}{OB}$ 106=y4 $\frac{10}{6} = \frac{y}{4}$

Y = 40/6

Y = 6.67 cm

Similarly, in ΔΔABE , we have

OCAB=OEOB $\frac{OC}{AB} = \frac{OE}{OB}$ 46.7=x6 $\frac{4}{6.7} = \frac{x}{6}$

6.7 x X = 6 x 4

X = 24/6.7

X = 3.75 cm

Therefore, x = 3.75 cm and y = 6.67 cm

Exercise 4.6: Triangles

1. Triangles ABC and DEF are similar.

(i) If area of (ΔABCΔABC) = 16 cm^2 , area (ΔDEF ΔDEF) = 25 cm^2 and BC = 2.3 cm, find EF.

(ii) If area (ΔABCΔABC) = 9 cm^2 , area (ΔDEF ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv) If area of (ΔABCΔABC) = 36 cm^2 , area (ΔDEF ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the area of two triangles.

Answer:

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

arΔABCarΔDEF =(BCEF)^2 $\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{BC}{EF})^2$ 1625 =(2.3EF)^2 $\frac{16}{25} = (\frac{2.3}{EF})^2$ 45 = 2.3EF

$\frac{4}{5} = \frac{2.3}{EF}$

EF = 2.875 cm

(ii) arΔABCarΔDEF =(ABDE)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

964 =(ABDE)$^2 \frac{9}{64} = (\frac{AB}{DE})^2$ 38 = AB5.1 $\frac{3}{8} = \frac{AB}{5.1}$

AB = 1.9125 cm

(iii) arΔABCarΔDEF =(ACDF)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AC}{DF})^2$

arΔABCarΔDEF =(198)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{19}{8})^2$ arΔABCarΔDEF =(36164)$\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{361}{64})$

(iv) arΔABCarΔDEF =(ABDE)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

3664 =(ABDE)$^2 \frac{36}{64} = (\frac{AB}{DE})^2$ 68 = AB6.2 $\frac{6}{8} = \frac{AB}{6.2}$

AB = 4.65 cm

(v) arΔABCarΔDEF =(ABDE)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{AB}{DE})^2$

arΔABCarΔDEF =(1.21.4)$^2 \frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{1.2}{1.4})^2$ arΔABCarΔDEF =(3649)$\frac{ar\Delta ABC}{ar\Delta DEF} = (\frac{36}{49})$

2. In the fig 4.178, ΔACBΔACB is similar to ΔAP QΔAPQ. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ. Also, find the Area of ΔACBΔACB: Area of ΔAP QΔAPQ.

Answer:

Given: ΔACBΔACB is similar to ΔAP QΔAPQ

BC = 10 cm

PQ = 5 cm

BA = 6.5 cm

AP = 2.8 cm

Find:

(1) CA and AQ

(2) Area of ΔACBΔACB: Area of ΔAPQΔAPQ

(1) It is given that ΔACBΔACB – ΔAPQΔAPQ

We know that for any two similar triangles the sides are proportional. Hence
ABAQ=BCPQ=ACAP $\frac{AB}{AQ}=\frac{BC}{PQ}=\frac{AC}{AP}$

ABAQ=BCPQ $\frac{AB}{AQ}=\frac{BC}{PQ}$ 6.5AQ=105 $\frac{6.5}{AQ}=\frac{10}{5}$

AQ = 3.25 cm

Similarly,

BCPQ=CAAP $\frac{BC}{PQ}=\frac{CA}{AP}$ CA2.8=105 $\frac{CA}{2.8}=\frac{10}{5}$

CA = 5.6 cm

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

arΔACQarΔAPQ=(BCPQ)2 $\frac{ar\Delta ACQ}{ar\Delta APQ}=(\frac{BC}{PQ})^2$

= (105)^2$(\frac{10}{5})^2$

= (21)^2$(\frac{2}{1})^2$

= 41 $\frac{4}{1}$

3. The areas of two similar triangles are 81 cm² and 49 cm² respectively. Find the ration of their corresponding heights. What is the ratio of their corresponding medians?

Answer:

Given: The area of two similar triangles is 81cm^2 and 49cm^2 respectively.

To find:

(1) The ratio of their corresponding heights.

(2) The ratio of their corresponding medians.

(1) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2)=(altitude1altitude2)^2 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = (\frac{altitude\ 1}{altitude\ 2})^2$ 8149=

(altitude1altitude2)^2 $\frac{81}{49} = (\frac{altitude\ 1}{altitude\ 2})^2$

Taking square root on both sides, we get

97=altitude1altitude2 $\frac{9}{7} = \frac{altitude\ 1}{altitude\ 2}$

Altitude 1: altitude 2 = 9: 7

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their medians.

ar(triangle1)ar(triangle2)=(median1median2)^2 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = (\frac{median\ 1}{median\ 2})^2$ 8149=

(median1median2)^2 $\frac{81}{49} = (\frac{median\ 1}{median\ 2})^2$

Taking square root on both sides, we get

97=median1median2 $\frac{9}{7} = \frac{median\ 1}{median\ 2}$

Median 1: median 2 = 9: 7

4. The areas of two similar triangles are 169 cm^{2}and 121 cm^{2}respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Answer:

Given:

The area of two similar triangles is 169cm2 and 121cm2 respectively. The longest side of the larger triangle is 26cm.

To find:

Longest side of the smaller triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

ar(largertriangle)ar(smallertriangle) = (sideofthelargertrianglesideofthesmallertriangle)^2

$\frac{ar\ (larger\ triangle)}{ar\ (smaller\ triangle)} = (\frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle})^2$ 169121 = (sideofthelargertrianglesideofthesmallertriangle)^2 $\frac{169}{121} = (\frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle})^2$

Taking square root on both sides, we get

1311 = sideofthelargertrianglesideofthesmallertriangle $\frac{13}{11} = \frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle}$

1311 = 26sideofthesmallertriangle $\frac{13}{11} = \frac{26}{side\ of\ the\ smaller\ triangle}$

Side of the smaller triangle = 11×2613 $\frac{11 \times 26}{13}$ = 22 cm

Hence, the longest side of the smaller triangle is 22 cm.

5. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer:

Given:

The area of two similar triangles is $25cm^2$ and $36cm^2$ respectively. If the altitude of first triangle 2.4cm.

To find:

The altitude of the other triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2) =(altitude1altitude2)^2 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = (\frac{altitude\ 1}{altitude\ 2})^2$ 2536 =(2.4altitude2)^2

$\frac{25}{36} = (\frac{2.4}{altitude\ 2})^2$

Taking square root on both sides, we get

56 = 2.4altitude2 $\frac{5}{6} = \frac{2.4}{altitude\ 2}$

Altitude 2 = 2.88 cm

Hence, the corresponding altitude of the other is 2.88 cm.

6. ABC is a triangle in which ∠ A = 90°, AN ⊥ BC, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCΔANC and ΔABCΔABC.

Answer:

Given:

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively.

To find:

Ratio of areas of triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(triangle1)ar(triangle2) =(altitude1altitude2)^2 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = (\frac{altitude\ 1}{altitude\ 2})^2$

ar(triangle1)ar(triangle2) =(69)^2 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = (\frac{6}{9})^2$ ar(triangle1)ar(triangle2) = 3681

$\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = \frac{36}{81}$ ar(triangle1)ar(triangle2) = 49 $\frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = \frac{4}{9}$

ar (triangle 1): ar (triangle 2) = 4: 9

Hence, the ratio of the areas of two triangles is 4: 9.

7. ABC is a triangle in which ∠A°,AN⊥BC$\angle A^{\circ},\ AN \perp BC$, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCandΔABC$\Delta ANC\, and\, \Delta ABC$.

Answer:

Given:

In ΔABCΔABC, ∠A=90°$\angle A = 90^{\circ}$, AN ⊥$\perp$ BC, BC= 12 cm and AC = 5 cm.

To find:
Ratio of the triangles ΔANCandΔABC$\Delta ANC\ and\ \Delta ABC$.

In ΔANCandΔABC$\Delta ANC\ and\ \Delta ABC$,

∠ACN=∠ACB$\angle ACN = \angle ACB$ (Common)

∠A=∠ANC$\angle A = \angle ANC$ (90°90°)

Therefore, ΔANC–ΔABC$\Delta ANC - \ \Delta ABC$ (AA similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Therefore,

Ar(ΔANC)Ar(ΔABC)=(ACBC)2$\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = (\frac{AC}{BC})^2$ Ar(ΔANC)Ar(ΔABC)=(5cm12cm)2

$\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = (\frac{5cm}{12cm})^2$ Ar(ΔANC)Ar(ΔABC)=25144 $\frac{Ar(\Delta ANC)}{Ar(\Delta ABC)} = \frac{25}{144}$

8. In Fig, DE || BC

(i) If DE = 4m, BC = 6 cm and Area (ΔADE)=16cm2$(\Delta ADE) = 16\, cm^2$, find the area of ΔABCΔABC.

(ii) If DE = 4cm, BC = 8 cm and Area $(\Delta ADE) = 25\,cm^2$, find the area of ΔABC.

(iii) If DE : BC = 3 : 5. Calculate the ratio of the areas of ΔADE and the trapezium BCED.

Answer:

In the given figure, we have DE $||$ BC.

In $\Delta ADE \text{ and } \Delta ABC$

$\angle ADE = \angle B$ (Corresponding angles)

$\angle DAE = \angle BAC$ (Common)

So, $\Delta ADE \text{ and } \Delta ABC$ (AA Similarity)

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

$$\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2} \quad \frac{16}{Ar(\Delta ABC)} = \frac{4^2}{6^2}$$

$$Ar(\Delta ABC) = \frac{6^2 \times 16}{4^2}$$

$Ar(\Delta ABC)$ = 36 cm^2

(ii) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

$$\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2} \quad \frac{25}{Ar(\Delta ABC)} = \frac{4^2}{8^2}$$

$$Ar(\Delta ABC) = \frac{8^2 \times 25}{4^2}$$

$Ar(\Delta ABC)$ = 100 cm^2

(iii) We know that

Ar(ΔADE)Ar(ΔABC) = DE²BC² $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{DE^2}{BC^2}$ Ar(ΔADE)Ar(ΔABC) = 3²5² $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{3^2}{5^2}$

Ar(ΔADE)Ar(ΔABC) = 925 $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{9}{25}$

Let the area of ΔADEΔADE = 9x sq units

area of ΔABCΔABC = 25x sq units

Now, Ar(ΔADE)Ar(trapBCED) = 9x16x $\frac{Ar(\Delta ADE)}{Ar(trapBCED)} = \frac{9x}{16x}$

Ar(ΔADE)Ar(trapBCED) = 916 $\frac{Ar(\Delta ADE)}{Ar(trapBCED)} = \frac{9}{16}$

9. In ΔABCΔABC , D and E are the mid- points of AB and AC respectively. Find the ratio of the areas ΔADEΔADE and ΔABCΔABC .

Answer:

Given:

In ΔABCΔABC, D and E are the midpoints of AB and AC respectively.

To find:

Ratio of the areas of ΔADEandΔABC$\Delta ADE \ and \ \Delta ABC$

It is given that D and E are the midpoints of AB and AC respectively.

Therefore, DE II BC (Converse of mid-point theorem)

Also, DE = 12$\frac{1}{2}$BC

In ΔADEandΔABC$\Delta ADE \ and \ \Delta ABC$

∠ADE=∠B$\angle ADE = \angle B$(Corresponding angles)

∠DAE=∠BAC$\angle DAE = \angle BAC$ (common)

So, ΔADE–ΔABC$\Delta ADE - \ \Delta ABC$ (AA Similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

Ar(ΔADE)Ar(ΔABC)=AD^2AB^2 $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{AD^2}{AB^2}$ Ar(ΔADE)Ar(ΔABC)=1^22^2 $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1^2}{2^2}$

Ar(ΔADE)Ar(ΔABC)=14 $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1}{4}$

10. The areas of two similar triangles are 100 cm2 and 49 cm2 respectively. If the altitude of the bigger triangles is 5 cm, find the corresponding altitude of the other.

Answer:

Given: the area of the two similar triangles is $100cm^2$ $100cm^2$ and $49cm^2$ $49cm^2$ respectively. If the altitude of the bigger triangle is 5cm

To find: their corresponding altitude of the other triangle

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding altitudes.

ar(biggertriangle1)Ar(triangle2) $\frac{ar(bigger\ triangle1)}{Ar(triangle2)}$=(altitudeofthebiggertriangle1altitude2$)^2$ $(\frac{altitude\ of\ the\ bigger\ triangle1}{altitude2})^2$

(10049)$(\frac{100}{49})$=(5altitude2$)^2$$(\frac{5}{altitude2})^2$

Taking squares on both the sides

(107 $(\frac{10}{7}$=(5altitude2)$(\frac{5}{altitude2})$

Altitude 2=3.5cm

11. The areas of two similar triangles are 121 cm2 and 64 cm2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Answer:

Given : the area of the two triangles is $121cm^2$ $121cm^2$ and $64cm^2$ $64cm^2$respectively. If the merdian of the first triangle is 12.1cm

To find the corresponding medians of the other triangle

We know that ratio of the areas of the two similar triangles are equal to the ratio of the squares of their merdians

(ar(triangle1)ar(triangle2) $\left(\frac{ar(triangle1)}{ar(triangle2)}\right.$ =(median1median2 $)^2\left(\frac{median1}{median2}\right)^2$

(12164 $\left(\frac{121}{64}\right.$ =(12.1median2 $)^2\left(\frac{12.1}{median2}\right)^2$

Taking the squareroot on the both sides

(118 $\left(\frac{11}{8}\right.$ =(12.1median2)$\left(\frac{12.1}{median2}\right)$

Median2=8.8cm

12. If ΔABC~ΔDEF $\Delta ABC \sim \Delta DEF$ such that AB = 5cm, area (ΔABCΔABC) = 20 cm2 and area (ΔDEF ΔDEF) = 45 cm2, determine DE.

Answer:

Given : the area of the two similar ΔABCΔABC=20cm²$20cm^2$ and ΔDEF ΔDEF =45cm²$45cm^2$ and AB=5cm

To measure of DE

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

ArΔABCArΔDEF =(ABDE $)^2 \frac{Ar\Delta ABC}{Ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

2045 $\frac{20}{45}$ = 5DE $^2 \frac{5}{DE}{}^2$

2045 $\frac{20}{45}$ = 5DE $\frac{5}{DE}$

DE²DE^2=25×4520 $\frac{25\times 45}{20}$

DE^2DE^2=2254 $\frac{225}{4}$

DE=7.5cm

13. In ΔABCΔABC, PQ is a line segment intersecting AB at P and AC at Q such that PQ || BC and PQ divides ΔABCΔABC into two parts equal in area. Find BPAB $\frac{BP}{AB}$.

Answer:

Given: in ΔABCΔABC, PQ is a line segment intersecting AB at P and AC at such that PQ||BC$PQ \parallel BC$ and PQ divides ΔABCΔABC in two parts equal in area

To find : BPAB $\frac{BP}{AB}$

We have PQ||BC$PQ \parallel BC$ and

Ar(ΔAPQΔAPQ) = Ar(quad BPQC)

Ar(ΔAPQΔAPQ) +Ar(ΔAPQΔAPQ) =Ar(quad BPQC) +Ar(ΔAPQΔAPQ)

2(Ar(ΔAPQΔAPQ) =Ar(ΔABCΔABC)

Now PQ||BC$PQ \parallel BC$ and BA is a transversal

In ΔABCΔABC and ΔAPQΔAPQ)

∠APQ=∠B$\angle APQ = \angle B$ (corresponding angles)

∠PAQ=∠BAC$\angle PAQ = \angle BAC$ (common)

In ΔABC~ΔAPQ$\Delta ABC \sim \Delta APQ$) (AA similarity)

We know that the ratio of the areas of the two similar triangles is used and is equal to the ratio of their squares of the corresponding sides.

Hence

ArΔAPQArΔABC=(APAB)^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{AB})^2$ ArΔAPQ2ArΔABC=(APAB)^2 $\frac{Ar\Delta APQ}{2Ar\Delta ABC} = (\frac{AP}{AB})^2$ 12=(APAB)^2 $\frac{1}{2} = \frac{(}{AP}AB)^2$ √12=(APAB)$\sqrt{\frac{1}{2}} = \frac{(}{AP}AB)$

AB=$\sqrt{2AP}$ $\sqrt{2AP}$

AB=$\sqrt{2(AB-BP}$ $\sqrt{2(AB-BP}$

$\sqrt{2BP}$ $\sqrt{2BP}$=$\sqrt{2AB-AB}$ $\sqrt{2AB-AB}$

BPAB $\frac{BP}{AB}$=$\sqrt{2}$-1$\sqrt{2}$ $\frac{\sqrt{2}-1}{\sqrt{2}}$

14. The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer:

Given: the areas of the two similar triangles ABC and PQR are in the ratio 9:16. BC=4.5cm

To find: Length of QR

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

ArΔABCArΔPQR=(BCQR)^2 $\frac{Ar\Delta ABC}{Ar\Delta PQR} = (\frac{BC}{QR})^2$

916 $\frac{9}{16}$ =(4.5QR)^2$(\frac{4.5}{QR})^2$

34 $\frac{3}{4}$=4.5QR $\frac{4.5}{QR}$

QR= 183 $\frac{18}{3}$ = 6cm

15. ABC is a triangle and PQ is a straight line meeting AB and P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of ΔAPQΔAPQ is one – sixteenth of the area of ΔABCΔABC.

Answer:

Given : in ΔABCΔABC, PQ is a line segment intersecting AB at P and AC at Q. AP = 1cm , PB = 3cm, AQ= 1.5 cm and QC= 4.5cm

To find Ar(ΔAPQΔAPQ)= 116×ΔABC$\frac{1}{16} \times \Delta ABC$)

In ΔABCΔABC

APPB $\frac{AP}{PB}$=AQQC $\frac{AQ}{QC}$

13 $\frac{1}{3}$= 13 $\frac{1}{3}$

According to converse of basic proportional theorem if a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Hence,

PQ||BC$PQ \parallel BC$

Hence in ΔABCΔABC and ΔAPQΔAPQ

∠APQ$\angle APQ$ =∠B$\angle B$ (corresponding angles)

∠PAQ$\angle PAQ$=∠BAC$\angle BAC$ (common)

ΔABC~ΔAPQ$\Delta ABC \sim \Delta APQ$ ArΔAPQArΔABC=(AP AB)^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = \frac{(}{AP} AB)^2$

ArΔAPQArΔABC=(AP(AB+BP))^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{(AB+BP)})^2$

ArΔAPQArΔABC=(14)^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{1}{4})^2$ (given)

ArΔAPQArΔABC=(116) $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{1}{16})$

16. If D is a point on the side AB of ΔABCΔABC such that AD : DB = 3 : 2 and E is a point on BC such that DE || AC. Find the ratio of areas of ΔABCΔABC and ΔBDE ΔBDE.

Answer:

Given In ΔABCΔABC, D is appoint on the side AB such that AD:DB=3:2. E is a point on side BC such that DE||AC$DE \parallel AC$

To find

ΔABCΔBDE $\frac{\Delta ABC}{\Delta BDE}$

In ΔABCΔABC,ΔBDEΔBDE,

∠BDE$\angle BDE$ =∠A$\angle A$ (corresponding angles)

∠DBE$\angle DBE$ =∠ABC$\angle ABC$

ΔABC~ΔBDE$\Delta ABC \sim \Delta BDE$

We know that the ratio of the two similar triangles is equal to the ratio of the squares of their corresponding sides

Let AD=2x and BD =3x

Hence

ArΔABCArΔBDE =(ABBD)2 $\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{AB}{BD})^2$ (AB+DABD)2 $(\frac{AB+DA}{BD})^2$ (3x+2x2x)2 $(\frac{3x+2x}{2x})^2$

ArΔABCArΔBDE =(254) $\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{25}{4})$

17. If ΔABCΔABC and ΔBDEΔBDE are equilateral triangles, where D is the midpoint of BC, find the ratio of areas of ΔABCΔABC and ΔBDEΔBDE .

Answer:

Given In ΔABCΔABC, ΔBDEΔBDE are equilateral triangles. D is the point of BC.

To find ArΔABCArΔBDE $\frac{Ar\Delta ABC}{Ar\Delta BDE}$

In ΔABCΔABC, ΔBDEΔBDE

ΔABC~ΔBDE$\Delta ABC \sim \Delta BDE$ (AAA criteria of similarity all angles of the equilateral triangles are equal)

Since D is the mid point of BC, BD : DC=1

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding sides.

Let DC=x, and BD= x

Hence

ArΔABCArΔBDE=(BCBD)2 $\frac{Ar\Delta ABC}{Ar\Delta BDE} = (\frac{BC}{BD})^2$

=(BD+DADC)2 $(\frac{BD+DA}{DC})^2$

=(1x+1x1x)2 $(\frac{1x+1x}{1x})^2$

ArΔABCArΔBDE=4:1 $\frac{Ar\Delta ABC}{Ar\Delta BDE} = 4:1$

18.Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Answer:

Given:

Two isosceles triangles have equal vertical angles and their areas are in the ratio of 36: 25.

To find:

Ratio of their corresponding heights

Suppose ΔABC ΔABC and ΔPQR ΔPQR are two isosceles triangles with ∠A=∠P $\angle A = \angle P$.

Therefore,

ABAC=PQPR $\frac{AB}{AC} = \frac{PQ}{PR}$

In ΔABC ΔABC and ΔPQR ΔPQR,

∠A=∠P $\angle A = \angle P$ ABAC=PQPR $\frac{AB}{AC} = \frac{PQ}{PR}$

∴ ΔABC ΔABC ~ ΔPQR ΔPQR (SAS similarity)

Let AD and PS be the altitudes of ΔABC and ΔPQR, respectively.

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\frac{ar\ \Delta ABC}{ar\ \Delta PQR} = (\frac{AD}{PS})^2$ $\frac{36}{25} = (\frac{AD}{PS})^2$ $\frac{AD}{PS} = \frac{6}{5}$

Hence, the ratio of their corresponding heights is 6: 5.

19. In the given figure. ΔABC and ΔDBC are on the same base BC. If AD and BC intersect at O, Prove that

$$\frac{Area\ of\ (\Delta ABC)}{Area\ of\ (\Delta DBC)} = \frac{AO}{DO}$$

Answer:

Given ΔABC and ΔDBC are on the same BC. AD and BC intersect at O.

Prove that : $\frac{Ar\Delta ABC}{Ar\Delta DBC} = \frac{AO}{DO}$

$AL \perp BC and DM \perp BC$

Now, in ΔALO and ΔALOwe have

$\angle ALO$ = $\angle DMO$ =$90°$

$\angle AOL$ =$\angle DOM$ (vertically opposite angles)

Therefore $\Delta ALO \sim \Delta DMO$

∴ $\frac{AL}{DM} = \frac{AO}{DO}$

$\frac{Ar\Delta ABC}{Ar\Delta BCD} = \frac{\frac{1}{2}\times BC\times AL}{\frac{1}{2}\times BC\times DM}$

$= \frac{AL}{DM}$

$= \frac{AO}{DO}$

20. ABCD is a trapezium in which AB || CD. The Diagonal AC and BC intersect at O. Prove that :

(i) ΔAOB ΔCOD$\Delta AOB\ \Delta COD$

(ii) If OA = 6 cm, OC = 8 cm,

Find:

(a) Areaof(ΔAOB)Areaof(ΔCOD) $\frac{Area\ of\ (\Delta AOB)}{Area\ of\ (\Delta COD)}$

(b) Areaof(ΔAOD)Areaof(ΔCOD) $\frac{Area\ of\ (\Delta AOD)}{Area\ of\ (\Delta COD)}$

Answer: Given ABCD is the trapezium which AB||CD$AB \parallel CD$

The diagonals AC and BD intersect at o.

To prove:

(i) ΔAOB~ΔCOD$\Delta AOB \sim \Delta COD$

(ii) If OA = 6 cm, OC = 8 cm

To find :

(a) ArΔAOBArΔCOD $\frac{Ar\Delta AOB}{Ar\Delta COD}$

(b) ArΔAODArΔCOD $\frac{Ar\Delta AOD}{Ar\Delta COD}$

Construction : Draw a line MN passing through O and parallel to AB and CD

Now in ΔAOBandΔCOD$\Delta AOB and \Delta COD$

(i) Now in ∠OAB$\angle OAB$ =∠OCD$\angle OCD$ (Alternate angles)

(ii) ∠OBA$\angle OBA$ =∠ODC$\angle ODC$ (Alternate angles)

∠AOB$\angle AOB$ =∠COD$\angle COD$ (vertically opposite angle)

ΔAOB~ΔCOD$\Delta AOB \sim \Delta COD$ (A.A.Acrieteria)

a) We know that the ratio of areas of two triangles is equal to the ratio of squares of their corresponding sides.

ArΔAOBArΔCOD =(AOCO)$^2 \frac{Ar\Delta AOB}{Ar\Delta COD} = (\frac{AO}{CO})^2$

=(68)$^2 (\frac{6}{8})^2$

ArΔAOBArΔCOD $\frac{Ar\Delta AOB}{Ar\Delta COD}$ =(68)$^2 (\frac{6}{8})^2$

b) We know that the ratio of two similar triangles is equal to the artio of their corresponding sides.

ArΔAOBArΔCOD =(AOCO)$^2 \frac{Ar\Delta AOB}{Ar\Delta COD} = (\frac{AO}{CO})^2$

=(6cm8cm)$^2 (\frac{6cm}{8cm})^2$ =(68)$^2 (\frac{6}{8})^2$

21. In ΔABCΔABC, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that PQ || BC. Find the ratio of the areas of ΔAPQΔAPQ and trapezium BPQC.

Answer: Given : In ΔABCΔABC , P divides the side AB such that AP: PB =1:2, Q is a point on AC on such that PQ ||$||$ BC

To find : The ratio of the areas of ΔAPQΔAPQ and the trapezium BPQC.

In ΔAPQΔAPQ and ΔABCΔABC

∠APQ=∠B$\angle APQ = \angle B$ (corresponding angles)

∠PAQ=∠BAC$\angle PAQ = \angle BAC$ (common)

So, ΔAPQ~ΔABC$\Delta APQ \sim \Delta ABC$ (AA Similarity)

We know that the ratio of areas of the twosimilar triangles is equal to the ratio of the squares of their corresponding sides.

ArΔAPQArΔABC=(APAB)^2 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = (\frac{AP}{AB})^2$ ArΔAPQArΔABC=1x^2(1x+2x)^2

$\frac{Ar\Delta APQ}{Ar\Delta ABC} = \frac{1x^2}{(1x+2x)^2}$ ArΔAPQArΔABC=19 $\frac{Ar\Delta APQ}{Ar\Delta ABC} = \frac{1}{9}$

Let Area of ΔAPQΔAPQ =1 sq. units and Area of ΔABCΔABC=9x sq.units

Ar[trapBCED]=Ar(ΔABCΔABC) – Ar(ΔAPQΔAPQ)

=9x-1x

=8x sq units

Now,

ArΔAPQAr(trapBCED) $\frac{Ar\Delta APQ}{Ar(trapBCED)}$ =xsqunits8xsqunits $\frac{xsqunits}{8xsqunits}$ =18 $\frac{1}{8}$

Exercise 4.7: Triangles

1. If the sides of a triangle are 3 cm, 4 cm, and 6 cm long, determine whether the triangle is a right-angled triangle.
   Sol:
   We have,
   Sides of triangle
   AB = 3 cm
   BC = 4 cm
   AC = 6 cm
   $\therefore AB^2 = 3^2 = 9$
   $BC^2 = 4^2 = 16$
   $AC^2 = 6^2 = 36$
   Since, $AB^2 + BC^2 \neq AC^2$
   Then, by converse of Pythagoras theorem, triangle is not a right triangle.

2. The sides of certain triangles are given below. Determine which of them right triangles are.
   (i) a = 7 cm, b = 24 cm and c = 25 cm
   (ii) a = 9 cm, b = 16 cm and c = 18 cm
   (iii) a = 1.6 cm, b = 3.8 cm and c = 4 cm
   (iv) a = 8 cm, b = 10 cm and c = 6 cm
   Sol:
   We have,

a = 7 cm, b = 24 cm and c = 25 cm

$\therefore a^2 = 49, b^2 = 576\ and\ c^2 = 625$

Since, $a^2 + b^2 = 49 + 576$

$= 625$

$= c^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

We have,

a = 9 cm, b = 16 cm and c = 18 cm

$\therefore a^2 = 81, b^2 = 256\ and\ c^2 = 324$

Since, $a^2 + b^2 = 81 + 256 = 337$

$\neq c^2$

Then, by converse of Pythagoras theorem, given triangle is not a right triangle.

We have,

a = 1.6 cm, b = 3.8 cm and C = 4 cm

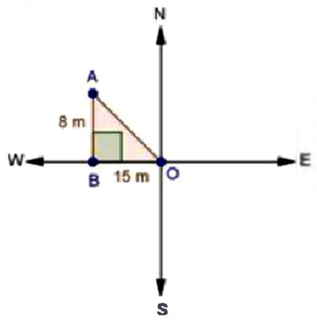
$\therefore a^2 = 64, b^2 = 100\ and\ c^2 = 36$

Since, $a^2 + c^2 = 64 + 36 = 100 = b^2$

Then, by converse of Pythagoras theorem, given triangle is a right triangle.

3. A man goes 15 metres due west and then 8 metres due north. How far is he from the starting point?

Sol:



Let the starting point of the man be O and final point be A.

$\therefore$ In ΔABO, by Pythagoras theorem $AO^2 = AB^2 + BO^2$

$\Rightarrow AO^2 = 8^2 + 15^2$

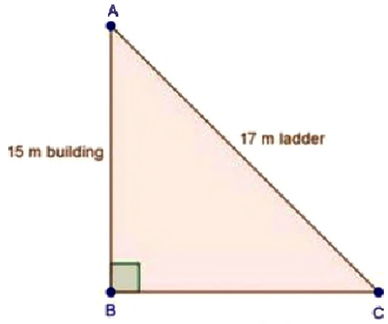
$\Rightarrow AO^2 = 64 + 225 = 289$

$\Rightarrow$ AO = $\sqrt{289} = 17m$

$\therefore$ He is 17m far from the starting point.

4. A ladder 17 m long reaches a window of a building 15 m above the ground. Find the distance of the foot of the ladder from the building.

Sol:



In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow 15^2 + BC^2 = 17^2$

$\Rightarrow 225 + BC^2 = 17^2$

$\Rightarrow BC^3 = 289 - 225$

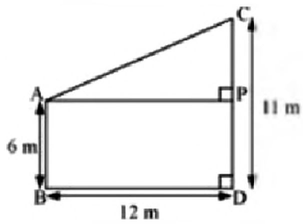
$\Rightarrow BC^2 = 64$

$\Rightarrow BC = 8\ m$

$\therefore Distance\ of\ the\ foot\ of\ the\ ladder\ from\ building = 8\ m$

5. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



Let CD and AB be the poles of height 11 and 6 m.

Therefore CP = 11 – 6 = 5 m

From the figure we may observe that AP = 12m

In triangle APC, by applying Pythagoras theorem

$AP^2 + PC^2 = AC^2$

$12^2 + 5^2 = AC^2$

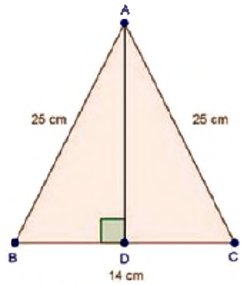
$AC^2 = 144 + 25 = 169$

AC = 13

Therefore distance between their tops = 13m.

6. In an isosceles triangle ABC, AB = AC = 25 cm, BC = 14 cm. Calculate the altitude from A on BC.

Sol:



We have

AB = AC = 25 cm and BC = 14 cm

In ΔABD and ΔACD

∠ADB = ∠ADC [Each 90°]

AB = AC [Each 25 cm]

AD = AD [Common]

Then, ΔABD ≅ ΔACD [By RHS condition]

∴ BD = CD = 7 cm [By c.p.c.t]

In ΔADB, by Pythagoras theorem

$AD^2 + BD^2 = AB^2$

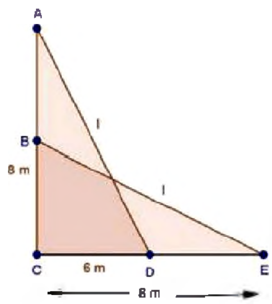
$\Rightarrow AD^2 + 7^2 = 25^2$

$\Rightarrow AD^2 = 625 - 49 = 576$

$\Rightarrow AD = \sqrt{576} = 24\ cm$

7. The foot of a ladder is 6 m away from a wall and its top reaches a window 8 m above the ground. If the ladder is shifted in such a way that its foot is 8 m away from the wall, to what height does its tip reach?

Sol:



Let, length of ladder be AD = BE = l m

In ΔACD, by Pythagoras theorem

$AD^2 = AC^2 + CD^2$

$\Rightarrow l^2 = 8^2 + 6^2$ ....(i)

$In\ \Delta BCE, by\ pythagoras\ theorem$

$BE^2 = BC^2 + CE^2$

$\Rightarrow l^2 = BC^2 + 8^2$ ....(ii)

$Compare\ (i) and\ (ii)$

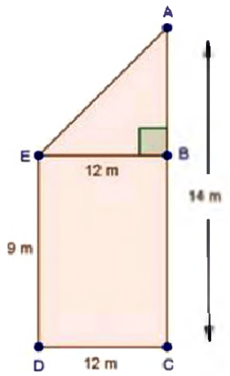
$BC^2 + 8^2 = 8^2 + 6^2$

$\Rightarrow BC^2 = 6^2$

$\Rightarrow BC = 6m$

8. Two poles of height 9 m and 14 m stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

Sol:



We have,
AC = 14 m, DC = 12m and ED = BC = 9m
Construction: Draw EB ⊥ AC
∴ AB = AC – BC = 14 – 9 = 5m
And, EB = DC = 12 m
In ΔABE, by Pythagoras theorem,
$AE^2 = AB^2 + BE^2$
$\Rightarrow AE^2 = 5^2 + 12^2$
$\Rightarrow AE^2 = 25 + 144 = 169$
$\Rightarrow AE = \sqrt{169} = 13\ m$
$\therefore Distance\ between\ their\ tops = 13\ m$

9. Using Pythagoras theorem determine the length of AD in terms of b and c shown in Fig. 4.219

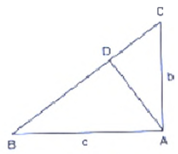


Fig. 4.219

Sol:
We have,
In ΔBAC, by Pythagoras theorem
$BC^2 = AB^2 + AC^2$
$\Rightarrow BC^2 = c^2 + b^2$
$\Rightarrow BC = \sqrt{c^2 + b^2}$ ...(i)
$In\ \Delta ABD\ and\ \Delta CBA$
∠B = ∠B [Common]

$\angle ADB = \angle BAC$ [Each 90°]

Then, $\Delta ABD \sim \Delta CBA$ [By AA similarity]

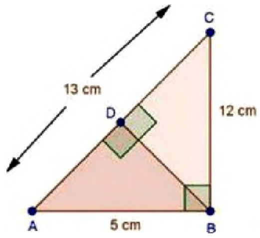
$\therefore \frac{AB}{CB} = \frac{AD}{CA}$ [Corresponding parts of similar Δ are proportional]

$\Rightarrow \frac{c}{\sqrt{c^2+b^2}} = \frac{AD}{b}$

$\Rightarrow AD = \frac{bc}{\sqrt{c^2+b^2}}$

10. A triangle has sides 5 cm. 12 cm and 13 cm. Find the length to one decimal place. of the perpendicular from the opposite vertex to the side whose length is 13 cm.

Sol:



Let. AB = 5cm. BC = 12 cm and AC = 13 cm. Then. $AC^2 = AB^2 + BC^2$. This proves that ΔABC is a right triangle. right angles at B. Let BD be the length of perpendicular from B on AC.

Now. Area $\Delta ABC = \frac{1}{2}(BC \times BA)$

$= \frac{1}{2}(12 \times 5)$

$= 30\ cm^2$

Also, Area of $\Delta ABC = \frac{1}{2} AC \times BD = \frac{1}{2}(13 \times BD)$

$\Rightarrow (13 \times BD) = 30 \times 2$

$\Rightarrow BD = \frac{60}{13}$ cm